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ON THE ELECTROSTATIC EFFECT OF A CHANGING MAGNETIC FIELD

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On the Electrostatic Effect of a Changing Magnetic Field.

By J. M. KUEHNE.

IN the development of Maxwell's Electromagnetic Theory there is implied, if not directly expressed, the existence of four distinct mutual effects between electrostatic and magnetic phenomena, some of which have to this day eluded experimental verification. While it may not be said that the value of the theory depends to any very serious extent upon the demonstrability of these assumed effects, yet a great theoretical interest attaches to the question whether the foundations of this theory, which has shown such phenomenal fruitfulness in directing research, can be experimentally established.

The four assumed effects are:—

1. A moving electric charge produces a magnetic field.
2. A moving magnetic pole produces an electric field.
3. A changing electric field produces a magnetic field.
4. A changing magnetic field produces an electric field.

In all four cases the difficulty in the way of experimental verification lies in the extreme smallness of the force which is finally to be observed and measured, together with the comparatively very large disturbing forces which must come into play when electric charges and magnetic fields of sufficient magnitude are used to make the effect sought for at all observable. This is shown at once by the fact that in the denominator of the formula in each case there enters the well-known v of Maxwell.

Of these four effects the first was demonstrated by Rowland in 1876, the third by Eichenwald in 1903. The converse effects (2) and (4) have, so far, resisted all attempts at experimental proof. The electrostatic effect of a changing magnetic field may be considered as derivable from Rowland's experiment, if the assumption is made that motion of the electrostatic lines of force relative to the æther is not an essential condition of a mutual magnetic and electrostatic effect; but the justification of such an assumption is by no means clear. It was the hope of giving a direct and unequivocal proof of the existence or non-existence of the last-named effect that prompted the present research.

Several previous attempts at the solution of this problem have been published, the results, however, being either non-committal or apparently directly contradictory. The first of these attempts was made by Sir Oliver Lodge* in 1889.

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* Phil. Mag. [5] xxvii. p. 469.

The experiment consisted of suspending two charged mica or gelatine vanes fastened to the ends of a light rod, after the manner of a very delicate torsion balance, inside and in the plane of a large magnetized ring placed vertically, and then reversing the magnetization of the ring synchronously with the natural period of oscillation of the vanes. The experiment was believed to have yielded positive results; but the fact that it was purely qualitative and that no data are available for deciding whether the observed deflexions were even remotely of the order of magnitude demanded by theory makes it at least very doubtful if one or more of the very numerous disturbing forces was not entirely responsible for the effects observed.

Crémieu* in 1900 published an experiment in which an attempt at a quantitative determination was made. His apparatus consisted of a charged disk of aluminium suspended horizontally by a light glass frame between the flat ends of two spools of wire, thus constituting a parallel plate condenser. A straight soft iron core passed through the two spools and through a large hole in the suspended disk. The frame supporting the disk was pivoted in jewelled bearings set into the ends of the bar magnet, and was supported partly by a very thin and long silver wire, partly by a float resting in a vessel of water underneath. By reversing the direction of magnetization of the magnet alternately with reversals of the charges on the condenser, a steady deflecting moment should be produced, whose value is

$$\mathrm{L}=\frac{d\mathrm{N}}{dt}\cdot\frac{\mathrm{Q}}{2\pi},$$

where Q is the charge on the disk, expressed in electromagnetic units, and N the magnetic flux through the magnet. With n reversals per second this becomes

$$\mathrm{L}=\frac{n\mathrm{NQ}}{\pi}.$$

Crémieu omits the factor 2π in the denominator, and calculates a deflexion, as observed by a telescope and scale at 110 cm. distance, of 100 to 140 mm., whereas he is unable to observe any deflexion whatever. From this he draws the conclusion that no such electromagnetic effect exists. Accepting the value (6 to 8×10^{-4} C.G.S. unit) of the deflecting moment calculated by Crémieu, it is by no means certain that even the slight friction of the jewelled bearings would

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* *Comptes Rendus*, cxxxi. p. 578.

not be sufficient to prevent all motion. More important than this friction is the directive moment due to the mutual electrostatic attraction of the condenser plates. It is mechanically impossible to construct plates so absolutely symmetrical that the directive force due to a non-homogeneous electrostatic field is not very many times as great as that of a delicate suspension fibre such as Crémieu used, and on the basis of whose moment of torsion alone the expected deflexion is calculated. The apparently negative result would therefore be unconvincing, even if, according to theory, a deflexion should take place. But this is not the case. A simple calculation* will show that the magnetic effect on the charging current, which passes down the suspended frame, through the magnetic field while at its maximum value, each time the charge on the plates is reversed, will produce a

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* Let the charging current dQ/dt pass along an elementary arc dl whose radius is r measured from the "pole" of the magnet as centre, and the plane of the arc passing through the axis of the magnet. The force perpendicular to the plane of the arc will be:

$$dF = \frac{dQ}{dt} . dl . H$$
$$= \frac{dQ}{dt} . dl . \frac{m}{r^2},$$

where H is the intensity of the magnetic field, and m is the strength of the pole.

The elementary moment around the axis of the magnet is then:

$$dL = \frac{dQ}{dt} \cdot dl . \frac{m}{r} . \sin\theta$$
$$= \frac{dQ}{dt} . m \sin\theta \, d\theta,$$

where θ is the angle between r and the axis of the magnet. Now since the average value of dQ/dt is $2nQ$, where n is the number of reversals of the charge per second, and since $m = N/4\pi$, where N is the total magnetic flux from pole to pole, we have:

$$dL = \frac{nQN}{2\pi} \sin\theta \, d\theta,$$

which is seen to be independent of r.

The total moment exerted on a conductor carrying a current of average value $2nQ$ from a point on the axis beyond the pole to a point on the middle of the magnet will be, regardless of the path,

$$L = \frac{nQN}{2\pi} \int_0^\pi \sin\theta \, d\theta = \frac{-nQN}{\pi},$$

which is the same in magnitude as the moment exerted on the charge Q by n reversals per second of the magnetic flux N. That the directions of the two moments are opposite is easily seen by applying the familiar "motor" and "dynamo" rules.

moment in the opposite direction whose value is the same as that due to the effect sought for.

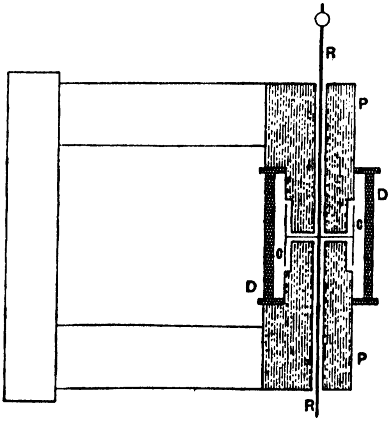
The only other published attempt is that of Whitehead* in 1905, following a suggestion by Kolaček. The method of Whitehead's experiment is so crude as to make it entirely incapable of detecting an effect as small as the one sought for. It consists of suspending a block of dielectric (rock-salt) by means of long silk fibres between two oppositely charged plates in the magnetic field of a solenoid, and looking for a ballistic throw when the direction of the magnetic field is suddenly reversed. The motion, as calculated, should be of the order of 10^{-6} or 10^{-7} cm., and is of course too small to be detected by even the most delicate optical means.

Present Research. First Method.

In my own attempt to solve the problem I have employed two distinct methods. The first of these was considerably simpler and more direct in principle, and although it just fell short of yielding the desired solution, yet on account of its bringing the problem more clearly to mind, and also better exhibiting the experimental difficulties in the way of an ultimate and unquestionable solution, I have taken the liberty to include a brief description.

The apparatus was constructed as follows:—A charged body in the form of a light hollow cylinder (C, fig. 1) of mica or paper, 6 cm. in diameter, and 6–9 cm. long, open at both ends, was fastened to a long slender rod (R) as axis,

Fig. 1.



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* *Phys. Zeit.* vi. p. 474 (1905).

being held coaxial with the rod by means of a single disk, also of mica or paper. The rod was suspended vertically by stretched phosphor-bronze ribbon attached above and below to torsion heads. The rod passed through holes drilled through the pole-pieces (PP) of a large electromagnet, which were so shaped and placed as to project into the suspended cylinder at both ends, leaving only a 6 mm. air-gap for the disk. In this way a very large magnetic field could be made to pass through the cylinder parallel to its axis, while the cylinder itself was free to rotate about its own axis. The cylinder was covered with gold-leaf to render it conducting, and was connected to earth together with the rod, suspension, and magnet core. It was charged inductively by means of a somewhat longer cylinder of hard rubber, 7·6 cm. internal diameter, set coaxial with it. The hard rubber cylinder was coated on its inner surface with tinfoil, and charged by being connected with one terminal of a high potential battery, the other terminal being earthed. If now the movable cylinder is charged + and the magnet is suddenly excited so as to make the upper pole a N pole, the cylinder would be expected to suffer a rotary impulse in the direction east, north, west, south. The magnitude of this impulse can be calculated as follows :—

Let N = the magnetic flux through the cylinder. Then $\frac{dN}{dt}$ = the E.M.F. induced in one turn of a conductor surrounding the cylinder, and the intensity of the electric field tangent to the cylinder is

$$F=\frac{1}{2\pi r}\cdot\frac{dN}{dt}.$$

Hence the instantaneous value of the moment exerted on the cylinder carrying a charge of Q electromagnetic units is

$$L=rQF=\frac{Q}{2\pi}\cdot\frac{dN}{dt},$$

or the total rotary impulse due to a change in the magnetic field of N lines is

$$I\omega=\frac{NQ}{2\pi},$$

where I is the moment of inertia of the cylinder and ω the initial angular velocity imparted to it.

If T is the period of one complete oscillation, the angular throw, assuming harmonic motion, is

$$\theta=\frac{T\omega}{2\pi}=\frac{TNQ}{4\pi^2 I}$$

$$\theta=\frac{TNCE'}{4\pi^2 I\times 9\times 10^{12}},$$

where C is the capacity in electrostatic units and E′ the charging E.M.F. in volts.

With a scale distance D the deflexion, as read by telescope and scale, is

$$d=\frac{DTNCE'}{2\pi^2 I\times 9\times 10^{12}}.$$

The first important difficulty encountered in the experiment was due to inability to find any material for making the movable cylinder and axis which was sufficiently non-magnetic not to be violently affected by the enormous magnetic fields used. Glass, wood, celluloid, aluminium, brass, and copper rods were tried, and numerous specimens of mica, and later paper, were used in making the cylinders. A thin rod of copper wire, carefully freed from surface contamination by leaving it in concentrated HCl for a day or two, and cylinders of pure unsensitized photographic paper, or good clear writing-paper, proved more nearly satisfactory than anything that was tried. With such a cylinder the purely magnetic deflexions could generally be brought within bounds, by carefully adjusting the position of the cylinder with reference to the magnet, by means of the adjustable torsion-heads.

A new difficulty arose when the cylinder was charged; for the two cylinders are in a position of unstable equilibrium with respect to the electric attraction between them, and only by having the suspension fibres stretched quite tight, and then carefully adjusting the position of the outer cylinder, was it sometimes possible to keep the suspended system approximately balanced, and moderately free from magnetic influence, while it was electrically charged.

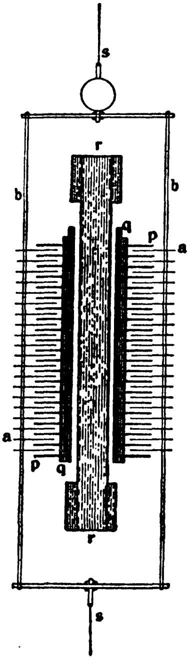
The experimental difficulties naturally increased with increasing strength of magnetic field, magnitude of charge, and sensitiveness of suspension. The largest total magnetic flux attained was $N=300{,}000$ lines, the highest charging potential was $E=3000$ volts, while the capacity, as calculated from the dimensions of the cylinders, was $C=10{\cdot}2$ electrostatic units. The other quantities entering into the formula were:—$T=10{\cdot}3$ sec., $I=18{\cdot}2$ g.cm.^{2}, and $D=318$ cm.

From this the deflexion due to the electromagnetic effect should be about ·1 mm., or the difference between deflexions in opposite directions ·2 mm. No such small difference could with certainty be determined, as the variations of individual readings were generally much larger*. Although slight improvements could probably be made and the magnitude of the deflexions somewhat increased, still the present method seemed incapable of yielding a decisive result, and a new and considerably different plan was adopted.

Second Method.

The form of apparatus which finally proved successful is constructed as follows:—One of the conductors of a parallel plate condenser is built up of 20 rings of thin varnished cardboard, inside diameter 10 cm., outside diameter 18 cm. (*a*, *a*, fig. 2). A strip 2 cm. in width along the inner edge of each ring is covered with silver leaf. The rings are fastened parallel to one another and about 9 mm. apart by a light wooden frame (*b*, *b*), and the frame is suspended by fine wire and phosphor-bronze ribbon (*s*, *s*) fastened above and below to torsion-heads and stretched tight, the rings themselves lying horizontal. The other portion of the condenser is stationary and consists of 21 rings of thin hard rubber (*p*, *p*), inside diameter 8 cm., outside diameter 14 cm., placed so as to alternate with the suspended rings. A 2 cm. strip along the outer edge of the hard rubber rings is covered on both sides with tinfoil, and the rings themselves are held in place by a hard rubber tube (*q*, *q*) of 8 cm. outside diameter. Through the centre of this tube, and not touching it, passes an iron core (*r*, *r*) about 6 cm. in diameter, built up of fine shellaced wire. This core forms part of a closed rectangular magnetic circuit

Fig. 2.



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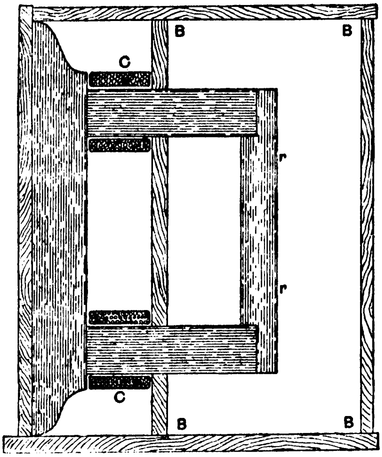
* On one occasion three series of readings gave remarkably consistent results, each series consisting of from 12 to 20 pairs of readings. However, the observed differences were very considerably larger than would be demanded by theory, and the results were never duplicated. The whole apparent consistence of results can, therefore, have been due only to a series of coincidences.

(fig. 3), the remainder of the circuit being built of best transformer sheet iron. The inner dimensions of the rectangle are: height 26 cm., width 23 cm. The condenser and the portion of the magnetic circuit passing through it are enclosed in a wooden box (B, B) lined with tinfoil, in order to protect the movable part of the condenser from air currents and electrostatic disturbances. The movable part of the condenser is earthed, the stationary part insulated. The magnetic circuit is energized by means of two exactly similar coils (C, C) placed around the horizontal parts of the circuit outside the box. The current used is a 110 volt, 60 cycle alternating current. The condenser is charged by the secondary of a potential transformer, the primary of which is connected in parallel with the magnetizing coils. The charging E.M.F. is thus very nearly 90° out of phase with the magnetic field. The effect of the magnetic field on the charging current passing to and from the movable part of the condenser along the suspension wire and frame is practically avoided by having the magnetic circuit closed so that there is never an appreciable magnetic field along the path of the current. The maximum result which this electromagnetic force can have is to entirely annul the effect sought for in the present experiment (see p. 463, footnote). In the present arrangement, therefore, it could have no more serious consequence than to reduce somewhat the observed deflexions. That this reduction has in fact been very slight is shown by the results of the experiment.

Fig. 3.



Calculation of the Effect.

Let N = magnetic flux through the condenser.
Q = quantity of charge, in electromagnetic units.
C = capacity, ,, ,, ,,
E = charging E.M.F., ,, ,, ,,
F = electric intensity, ,, ,, ,,
f = force, in dynes.
L = moment, in dyne-cm.
n = frequency of alternation.
θ = phase angle.

Also let $\dot{\mathrm{N}}$, $\dot{\mathrm{E}}$, &c. indicate instantaneous values.

$\hat{\mathrm{N}}, \hat{\mathrm{E}}$,, ,, maximum ,,
$\overline{\mathrm{N}}, \overline{\mathrm{E}}$,, ,, average ,,
N, E ,, ,, effective ,,

The instantaneous E.M.F. induced in a single turn of a conductor encircling the magnetic field is $d\dot{\mathrm{N}}/dt$, and the mean value at any instant of the electrostatic intensity around a circle of radius r, is

$$\dot{\mathrm{F}} = \frac{d\mathrm{N}}{dt}\,\frac{1}{2\pi r}.$$

The force exerted on the quantity $\Delta\dot{\mathrm{Q}}$, distributed evenly along an elementary annulus of radius r, is

$$\Delta\dot{f} = \frac{d\dot{\mathrm{N}}}{dt}\cdot\frac{\Delta\dot{\mathrm{Q}}}{2\pi r}.$$

The elementary moment is

$$\Delta\dot{\mathrm{L}} = \frac{d\dot{\mathrm{N}}}{dt}\cdot\frac{\Delta\dot{\mathrm{Q}}}{2\pi},$$

and is independent of the radius. Hence the total moment exerted on any number of charged annuli of any width is

$$\begin{aligned}\dot{\mathrm{L}} &= \frac{d\dot{\mathrm{N}}}{dt}\cdot\frac{\dot{\mathrm{Q}}}{2\pi}\\ &= \frac{d\dot{\mathrm{N}}}{dt}\cdot\frac{\mathrm{C}\dot{\mathrm{E}}}{2\pi}\end{aligned}$$

Assuming both $\dot{\mathrm{N}}$ and $\dot{\mathrm{E}}$ to vary according to the sine law, we have

$$\begin{aligned}\dot{\mathrm{E}} &= \hat{\mathrm{E}}\sin\theta\\ \dot{\mathrm{N}} &= \hat{\mathrm{N}}\cos\theta.\end{aligned}$$

Hence

$$\dot{\mathrm{L}} = \frac{\hat{\mathrm{N}}\mathrm{C}\hat{\mathrm{E}}}{2\pi}\sin^2\theta\frac{d\theta}{dt}$$

$$= \hat{\mathrm{N}}\mathrm{C}\hat{\mathrm{E}}n\sin^2\theta,$$

since $d\theta/dt = 2\pi n$.

Now the average moment over one complete cycle is

$$\bar{\mathrm{L}} = \frac{\hat{\mathrm{N}}\mathrm{C}\hat{\mathrm{E}}n}{2\pi}\int_0^{2\pi}\sin^2\theta d\theta$$

$$= 1/2\,\hat{\mathrm{N}}\mathrm{C}\hat{\mathrm{E}}n.$$

Since this formula expresses the moment directly in terms of the magnetic flux and the charge, it would seem to be the most natural one to use. It was in fact employed in all the earlier calculations of the present experiment. The quantities $\hat{\mathrm{N}}$ and $\hat{\mathrm{E}}$ have to be expressed in terms of measurable quantities as follows:

$$\hat{\mathrm{E}} = \mathrm{R}\sqrt{2}\,\mathrm{E}_1 \times 10^8,$$

where E_1 is the effective E.M.F. in volts impressed on the primary of the transformer, and R is the ratio of transformation.

$$\text{Av. } \frac{d\dot{\mathrm{N}}}{dt} = 4n\hat{\mathrm{N}} = \frac{\cdot 901\mathrm{E}_2 \times 10^8}{\mathrm{T}},$$

where n is number of complete cycles per second, and E_2 is the effective E.M.F. in volts induced in a coil of T turns. From this

$$\hat{\mathrm{N}} = \frac{\cdot 901\mathrm{E}_2 \times 10^8}{4n\mathrm{T}}.$$

On account of its greater convenience of application to the experimental data the following formula was given the preference in all the later computations.

Let E_2 = the effective E.M.F. induced in a single turn of a conductor encircling the magnetic field. Then the mean value around the circle of the "effective" electric intensity is

$$\mathrm{F} = \frac{\mathrm{E}_2}{2\pi r}.$$

Also let E_1 = the effective charging E.M.F. Then since

E_1 and E_2 are very nearly of the same phase the force is

$$f = \frac{E_2}{2\pi r} \cdot CE_1,$$

or the moment

$$L = \frac{E_2 CE_1}{2\pi}.$$

The two formulæ must evidently yield the same result, but the second has the manifest advantage of requiring merely two voltmeter readings, together with a determination, once for all, of the capacity.

Method of Observation.

The capacity of the condenser was determined by charging and discharging by means of a rated tuning-fork; using a storage-battery of known E.M.F. for charging, and measuring the rate of discharge by means of a galvanometer of known sensitiveness. The following is a specimen determination:—

Sensitiveness of galvanometer...	525×10^{-10} amp./cm.
E.M.F. applied	5·04 volts.
Frequency of tuning-fork	89·1 vibr./sec.
Deflexion produced	8·50 cm.
∴ Capacity	$9 \cdot 9 \times 10^{-10}$ farad.

The induced E.M.F. (E_2) was obtained by means of a coil of 50 turns and of the same diameter as the condenser rings, placed in the position occupied by the charged condenser rings. A table was prepared giving the values of E_2 when different E.M.F.'s were impressed on the magnetizing coils, so that during the experiment it was only necessary to observe the voltmeter which was left applied to the magnetizing circuit.

The charging E.M.F. (E_1) was determined by observing the E.M.F. impressed on the primary of the potential transformer, whose ratio of tranformation (20 : 1) was known.

Both E_1 and E_2 were thus made to depend on the same A.C. voltmeter, and this in turn was compared with a Weston laboratory standard voltmeter.

In attempting to observe the electromagnetic deflexions with this form of apparatus the only serious difficulty encountered was due to the electrostatic attraction between the condenser parts. This caused a directive moment to be exerted on the movable part, which was usually so large as to completely overshadow the effect of the suspension wire, although a strong suspension of manganin wire ·13 mm. in diameter and about 22 cm. long was used. Sometimes days were spent in the attempt to so adjust the condenser as to leave the movable part reasonably free to rotate, so that there was a possibility for the feeble electromagnetic force to manifest itself. The moment of inertia of the suspended rings was roughly 11000 g.cm.^{2}, and when the condenser was adjusted so that the period was as large as 8–10 seconds, the deflexions could be easily and unmistakably observed. Having balanced the condenser with an E.M.F. of about 40 volts impressed on the primary of the potential transformer (which means a charging E.M.F. of about 800 volts), and then suddenly applying an E.M.F. of about 100 volts to the magnetizing circuit (which would result in an induced E.M.F. of about ·44 volt per turn, indicating a maximum magnet flux of about 170,000 lines) the suspended body could be observed by means of telescope and scale to undergo a steady deflexion of some millimetres. Now reversing the connexions of the magnetizing circuit while the charging circuit remained unaltered, thus changing the relative phase of magnetizing and charging E.M.F.'s by 180°, a deflexion in the opposite direction was noted. These deflexions were always and unmistakably observable whenever the charged body was at all delicately poised, and were reversed whenever the connexion of either circuit was reversed. It was also an easy matter to trace out the connexions of the magnetizing coils and the transformer, and thus determine that the deflexions were taking place in the direction demanded by theory for the effect of the changing magnetic field on the charged body, and opposite to that which would be due to the static magnetic field acting on the charging current. The purely magnetic effect (*i. e.* the effect due to unsymmetrical magnetic properties of the suspended body), was always very small, and since it is completely eliminated by the method of observation adopted, no further attention was paid to it.

The angular magnitude of the deflexions was of course dependent upon the restoring couple, and this was due chiefly to the electrostatic attraction between the imperfectly symmetrical charged bodies. By adjusting the torsion-heads, which were capable of universal adjustment, lateral as well as rotational, and by shifting the stationary part of the condenser, the sensitiveness was on a few occasions increased so far as to admit of deflexions of from 6–10 cm. on the scale. But this condition would never continue long, and furthermore the zero was then so unsteady that reliable quantitative measurements could not be made under the circumstances. The most consistent readings for quantitative work were obtained when the sensitiveness was such as to give a difference in scale readings of ·5 to 2 cm. when the magnetizing current was reversed.

At first it was thought that the value of the restoring couple could be deduced from a knowledge of the moment of inertia of the swinging body and its observed period. Since, however, the restoring couple is due almost entirely to the electrostatic field the motion could not be simple harmonic, and the period therefore bears no known simple relation to the directive moment. The plan finally adopted was to determine the moment of torsion of the upper suspension wire by observing the period with a body of known moment of inertia suspended, and then making series of deflexions due to the electromagnetic effect alternately with similar series due to turning the torsion-head back and forth through a known angle. The angle of rotation of the torsion-head was read by telescope and scale, using the same scale on which the torsion deflexions and electromagnetic deflexions were read. From these readings the moment actually exerted by the electromagnetic effect can be calculated and compared with that demanded by theory.

The following is a specimen set of observations:—

Charging E.M.F.......... $E_1 = 828$ volts, " effective."
Induced E.M.F. .... $E_2 = \cdot 490$ volt, ,,
Capacity .... $C = 9\cdot 9 \times 10^{-10}$ farads.
Moment of torsion of upper suspension $\phi = 58\cdot 2$ dyne cm. per radian.
Scale distance $D = 388$ cm.

	A.	B.	A−B.
Torsion deflexions.	6·32	4·43	1·94
	6·42	4·76	1·49
	6·08	4·90	1·29
Electro-magnetic deflexions.	6·30	5·40	1·14
	6·78	5·10	1·43
	6·28	5·30	1·12
	6·56	5·40	1·13
	6·51	5·40	1·00
Torsion deflexions.	6·29	4·94	1·32
	6·24	4·76	1·44
	6·15	4·60	1·44
Electro-magnetic deflexions.	5·92	4·74	1·12
	5·80	5·00	1·00
	6·20	5·12	1·13
	6·30	5·16	1·20
	6·43	5·25	1·08
Torsion deflexions.	6·23	4·96	1·41
	6·51	5·67	·92
	6·67	5·61	1·15
	6·85		

In the groups "electromagnetic deflexions" column A gives the scale readings with both charging and magnetizing circuits closed, and column B the readings taken alternately with the A readings, but with the magnetizing current reversed. A − B therefore gives twice the electromagnetic deflexion. Each "reading" is the point of rest determined from three successive turning-points. The differences A − B are obtained by subtracting each B reading from the mean of the preceding and following A readings. In the groups "torsional deflexions" the A readings are taken under the same conditions as the A readings in the other groups, and the B readings are obtained by turning the upper torsion-head over 20 cm. of the scale, so as to produce a deflexion in the same direction, and of nearly the same magnitude, as that due to reversing the magnetic field. Both the electric and magnetic fields were left unchanged while the torsion deflexions were made.

If now $2d$ is the mean A − B for the electromagnetic, and $2d'$ the mean for the torsional deflexions, D the scale distance, and ϕ the moment of torsion of the suspension wire, then the observed moment due to the electromagnetic effect is

$$L=\frac{d}{d'}\frac{10}{2D}\phi.$$

Hence from the above set the observed moment is

$$L'=\frac{1{\cdot}13\times 5\times 58{\cdot}2}{1{\cdot}34\times 388}=\cdot 63 \text{ dyne cm.}$$

while from theory

$$L=\frac{828\times 9{\cdot}9\times\cdot 49}{2\pi\times 10^3}=\cdot 64 \text{ dyne cm.}$$

In the following table of results:

E_1 = the "effective" charging E.M.F. in volts.

E_2 = the "effective" E.M.F. in volts induced in one turn of a conductor encircling the magnetic field.

$2d$ = the (double) deflexion due to the electromagnetic effect, in cm.

$2d'$ = the deflexion due to turning the torsion-head through an angle 20/2D.

L = the moment in dyne cm. calculated from theory.

L′ = the measured moment in dyne cm.

The last column gives the percentage deviation of the observed from the theoretical value of the moment. The capacity, scale distance, and moment of torsion of the upper suspension are the same as in the foregoing specimen set.

E_1.	E_2.	2*d*.	2*d*'.	L.	L'.	Per cent. error.
600	·345	1·16	3·34	·33	·26	−21
606	·342	2·70	5·72	·33	·35	+6
606	·490	1·29	2·13	·47	·45	−4
600	·492	·73	1·24	·47	·44	−6
956	·346	·65	1·02	·52	·48	−8
956	·348	2·06	3·36	·525	·46	−13
806	·444	1·59	2·24	·565	·53	−7
772	·485	1·01	1·33	·59	·57	−4
828	·490	1·13	1·34	·64	·63	−2
840	·494	·53	·62	·655	·64	−2
848	·495	·65	·73	·66	·67	+1
864	·485	1·27	1·31	·66	·73	+10
916	·490	2·39	2·70	·71	·66	−7
956	·474	1·43	1·53	·716	·70	−2
956	·490	·87	1·02	·74	·72	−3
956	·492	1·78	1·86	·74	·72	−3
1294	·480	·45	·32	·98	1·05	+7
1314	·498	·40	·29	1·03	1·04	+1

Av. L' 3 per cent. too small.

Conclusion.

Considering the experimental difficulties involved, the form of apparatus used, and the smallness of the force measured, the accuracy of the foregoing results is all that could be expected. The observed moment shows a decided tendency towards lower values than the calculated amount. This is possibly due partly to the effect of the magnetic field on the charging current, partly to a phase difference other than 90° between the electric and the magnetic field. These are the only two sources of error not eliminated by the method of observation, and since both act in the same direction of diminishing the observed deflexion, the sum of their effects is shown by the results to be only about 3 per cent. of the quantity measured, which is not far from the probable experimental error. This can hardly be considered as casting any doubt upon the result. However, an attempt will be made to determine the phase difference, so that it can be taken into account in the calculation.

The experiment as it stands shows that a charged body in a changing magnetic field, or perhaps more exactly in a field swept by magnetic lines of force, is subjected to a mechanical force, whose magnitude and direction are in agreement with that demanded by the electromagnetic theory of Maxwell.

The first part of the work was done in the laboratories of the University of Chicago, the second part at the University of Texas. To the members of the Physics Faculty of the University of Chicago I wish to express my thanks for the appreciative interest they have continually shown in the progress of my work.

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