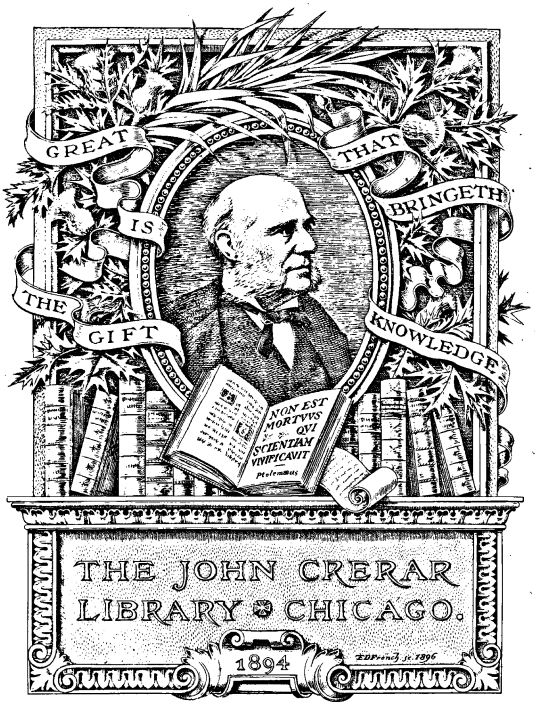

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PHYSICS

AN ELEMENTARY TEXT-BOOK FOR UNIVERSITY CLASSES

BY

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PREFACE.

In the following pages I have endeavoured to provide for the use of junior students, and more especially Medical students in their first year of study, a text-book on Physics, neither too large nor too elementary. The subject is developed as logically as possible along essentially practical lines, the principles of the science being elucidated by reference to familiar facts of observation and to experiments of the simplest character.

The book is divided into two parts for reasons which will be found indicated in the Introduction to Part II. The section devoted to Electricity and Magnetism has appeared in separate form—a fact which will explain the somewhat greater space apportioned to them than might have been deemed necessary in a book on general physics of the size contemplated. These subjects are, however, becoming daily of greater importance; and in the present case their somewhat fuller treatment has not led to any curtailment of the space apportioned to the other branches. It cannot be too strongly insisted that through Part I., in which are developed the far-reaching principles of Newton and Carnot, lies the only true approach to an intelligent acquaintance with Part II.

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To many of the chapters exercises have been appended. These the student will find of service in strengthening his grasp of the laws and methods of which they are examples.

My thanks are due to Professor Tait and Mr R. T. Omond for their kindness in reading and correcting the proof-sheets. Their suggestions and criticisms have been of the greatest value. But the extent of my indebtedness to Professor Tait cannot be gauged. It is that of a pupil to a master, whose friendly sympathy has been at all times stimulating and educative in the highest degree.

C. G. KNOTT.

EDINBURGH UNIVERSITY,
January 1897.

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PHYSICS.

PART I.

MATTER AND ENERGY.

PROPERTIES

OF

MATTER.

CHAPTER I.

THE PHYSICAL UNIVERSE.

§ 1. Broadly speaking, the various entities of the universe may be grouped under two great classes—living or *vital*, and non-living or *physical*. It is with the latter that Physics or Natural Philosophy deals. But the obvious distinction between the vital and the physical is not one of mutual exclusion; for all vital phenomena depend more or less upon purely physical processes. Matter which is associated with life does not cease to possess most of the properties that characterise it when life is absent. Certain other properties, peculiarly vital, come indeed into evidence; but living matter is still matter in the physical sense. The physical universe is the foundation upon which life builds its complicated structures; and it is hopeless to expect to arrive at a clear understanding of vital processes without first becoming familiar with the less complex processes that are known as physical. The energy with which life works is derived primarily from purely physical or chemical combinations; the mechanism through which it works is purely material.

The natural philosopher assumes, what is practically taken for granted by everyone, that matter in the ordinary acceptance of the term is a real thing. Its existence is a legitimate inference from the evidence of his senses. These inform him that matter exists in a great many forms, with an endless variety of properties and functions. It occurs as solid, liquid, or gas; it is sometimes transparent, sometimes opaque; it is hard, soft, brittle, tough; its tints and hues are too numerous to discriminate by name. Nevertheless, in spite of all this diversity, man's reason has convinced him that matter is a unity. Certain broad characteristics are common to all sorts; and any one of these *General Properties*, as they are called, may be accepted as a description if not a definition of matter.

§ 2. **General Properties of Matter.**—The most obvious general property of matter is that **it occupies space.** Every portion of matter has consequently a certain *volume* or bulk and a certain *form* or figure. On these properties is based the abstract science of Geometry. Again, if one portion of matter occupies a certain region of space, it does so to the exclusion of all other matter; or, no two portions of matter can occupy the same region of space at the same time. This property is known as *impenetrability.* In all cases of interpenetration of matter, as when a bullet buries itself in a plank, the penetrating matter makes way for itself by pushing aside the particles of the matter penetrated. A fish passes through the water by displacing it. Alloys are a fine form of interpenetration, the individual particles of the one substance mixing so closely with those of the other that our senses cannot distinguish them. These and many other examples, in which one kind of matter seems to penetrate into another, do not disprove the property of impenetrability. They show more clearly in what sense the term must be used, and prove that the mutually impenetrable *particles* of matter are in many cases (probably in all) separated from each other by regions of space unoccupied by matter. In

other words, matter is *porous*. The porosity of matter can be demonstrated experimentally, as when hydrogen passes through palladium and red-hot iron. So far, vitreous substances have not been experimentally proved to be porous; but there is no reason to doubt that they also have a coarse-grained structure.

§ 3. **Inertia.**—Another general property of fundamental importance is **Inertia.** In order to set a piece of matter in motion or bring it to rest we must act upon it. And, generally, when matter changes its state of motion or rest it is because of the presence of other matter influencing it in some way. This is the experience which is made the basis of the branch of Physics known as **Dynamics.** A fuller discussion will be found in Chapter IV.

§ 4. **Energy.**—This inertia, inertness, or passivity, which is ascribed to matter *as we know it*, compels us to search for some other real thing in the physical universe if we are to explain the *activity* which is so characteristic of it. This other real thing has been called Energy. The energy of a material system is its power of doing work, its power of being active. A bent bow, a wound-up watch-spring, a loaded gun, a raised weight—all are examples of material systems which are capable of doing work of a definite kind. All possess energy. Now, any change whatever that may occur in the physical universe means that work of some kind is being done, that some material system is consequently losing energy. What really happens is that energy is transferred from the system or body which is doing the work to the system or body upon which the work is being done. During this process there is neither loss nor gain of energy to the universe as a whole. The full significance of this process of energy-transference cannot be appreciated until the whole range of Physics has been studied. Meanwhile, we pass to the statement of a third general property of Matter, which is also a property of Energy.

§ 5. **The Conservation of Matter and of Energy.**—It is

not possible, by physical processes, either to destroy or create matter. This is the axiom on which the whole science of Chemistry is based. Through whatever transformations any portion of matter may pass, its quantity, as measured by recognised physical processes, never changes. It is this experimental truth which convinces our reason that matter is something objectively real. It is an axiom which probably no intelligent man ever seriously doubted, and which the utmost refinements of modern research have only served to establish on a firmer footing.

But this same principle of conservation holds also for Energy; and on this ground energy is placed in the same category with matter as being something objectively real. With the recognition of energy as a real conserved entity, physical science entered upon a new era in the early half of this century. The various branches of physics, which till then had been developed each along its own lines, and largely independent of the others, became knit together as one inseparable whole. It is still convenient for purposes of reference to adhere to the old subdivision into Properties of Matter (including Dynamics), Heat, Light, Sound, Electricity, and Magnetism. From the modern standpoint, however, these are simply different ways in which matter and energy are associated together; and it should be our aim to consider these varied phenomena of the physical universe in the light of the great principle of conservation.

§ 6. **Contingent or Special Properties of Matter.**—We readily distinguish different kinds of matter, and we effect the distinction by means of some one or more special properties. A few of these will be touched upon in this introductory chapter; but the more important will be treated later in their *energetic* relations. The mere mention of some of these contingent properties will serve a useful purpose in showing how particular properties of matter conduct us at once into one of the other recognised subdivisions of the subject.

(*a*) *Warm* and *Cold.* These have to do with properties which appeal to the sensation of touch. They give a hint as to the meaning of *temperature;* and, when thoroughly investigated, lead to a discussion of the nature, effects, and modes of transference of *Heat.*

(*b*) *Black* and *White; Red, Blue, Green,* &c. These appeal to our sense of sight, and introduce us at once to the study of Light. And closely connected therewith are other familiar properties, such as *transparency, translucency, opacity, phosphorescence, fluorescence,* &c.

(*c*) *Elasticity, Compressibility, Rigidity.* Here we are introduced to a group of dynamic properties of matter, very important in practical applications. Elasticity also is that property which is involved in the propagation of sound and certain other forms of wave-motion.

(*d*) *Malleable, Ductile, Plastic, Friable, Hard, Soft.* These have to do with what are known as the Molecular Forces—that is, the forces which exist between the molecules or ultimate particles of matter. In virtue of the possession of one or other of the associated properties, certain substances become of peculiar use for special purposes. For example, gold is pre-eminently malleable and can be beaten out into gold-leaf as thin as $\frac{1}{300000}$ of an inch. Although opaque in the lump, it can be obtained as a thin transparent sheet through which ordinary objects appear green. Again, silver is the most ductile of the metals, and can be drawn out in the form of very fine wire. Quartz at its point of fusion is pre-eminently ductile; and by an ingenious process Boys has made quartz fibres so thin as to be invisible. Of plastic substances we may mention putty, clay, hot guttapercha; but it is demonstrable that many substances, which under ordinary circumstances are rigid, unyielding solids, are plastic when a suitable force is applied. For example, cobblers' wax has all the appearance and many of the properties of a brittle solid; yet bullets placed on its surface gradually sink, and corks placed beneath gradually float up through it. The familiar terms *hard* and *soft* are

not capable of a simple definition precise enough to give a quantitative estimate of the degree of hardness or softness. In mineralogy the quality is tested by scratching, the harder substance being able to scratch the less hard. According to this mode of estimating hardness, talc is the softest and diamond the hardest of all natural substances; and the hardness of any new mineral is determined by its position in a scale formed arbitrarily by a series of minerals of increasing hardness. The complete series is—(1) talc, (2) rock-salt, (3) calcite, (4) fluor-spar, (5) apatite, (6) orthoclase, (7) quartz, (8) topaz, (9) corundum, (10) diamond. A substance that scratches quartz, but is scratched by topaz, would be said to lie between 7 and 8 in the scale of hardness. The artificial compound carbide of silicon is, in this sense, as hard as, if not harder than, diamond.

(*e*) *Viscous* and *Mobile*. These terms are applied to fluids according as they yield slowly or quickly to forces producing change of shape. Tar, treacle, glycerine, may be cited as examples of highly viscous liquids; water, alcohol, ether, and gases as fluids of great mobility. Viscosity is, however, a property of all fluids, and is the effective agent in many important phenomena. The gradual subsidence of waves on water or of currents in air is a direct result of viscosity. The little drops of water that form the clouds fall through the air at a very slow rate simply because the air is viscous, and consequently offers resistance to any body moving through it. Stokes calculates that a drop of water, a thousandth of an inch in diameter, cannot fall in still air at a speed greater than an inch or two per second, and that the terminal velocity (see below, § 37) for such small drops is proportional to the square of the diameter of the drop.

(*f*) *Flexible* and *Rigid* refer to what might be called accidental properties of matter, inasmuch as they depend upon the *form* which the substance happens to have. A glass rod is a good type of the inflexible or rigid substance. Yet the same glass can be drawn out in the form of a fibre or thread so

thin as to have great flexibility. Flexibility should never be confused with elasticity. Flexibility depends on the form of the body, whereas elasticity is a property of the substance of the body. Every part of a homogeneous body has the same density, elasticity, malleability, thermal conductivity, and so on; but such properties as flexibility, transparency, electric resistance, &c., vary with the dimensions and form of the part considered. The former are known as *specific* properties. Once they are determined, the corresponding accidental properties can in many cases be calculated, when the shape and size of the body are known.

(*g*) *Inductive Capacity, Magnetic Permeability, Electric Conductivity.* These properties of matter do not appeal directly to any of our senses, but are inferences from certain phenomena that are produced experimentally. They have become of practical importance in connection with telegraphy and electric lighting.

These—and the list might be easily added to—will suffice to show the great variety of properties of matter with which the student of Physics has to deal. The more important of them require for their elucidation a clear apprehension of the principles of **dynamics,** without which the exact science of Physics would be non-existent.

SUMMARY.

§ 1. The subject stated. Distinction between Vital and Physical.

§ 2. General Properties of Matter.—Form. Bulk. Impenetrability. Porosity.

§ 3. Inertia.—Basis of Dynamics.

§ 4. Energy.—Activity.

§ 5. The Conservation of Matter and of Energy.

§ 6. Contingent or Special Properties of Matter.—Temperature. Colour. Elasticity: compressibility, malleability, viscosity, flexibility, &c. Electric and Magnetic Properties.

CHAPTER II.

SPACE, MASS, TIME, AND THEIR MEASUREMENTS.

§ 7. **Length.**—Accurate measurement of quantity is the basis of all true science. In Physics, which is the foundation of all the experimental and observational sciences, every principle or law is a quantitative statement, more or less laboriously deduced from measurements made or observations taken.

Any given quantity is measured by comparing it with a chosen quantity of the same kind, called the Unit. For example, if we choose the foot as the unit of length, all other lengths must be expressed in feet. In ordinary practice, the unit chosen is largely a matter of convenience or of custom. Long distances are expressed in miles, short lengths in inches; heights are commonly given in feet, moderate lengths and distances in yards. Each of these familiar lengths can be expressed in terms of any of the others, and any given length can be expressed in terms of any one of them. Thus, a furlong is $\frac{1}{8}$ of a mile, or 220 yards, or 660 feet, or 7920 inches. The numerical value or measure of the quantity depends upon the unit chosen.

In the metric system, the system now in universal use for purely scientific purposes, the standard unit of length is the French metre (or the hundredth part of this, the centimetre). The metre is defined as a length marked off upon a particular rod of platinum at the temperature of melting ice. Similarly, the British standard unit of length is the length of a particular rod of bronze, deposited in the office of the Exchequer, when its temperature is 62° Fahr. This is called the standard yard, 1760 of which make the statute mile. Our standard units of length are, in fact, fixed arbitrarily.

§ 8. **Mass.**—A second important kind of quantity is that which we associate with such words as pound, ton, kilogramme, &c. We usually measure this kind of quantity by the familiar process of weighing on a balance, which is the recognised way of comparing *quantities of matter.* Three pounds of sugar, ten tons of iron, twelve ounces of gold, &c., all express to the mind definite quantities of the substances named. The scientific term for quantity of matter measured in this way is **mass.** In Chapter IV. the full significance of the term will be discussed. In the meantime it is important to notice that all substances or all kinds of matter possess mass; and that, as regards mass, a definite numerical relation can always be established between any two portions of matter however different they may otherwise be. One pound of lead has exactly the same mass as one pound of air, although they have very different bulks, and otherwise have very little in common.

Just as in the case of length, so here it is necessary to fix arbitrarily a definite unit of mass, in terms of which all other masses can be expressed. The British unit of mass is the standard pound, which is defined to be the mass of a particular piece of platinum preserved in the office of the Exchequer. In France the kilogramme is defined to be one thousand times the mass of a cubic centimetre of pure water at a temperature of 4° Centigrade (39·2° Fahrenheit); but practically it is the mass of a particular piece of platinum known as the 'Kilogramme des Archives.'

§ 9. **Time.**—A third quantity of importance is time, which necessarily enters into all questions involving motion or succession. In physical science we are not concerned with what time is, but only with its measurement. It is a familiar experience that certain events occur at the same instant, while others occur at quite different instants. It is necessary to have some convenient way of measuring the interval of time between the instants at which two different events have

occurred. A man may refer somewhat vaguely to a disaster as having happened when he was a schoolboy. A Chinese historian may speak of a great famine as having occurred in the reign of a certain emperor. But it would be impossible to connect these distinct events in time unless we had some means of expressing them in terms of the same unit. This is supplied us at once by the succession of the seasons which makes up what we call a *year*, and the succession of light and darkness which constitutes the *day*. These intervals depend upon the relative motions of the sun and earth. At a certain instant every day, at any chosen place, the sun *culminates*—that is, reaches its highest point in the heavens. Between two successive culminations a certain interval of time elapses, which is called the Solar Day. During this interval the pulse of a normally healthy man will beat a certain number of times, which we may suppose to be counted. Or, better still, we may use some machine which gives pulsations or beats in much the same manner, but which is not subject to such disturbances or variations as affect our heart pulsations. The pendulum of an ordinary clock is such a machine. It, with the accompanying mechanism of the clock, enables us to divide the day into convenient intervals of twenty-four hours, each hour into sixty minutes, and each minute into sixty seconds. The pendulum is in fact constructed to beat $24 \times 60 \times 60$, or 864,000 times in one day. It is found, however, that there are very few solar days during which this exact number of beats is obtained. Sometimes there are fewer beats, sometimes there are more. This is due to the variable speed of the earth in its orbit, a fact which makes the solar day in winter a little longer than the solar day in summer. Consequently the second is not defined in terms of any particular natural day, but in terms of the mean value of the day throughout the year. It is, in short, the $\frac{1}{864000}$ part of the Mean Solar Day.

The second is the unit of time adopted in our scientific system of units, although for practical purposes we employ

the minute, the hour, the day, the month, the year, the century, &c., according to circumstances.

§ 10. **Fundamental and Derived Units.**—The three units just discussed are called the Fundamental Units, since in terms of them all other scientific units of quantity can be expressed. These others are distinguished as derived units. Consider, for example, the units of area and of volume, which might be chosen quite arbitrarily just as the unit of length is chosen. Indeed, to some extent this has been done, as in our acre and gallon; and then, for completeness, we require to know how these are related to our unit of length. The simplest and most serviceable way is, however, to define the units of area and of volume once for all, by means of some geometrical relation connecting them with the unit of length. Thus, the scientific unit of area is the area of the square whose side is unity; and the unit of volume is the volume of the cube whose side is unity. With the foot as the unit of length, the unit of area is the square foot, and the unit of volume the cubic foot. In the same way the fundamental unit known as the centimetre gives rise to the derived units of area and volume known respectively as the square centimetre and the cubic centimetre. One metre contains 100 centimetres; hence, 1 sq. metre contains 100×100 or 10^4 sq. centimetres, and 1 cub. metre contains 10^6 or one million cub. centimetres. In like manner 1 sq. yard contains 9 sq. feet, and 1 cub. yard 27 cub. feet.

Other examples of derived units will be taken up in due course.

The choice of the quantities which are to serve as fundamental units is altogether arbitrary. Other quantities might be chosen—e.g. length, speed, and force; but these would not form such an intelligible system to the untrained mind as the quantities length, time, and mass. There is a great gain in establishing a scientific system of measurement upon familiar notions.

§ 11. **The Yard and Metre.**—By a very careful comparison the metre has been found to contain 39·370432 inches, so

that the metre is longer than the yard in the ratio of 1·093623 to 1, or roughly 11 to 10. The yard was no doubt originally chosen as being a convenient length for making ordinary measurements with. In this respect the metre is not quite so handy for use. Originally the metre was intended to be the ten-millionth part of the earth's quadrant, measured from equator to pole. Having measured the earth's quadrant in terms of the old French ell, the French surveyors constructed the standard metre. But the survey was not quite accurate; and it is now known that the earth's quadrant contains $1{\cdot}00074 \times 10^7$ metres.

The supreme advantage of the metre for scientific purposes lies in its decimal division, which greatly facilitates calculation. The names given to the subdivisions—decimetre, centimetre, millimetre—are very significant, since they are the tenth, hundredth, and thousandth part of the metre respectively. Similarly, the kilometre simply means the 'thousand-metre' length. Its value is 3280·9 feet or 0·621 mile; so that 1 mile = 1·61 kilometres nearly. The millimetre is a very useful unit for measuring small lengths. It is a little smaller than $\frac{1}{25}$ of an inch $\left(\text{almost exactly } \frac{1}{25{\cdot}4}\right)$, so that 3 millimetres are a little less than the eighth of an inch.

An important derived unit of volume in the metric system is the *litre*, which is defined as being equal to 1000 cubic centimetres or 1 cubic decimetre. A cubic foot contains 28·316 litres, and a gallon is equal to 4·54 litres. Note that a litre of water at 4° C. weighs almost exactly 1000 grammes or 1 kilogramme.

The British system of inch, foot, yard, and mile—in spite of the lack of method in the numerical ratios existing among them—has for ordinary practical purposes certain advantages not possessed by the metric system. The inch, foot, and yard fall in with human needs more readily than the centimetre, decimetre, and metre, and are besides much shorter words. Also, there is distinctly less chance of error in calculation.

§ 12. **The Pound and Gramme.**—The gramme was originally intended to be the mass of a cubic centimetre of distilled water at 4° C. The standard gramme is, however, the thousandth part of the 'kilogramme des Archives,' and the mass of a cubic centimetre of water at 4° C. (the temperature of maximum density) is 1·000013 grammes. For most practical purposes the intended relation may be assumed to be true. This extremely convenient relation between the mass and bulk of water gives to the metric system of weights and measures a scientific advantage not possessed by the British system, in which the foot and pound take the place of the centimetre and gramme. On the other hand, the pound is an essentially practical unit, the gramme being far too small for everyday purposes and the kilogramme a little too large. The pound contains 453·59 grammes or ·45359 kilogrammes; hence the kilogramme is 2·2046 pounds. Roughly speaking, half a kilogramme is one-tenth greater than a pound.

When dealing with very small masses it is convenient to use the *milligramme* as the unit. One grain (7000 of which make the standard pound) contains 64·8 milligrammes.

§ 13. **The Difficulty of Accurate Measurement.**—To obtain a scientifically accurate measurement of any quantity is by no means an easy matter. Even in the simplest of all, the comparison of lengths, there are great difficulties if anything like accuracy is demanded.

In the first place, a copy of the standard must be made with great care, and this must be subdivided into a convenient number of *equal* subdivisions. For example, the metre must be divided into 100 centimetres, and at least one of these into 10 millimetres. This equal subdivision is effected by means of a **Dividing Engine,** the indispensable part of which is a good **Screw.**

The essential feature of a screw is that its motion forwards or backwards is proportional to the amount of rotation given to it. For every complete turn, in whatever position effected, a perfect screw moves through a definite distance called

the *pitch*. It is with what are called fine screws—that is, screws of small pitch—that we effect the accurate subdivision of scales and the accurate measurement of small lengths. All instruments of precision are provided with screws in some form or other. By these a very slight forward or backward motion can be measured in terms of the rotation given to the screw. For example, if a screw has a pitch of one millimetre, and is rotated through the one-hundredth part of a complete turn, its translational motion will be through the hundredth part of a millimetre, or the $\frac{1}{2540}$ part of an inch.

§ 14. **The Spherometer.**—One of the best examples of an instrument which utilises the principle of the screw to measure small lengths is the **Spherometer** (fig. 1), so called because it is adapted for measuring the curvature of lenses. It is also employed for measuring the thickness of thin plates. It consists of a fine screw (A) which works vertically in a nut supported in the centre of a tripod stand (fig. 1). The lower end of the screw terminates in a blunt point, and to the upper end is fixed a flat circular plate (B), carefully graduated round the rim. In using the instrument the operator first finds the position in which the blunt point of the screw is in the same horizontal plane with the three feet. When the point of the screw is slightly too low, the spherometer will rock from one set of three legs to another set of three. Just at the instant this rocking ceases, as the screw is slowly screwed upwards, the three feet are assumed to be on the same plane with the point of the screw. The position of the screw is then noted. The plate whose thickness is to be measured is now inserted under the point of the screw. At once, of

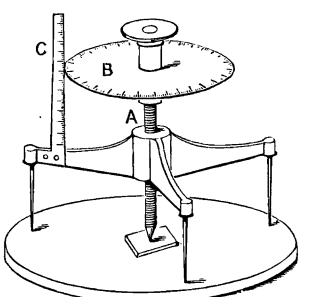


Fig. 1.

course, rocking again begins. Consequently the screw is screwed upwards until, as in the first adjustment, rocking just ceases. When this is so, the point of the screw will just be resting on the plate, and will be higher than the plane through the three legs by a distance equal to the thickness of the plate. This distance is at once measured by the number of revolutions (shown on the vertical scale C) and parts of a revolution which must be given to the screw to enable it to pass from the one position to the other.

The ordinary wire gauge for measuring the diameters of wires is constructed on essentially the same principle. The wire is gripped between two jaws, one of which is adjusted by means of a screw. The number of turns of the screw required to close the jaws after the wire has been removed measures the thickness of the wire. Ciceri Smith has greatly improved this convenient apparatus by fitting it with a delicate automatic recorder, which gives the true thickness of a wire at once in figures to thousandths of an inch.

§ 15. **The Vernier.**—In many cases the fine screw is simply used to give a sufficiently delicate adjustment, such, for example, as may be required for focussing a telescope or microscope, or for adjusting one scale relatively to another, as in barometers, cathetometers, sextants, theodolites, spectrometers, and all instruments of precision in which lengths or angles are measured. In these latter instances the one scale is the normal fixed scale on which the necessary graduations are made. The other scale, which slides in close contact with the first, has a zero mark on it; and the true reading is the position on the fixed scale opposite to which this zero mark stands. This position may be exactly at a division of the fixed scale, in which case the reading is given at once. It may, however, be between two divisions. How then is the exact reading to be obtained? If great accuracy is not demanded, the eye may judge what fraction of a complete division the mark is distant from either of the scale divisions; but for great accuracy the movable scale is graduated in a

very ingenious manner, which enables the reader to measure accurately to tenths or twentieths or even fiftieths of one of the smallest scale divisions. This additional scale is called a **Vernier.**

The simplest type of Vernier is one which measures to tenths. It is constructed so that each of its divisions is greater or less than the scale division by one-tenth of the latter. This may be done in either of two ways: *Ten* vernier divisions may be made to occupy the same distance as *nine* scale divisions (fig. 2); or *ten* vernier divisions the same distance as *eleven* scale divisions.

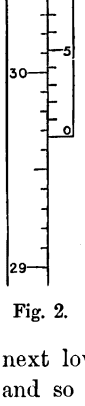


Fig. 2.

To use the instrument the vernier must be slid along parallel to the scale until its zero line comes opposite the position to be measured. Fig. 2 shows the position of the vernier for the reading 29·67. Here the zero of the vernier lies between the graduations 29·6 and 29·7. Running our eye up the scale, we see that the seventh graduation on the vernier is exactly opposite a graduation on the scale. Hence the sixth on the vernier must be higher than the next lower graduation on the scale by $\frac{1}{100}$, the fifth by $\frac{2}{100}$, and so on to the zero of the vernier, which will be found accordingly to lie $\frac{7}{100}$ higher than 29·6.

§ 16. **The Tri-dimensional Character of Space.**—The geometrical relations between the units of length, area, and volume have already been touched upon. The derived units of area and volume are defined in terms of the fundamental unit of length in such a manner that any change in the fundamental unit involves corresponding changes in the derived unit. For example, if it be desired to use a unit of length n times the unit originally chosen, then the corresponding unit of area will be $n \times n$ or n^2 times the original unit of area, and the new unit of volume n^3 times the original unit of volume. Otherwise expressed, the unit of length enters in a twofold manner

into the unit of area and in a threefold manner into the unit of volume. The reason of this is sufficiently obvious from the geometry of the line, square, and cube, and is intimately connected with the question of space dimensions.

A line has only one dimension—length; an area has two dimensions—length and breadth; a volume has three dimensions—length, breadth, and thickness. The line need not be straight, but may be curved or crooked; it still retains its one-dimensional character. The area need not be plane, but may be a part of any surface; it still retains its two-dimensional character. In other words, a line is space of one dimension and a surface is space of two dimensions; similarly, a volume is space of three dimensions. It is in this space of three dimensions that the physical universe exists.

The tri-dimensional character of ordinary space is demonstrated by the fact that three numbers are necessary and sufficient to fix the position of any point in space relatively to any other point.

Unless we have a chosen point of reference, it is not possible to fix position at all. For position is purely relative, and has nothing absolute about it. But more is required than a mere point of reference, if positions are to be accurately determined. An object described as being six feet distant from a particular pillar or post is quite undetermined in position. It may lie in any of an infinite number of possible directions. Hence, in addition to the origin or point of reference, we must agree upon certain directions as directions of reference, or upon some equivalent geometrical arrangement.

The numbers necessary to fix a position may be measured in a variety of ways, of which two are specially important. The one is by means of *Rectangular Co-ordinates*, which simply mean the distances of the point from three chosen planes which pass through the point of reference and are mutually perpendicular one to the other. Thus, the position of the top of a spire from any station is determined if we know (say) its distance north, its distance east, and its height—these being the distances respectively from a vertical plane facing north, a vertical plane facing east, and a horizontal plane, all passing through the station. Any one of

these numbers determines a plane in which the point must lie. Any two of them determine a straight line which is the intersection of the two corresponding planes; and in this line the point must lie. The third number will then determine the point as the point of intersection of the third plane and this line.

The other important method depends upon the conception of position as involving (1) distance, (2) direction. For example, let the top of the spire be 1000 feet distant from the chosen station or point of reference. With this point as centre, imagine a spherical surface described with a radius of 1000 feet. The top of the spire must lie somewhere on this surface. Its position will be determined if we know the *direction* of the line joining it to the centre. Imagine a vertical line OV (fig. 3) drawn through the centre, and let OS be the line that passes through the top of the spire S. If now the angle VOS is given, the point S will be confined to a certain line on the spherical surface, the line being a circle which is the intersection of the original sphere with a right cone O*abc* whose apex is O and whose semivertical angle is VOS. Finally, to determine the position of S on this line we must know the angle between the vertical plane containing VOS and any chosen vertical plane of reference, say that which passes north and south. This is the third number, and the point S is fixed as the point of intersection of three surfaces—a sphere, a cone, and a plane.

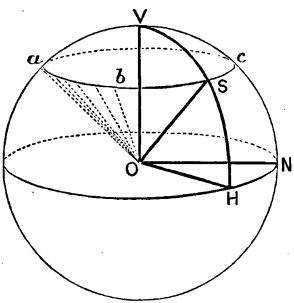


Fig. 3.

The angle VOS is commonly called the *Zenith Distance.* Its supplement, obtained by subtracting it from 90°, is the *Altitude* (HOS). The other angle described above is called the *Azimuth* (NOH). The azimuth and altitude together determine the direction in which the object lies; and the position is then completely defined when the distance is given.

Geographers have long used an exactly similar system of angular co-ordinates in describing the position of any place on the earth's surface. These are the longitude and latitude. They are determined with reference to the polar axis of the earth and a chosen plane passing through this axis and Greenwich Observatory, by an exactly similar geometrical construction as that just used in determining azimuth and altitude with reference to the vertical

line and the north-and-south plane drawn through it. To determine the position of any point on the earth's surface requires only these two numbers. The surface being given, the third co-ordinate (the distance, namely) is fixed once for all.

§ 17. **Contour Lines.**—In maps or charts, lines corresponding to certain longitudes and latitudes are drawn, and with reference to these the surface details of any country can be laid down. For most practical purposes this two-spread representation is sufficient. Rivers, roads, railways, boundary-lines, fences, &c., can all be represented by properly-drawn lines. But obviously it is impossible to represent on this flat surface the *relief* of the country—that is, the difference in height of contiguous points. For this relief involves the third dimension. It depends directly on the distance from the centre of the earth. How is this third dimension to be indicated on a flat surface? The representation is effected graphically by means of *contour lines*, the principle of which is of wide application in all branches of physical science.

Neglecting for the moment the tidal wave, we may suppose the ocean surface at any instant to be a level surface, so that all points along the boundary between sea and land are at one and the same level. From this level, technically called the sea-level, all heights are measured. Standing on the sea-level line, and looking in towards the land, an observer will generally see some point on the land which is at the same height as his eye above the sea-level, say 5 feet. We may suppose him to be possessed of some levelling instrument which can indicate exactly such a point. From any one stance a great number of such points may be picked out; from some other similar sea-level stance another set of points at the level 5 can be found; and so on, until all round the island or continent a sufficient number of points have been picked out and marked, all these points having the common property of being at the same height of 5 feet above the sea-level. Should the land sink 5 feet, these points would all lie on the new sea-level line. Any line which

passes through all points at the same level is called a Contour Line. The points found in the manner just described lie on the Contour Line 5. The sea-level line is the Contour 0 (zero).

Having thus found the Contour 5, the observer can proceed by an exact repetition of the process to find the contour 10; then 15, 20, and so on, until the whole land has been surveyed and contoured. On the chart, or map, these successive contour lines are drawn and named; and a map so constructed gives information regarding heights just as clearly as it gives information regarding distances. The completeness of the information will depend upon the number of successive contours drawn. In fig. 4, a simple case is shown of a small island contoured for every 100 feet. Where the contiguous contours are wide apart, the slope is small; where they come close together, as at B, the land is precipitous. In the centre of the island is a lake, whose margin is itself a contour line—the contour of 120. The contour of 200 consists of two isolated portions, and one of these is a figure of 8. The point C which is at the cross is a col or saddleback. It lies on the watershed which separates the basin of the lake and connected river from the basins of all the other rivers or streams. This watershed is shown by the dotted line. Every watercourse, being a line of quickest descent, must cut the contours at right angles. Hence, if the contours were sufficiently detailed, the watercourses could be drawn in. The elevation of the island along

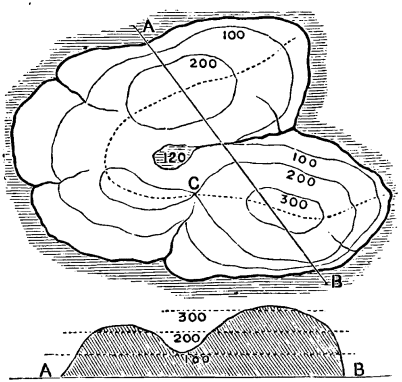


Fig. 4.

the line AB is given in the second figure. This elevation can be drawn from the contoured map if the contours are numerous enough. The ocean depths can also be similarly contoured.

Instead of *height* we may consider some other property common to contiguous points, and draw lines such that each line will pass through points which have the contemplated property in equal measure. The method is invaluable in meteorology, being peculiarly characteristic of all so-called weather-charts. At certain definite hours of every day the Meteorological Office receives telegraphic information of the observed temperature and pressure at the various stations. From these observations *isotherms* or lines of equal temperature and *isobars* or lines of equal pressure are drawn on charts, and convey to the trained eye an amount of information which the merely tabulated numbers could never give. The same method is adopted in the preparation of *mean* monthly and annual isotherms and isobars over the whole earth or any limited part of it. An excellent example of a general character is shown below in Part II., Chapter V.

But the principle is of wider application, and may be adapted to represent physical relations between any three measurable quantities. Thus, diagrams can be constructed giving a graphical representation of the relations between temperature, volume, and pressure of a substance. Volume and pressure take the place of latitude and longitude, and temperature takes the place of height—so that corresponding to contour lines we have lines of equal temperature. See below, under Heat (Chapter XIV.).

EXAMPLES.

1. Draw the contours of (*a*) a hemispherical, (*b*) a conical hill. If part of the hemisphere is removed by cutting it along a plane which passes through the summit and a point on the rim, show what change is produced in the distribution of the contour lines.

2. Under what conditions may one contour cross another of a different height? Show that if there is one crossing, there must be at least two.

3. When is a contour a watershed? And when is a double point on a contour not a col?

4. A scale is graduated in half-degrees. Show how to construct a vernier that will read to minutes.

5. In general, if m vernier divisions correspond to n scale divisions, the fraction of a scale division which the vernier enables us accurately to read is $(n \sim m)/m$.

6. Compare the pitches of two screws, one of which advances half an inch in ten turns, and the other a centimetre in eight turns.

SUMMARY.

§ 7. Unit of Quantity.—Length: Yard and Metre.

§ 8. Mass or Quantity of Matter.—Unit mass arbitrary. Pound and Kilogramme.

§ 9. Time.—The Year. Solar Day. Mean Solar Day. Hour, Minute, and Second.

§ 10. Fundamental and Derived Units.—Area. Volume.

§ 11. Yard and Metre.—Centimetre. Litre.

§ 12. Pound and Gramme.

§ 13. Difficulty of Accurate Measurement.—The Screw.

§ 14. Spherometer.

§ 15. Vernier.

§ 16. Tri-dimensional Character of Space.—Rectangular co-ordinates. Zenith Distance. Azimuth and Altitude. Latitude and Longitude.

§ 17. Contour Lines.—Isotherms. Isobars.

CHAPTER III.

KINEMATICS, OR THE SCIENCE OF PURE MOTION.

§ 18. **Speed.**—When matter is in motion it is moving with a definite rate of motion or speed—that is, it takes a certain definite interval of time to pass from one definite position to another. This change of position is accomplished along a **Path.** If we suppose the path to be given, then clearly one number is sufficient to fix the relative position of two points on the path. We consider the one point (O) to be fixed. It is the origin or zero point from which the position of every other point is to be reckoned. The second point we consider to be the position of a particle of matter which is moving along the path.

By way of illustration we may suppose the path to be a railway track, O (fig. 5) the station from which the train has started, and P, Q certain positions on the track through which the train passes at necessarily different instants. In familiar language we speak of the train travelling at the rate of (say) 30 miles per hour. This does not mean that the train actually describes 30 miles in one hour, for it may move only for a few minutes. The meaning is that if the train were to keep up this particular speed for a sufficiently long time it would in one hour describe the 30 miles. Thirty miles per hour is the same as half a mile or 880 yards per minute, or 44 feet per second. Thus, the same speed may have quite different numerical expressions depending upon the particular units of length and of time employed.



Fig. 5.

§ 19. **Average, Uniform, and Variable Speeds.**—As the particle moves from position P to position Q (fig. 5), it describes the space PQ. Let the length of path so described be s in terms of the chosen unit of length. Also let t be the

measure of the time taken to describe this space s. Then the ratio s/t measures the **Average Speed** of the particle as it passes from P to Q.

Definition: Average Speed is measured by the ratio of the space described to the time taken to describe it.

If the average speed is the same through whatever interval of time it may be reckoned, then the particle is moving uniformly. Its speed is uniform or constant, and is the same as the average speed. In other words, a body moves with uniform speed when it describes equal spaces in equal times, or when the space described is proportional to the time taken to describe it.

In general, however, it is found that the average speed of a moving particle has different values when measured through different intervals of time. The speed is then said to be **variable.**

For example, consider the case of a railway train travelling 400 miles in ten hours. This gives an average speed of 40 miles an hour over the whole ten hours' interval. But between two stations on the line which are 50 miles apart the train passes in one hour. During this hour the average speed is 50 miles per hour. But one of the mile intervals between the stations is travelled over in one minute. During this minute the average speed is 60 miles an hour. In another minute the space covered is only half a mile, and the average speed accordingly 30 miles an hour. And now, because in each of these cases we have taken a fairly short interval of time with a corresponding space described, we get a better measure of the actual speed which the train has at some definite point within the limits of the particular mile under consideration. But we get a still better approximation to the actual value of the speed at a given point by noting the time taken to pass between the two contiguous telegraph poles which are nearest to the point. And we can carry the process on in imagination, taking a shorter and shorter piece of the path and a correspondingly shorter time.

Thus we see that, in order to measure the actual speed of a particle at any required point of its path, we first measure the average speed over a portion of the path which contains

this point, and then consider what this average speed becomes as the portion contemplated is taken shorter and shorter. The **Unit of Speed** implicitly defined in these statements is that speed which would enable the particle to pass over unit length in unit time.

§ 20. **Graphic Construction showing relation between Space described, Speed, and Time taken.**—Let us suppose that the successive positions of the moving particle at successive equal intervals of time are noted, and the average speeds during these intervals calculated. Along the horizontal axis OX (fig. 6) mark off distances representing the successive intervals of time. At the middle point of each interval so marked off draw a perpendicular line, whose length, drawn to an arbitrary scale, is equal to the average speed during that interval. A series of points will thus be obtained which will indicate more or less approximately the manner in which the speed of the particle is changing in time. The rectangles shown by the dotted lines, being the product of the average speed and the time, evidently represent the spaces described.

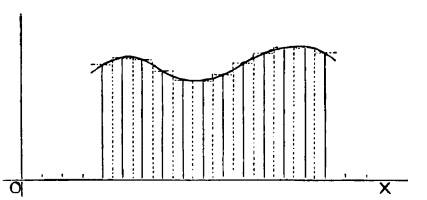


Fig. 6.

If now we take shorter and shorter intervals of time, the points will come closer and closer, and ultimately the succession of lines joining them will form a continuous curve. Any point P (fig. 7) on this curve measures, by its distance PM from the horizontal axis, the speed which the particle has at the time represented by OM or NP, the distance of P from the vertical axis. Also, it is easy to see that the space described during the

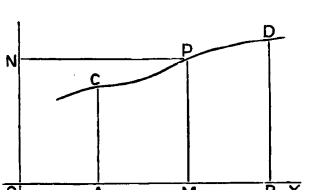


Fig. 7.

interval AB is measured by the area ACDB, which is bounded by the initial and final speeds AC and BD, the speed graph CPD, and the time axis.

§ 21. **Change of Speed: Tangential Acceleration.**—Let *pm*, *qn* (fig. 8) represent the speeds at successive instants. Then the change of speed in the interval *mn* is

$$qr = qn - pm.$$

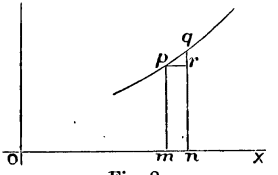


Fig. 8.

Hence the average change of speed is the ratio of *qr* to *mn*—i.e. the change of speed divided by the interval of time in which the change takes place. This we shall call the average acceleration in the direction of motion or **average tangential acceleration.**

The qualifying adjective 'tangential' is introduced for a reason which will appear later. Meanwhile, it is enough to note that, since the path is supposed to be given, the motion at any point must always be along the tangent at that point, and any change in the speed must also be along the tangent. When there is no risk of misunderstanding, the adjective may be dropped.

Now, just as we pass from average speed to the actual speed by considering shorter and shorter intervals of time, so by an exactly similar process we pass from average tangential acceleration to the actual tangential acceleration. In short, tangential acceleration bears the same relation to speed which speed does to space described. It is the rate of change of speed.

A railway train starting from rest gets up a speed of 30 miles an hour in one minute. The average tangential acceleration is 30 miles per hour in 1/60th of an hour, or 1800 miles-per-hour per hour. Expressed otherwise, this tangential acceleration is 44 feet per second in 60 seconds, or 44/60 feet-per-second per second.

A stone falling towards the earth has its speed increased every second by 32 feet-per-second. Hence the acceleration is 32 feet-per-second per second.

It is important to notice the manner in which speed and acceleration involve the fundamental units of space and time.

The measure of a speed is the ratio of the measure of a length to the measure of a time. Hence, just as an area is of two dimensions in length, so we speak of a speed as being of one dimension in length directly and one dimension in time inversely; or the dimensions of speed are +1 in length and −1 in time. Similarly, the dimensions of acceleration are +1 in speed-dimension and −1 in time—that is, +1 in length and −2 in time. By bearing this in mind we readily effect the change from one system of units to another. Thus, in the example given above, we have an acceleration of 1800 in mile and hour units: what is its value in foot and second units? Applying the principle of dimensions we find—

$$1800\ \frac{(\text{mile})}{(\text{hour})^2} = 1800\ \frac{5280\ \text{feet}}{(60 \times 60\ \text{seconds})^2}$$

$$= \frac{1800 \times 5280}{60 \times 60 \times 60 \times 60}\ \frac{\text{foot}}{\text{sec.}^2} = \frac{44}{60}\ \text{ft.-per-sec. per sec.}$$

A *speed* of 1800 miles per hour becomes

$$1800\ \frac{\text{mile}}{\text{hour}} = 1800\ \frac{5280\ \text{feet}}{60 \times 60\ \text{seconds}} = 2640\ \text{ft. per sec.}$$

§ 22. **Constant Tangential Acceleration.**—When, whatever be the interval chosen, the average tangential acceleration has always the same value, the acceleration is constant. In other words, the speed increases by an amount proportional to the interval of time. In such a case the speed graph becomes a straight line, as indicated in fig. 9.

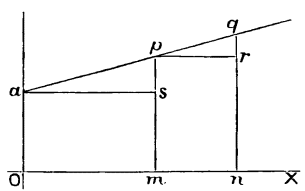


Fig. 9.

Here O*a* is the speed at time zero, *mp*, *nq* the speeds at times O*m*, O*n*. The acceleration is measured by the ratio *ps*/O*m* or *qr*/*mn*. Obviously the average speed during the interval *mn* is simply the arithmetical mean of the initial and final speeds *mp*, *nq*. Representing these speeds by the symbols

u and v, and the interval of time by t, we find for the acceleration the value

$$a = \frac{v - u}{t},$$

and for the space described the value

$$s = \tfrac{1}{2}(v + u)t.$$

Multiplying the quantities a and s we get

$$as = \tfrac{1}{2}(v^2 - u^2),$$

an important result, expressing the relation between the initial and final speeds, the acceleration and the space described.

The most important case of a constant acceleration is the case of falling bodies. It is usual to represent this acceleration, which is due to gravity, by the letter g; its value is about 32·2 feet-per-second per second. Suppose a body to fall from rest for three seconds; then the speed acquired after that interval is evidently (since $u = o$)

$$v = gt = 32{\cdot}2 \times 3 = 96{\cdot}6 \text{ feet per second,}$$

and the space described is

$$s = \tfrac{1}{2}vt = 48{\cdot}3 \times 3 = 144{\cdot}9 \text{ feet.}$$

Again, let the body be projected upwards with a speed of 80·5 feet per second. How high will it rise, and how long will it take to reach the highest point? The time taken to ascend is evidently $v/g = 80{\cdot}5/32{\cdot}2 = 2\tfrac{1}{2}$ seconds. The height reached is the product of the time and the average speed—that is, $40{\cdot}25 \times 2\tfrac{1}{2} = 100{\cdot}6$. This could also be calculated from the third formula

$$gs = \tfrac{1}{2}v^2$$

or

$$s = 80{\cdot}5 \times 80{\cdot}5 \div 64{\cdot}4 = 100{\cdot}6.$$

In this example, the acceleration is *negative*, since the speed is diminishing.

§ 23. **Direction of Motion, Velocity.**—The complete specification of the motion of a particle implies not only amount but also direction. Thus, two particles may have the same **speed**, say, 10 feet per second; but, if they are moving in different directions, they have different motions. It leads to greater clearness of expression to have a single word for

this complete conception of the motion of a particle; and here we shall follow Tait's custom and employ the word **Velocity** in this sense. Speed is the magnitude of the Velocity. Numerically they measure the same quantity; but Velocity has a fuller significance, being a quantity which has a direction as well as a magnitude. Two particles have equal velocities only when they are moving in parallel directions with equal speeds.

When the path of a particle is given, as we have supposed to be the case in previous paragraphs, the further knowledge of the speed is sufficient fully to specify the motion. But this is because a knowledge of the path necessarily implies a knowledge of the direction of the motion. In any position, the particle, if moving at all, is moving along the tangent to the path at the position momentarily occupied. Thus (fig. 10), the direction of motion of the particle at P is along the tangent PT. Draw from any point *o* a line *op* in the direction of the motion at P and equal in length to the speed. Then the line *op* is a complete specification of the velocity at P. Similarly *oq* is a complete specification of the velocity at Q.

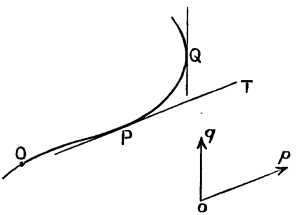


Fig. 10.

§ 24. **Change of Velocity.**—As the particle passes from position P to position Q, its velocity changes from *op* to *oq*. What then is the *change* of velocity? What velocity must we add to *op* so as to get *oq*? In other words, we wish to compound with the given velocity *op* another velocity such that the particle will have the actual velocity *oq*. The rule for doing this follows at once from the consideration that a velocity is essentially a displacement or change of position. As we have just seen, the velocity is in the direction of the displacement, and the magnitude or speed is measured by the ratio of the (small) space moved through to the corresponding

time taken. We may therefore regard *op*, *oq* as magnified representations of very small displacements in the vicinity of the positions P and Q respectively. Now, a displacement simply means a change of position, and the change of position *oq* is exactly the same whether we travel from *o* to *q* direct or by way of *p*. That is,

displacement *oq* = displacement *op* + displacement *pq*.

But what is true for the displacements will be true for the velocities, which are derived from them by dividing by the time interval. Hence

Velocity *oq* = velocity *op* + velocity *pq*.

Hence *pq* represents the change of velocity. This leads to the whole question of the

§ 25. **Composition and Resolution of Velocities.**—Consider first a few familiar examples.

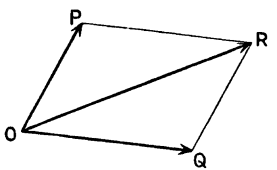


Fig. 11.

Let OP (fig. 11) represent the velocity with which a boat would sail in still water, and let OQ represent the velocity of the water itself. What is the velocity of the boat? Suppose the velocities constant, so that OP, OQ may be regarded as the spaces described in unit time. Then, as the boat moves from O to P in the water, the water flows from O to Q. Hence the boat will be found at R, and its velocity will be represented by OR.

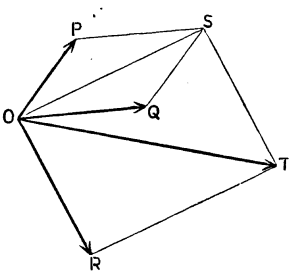


Fig. 12.

Again, let OP represent the velocity of a man crossing the deck of a ship, OQ the velocity of the ship in the water, and OR the velocity of the water current. What is the actual velocity of the man? First, by compounding OP and OQ (fig. 12) we get OS, the velocity of the man relatively to the water. Then by compounding OS and OR we get OT, the required actual velocity. Or more directly thus: from P the extremity of OP draw PS equal and parallel to OQ, and

then from S draw ST equal and parallel to OR; OT, which joins the beginning and end, is the resultant velocity of OP, PS, ST—that is, of OP, OQ, OR.

Generally, then, we compound two given velocities by forming the parallelogram whose sides are these velocities, and drawing the diagonal contiguous to those sides. This diagonal represents the resultant of the two given velocities. The construction is commonly known as the **Parallelogram of Velocities.** There is no difficulty extending the construction so as to find the resultant of any number of simultaneous velocities.

Just as any number of velocities can be compounded into a single resultant velocity, so a single velocity can be decomposed in an infinite number of ways into two or more component velocities. All we have to do is to form a closed polygon of which the given velocity is one side; then the other sides will represent a set of velocities of which the given velocity is the resultant. Thus, in fig. 13, the velocity AG can be decomposed into the velocities AB, BC, CD, DE, EF, and FG. These need not be in one plane, but may form a polygon in space.

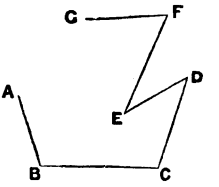


Fig. 13.

§ 26. **Resolution of Velocities.**—The most important mode of decomposing any velocity is into components which are perpendicular to one another. Confining our attention meanwhile to one plane, we first of all choose a particular line in the plane with reference to which the direction of the given velocity is determined. Let OX (fig. 14) be the line of reference or axis, and let OY be the axis perpendicular to it. Let OP be the given velocity. From the end P drop perpendiculars PM, PN on the axes

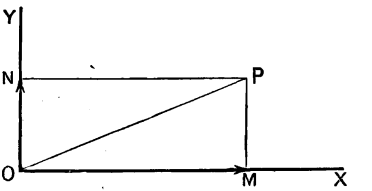


Fig. 14.

OX, OY. Then OM, ON are the components of OP. The component OM is called the resolved part of OP in direction OX; and ON the resolved part of OP in direction OY.

Generally, to resolve a velocity in any required direction is to find its component in that direction, the other component being understood to be in the perpendicular direction. The construction in every case is as follows: Through the one end of the line representing the velocity draw a line parallel to the required direction; and from the other end of the velocity drop a perpendicular on the line so drawn. The intercept on the line is the resolved part of the velocity in that direction.

Velocities are quantities having direction. They can be represented by directed lines. Such directed lines are called generically **Vectors**; and all quantities so representable are called Vector quantities. The Displacement or step is the simplest type of vector quantity; but there are many other quantities which have the same directed quality. All such vector quantities are compounded like velocities. They all obey the 'parallelogram' law, which it is customary now to call the law of vector addition.

§ 27. **Change of Direction, Angular Speed.**—It is necessary to consider carefully the method of measuring direction. In the first place let us confine our attention to one plane—say the plane of the paper. It is clear that the direction of any line is completely determined by the angle which it makes with the fixed line of reference. If this angle changes, the direction changes, and the change of direction is measured by the change of angle.

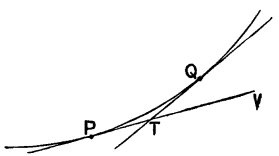


Fig. 15.

In the case of a particle moving along a path, the direction of its motion is, in general, constantly changing. Thus, as the particle moves from P to Q (fig. 15), the direction of motion changes by an amount equal to the angle between the tangents at P and Q—that is, the angle QTV. The ratio of this change of angle to the time taken to effect the change is called the

Average Angular Speed of the tangent to the curve. By measuring this average through shorter and shorter times we obtain better and better approximations to the value of the Angular Speed of the tangent at any given instant.

The simplest, and at the same time a very important case, is presented by the rotation of a straight line about a point fixed in it. The spoke of a wheel on a fixed axle, the hand of a clock or watch, any handle or crank producing rotatory motion are familiar illustrations. Let OP (fig. 16) be rotating about the fixed point O in the plane of the paper. Then, if OP remains constant in length, P will describe a circle about O. In a particular time it will describe a definite length of arc PQ. The ratio of this arc PQ to the radius OP is the true measure of the angle POQ. When the arc is numerically equal to the radius the subtended angle is the unit angle or *Radian;* and in terms of this unit and not in terms of the more familiar 'degree' we shall suppose all angles to be measured. When this mode of measurement is adopted, a very simple relation exists between the *linear speed* of P and the *angular speed* of OP. For, since the angle is measured by the ratio of the circular arc to the radius, any change of angle will be similarly measured by the ratio of the change of arcual position to the radius, and hence at once we get the relation,

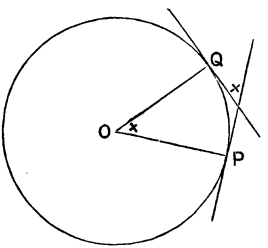


Fig. 16.

Angular speed of OP × OP = linear speed of P.

For example, if a railway train is travelling at 30 miles an hour on a curve whose radius is one mile, the angular speed of any line in the train is 30 per hour, $\frac{1}{2}$ per minute, $\frac{1}{120}$ per second. Since the radian is fully 57°, this angular speed of $\frac{1}{120}$ means that every second the direction of motion changes by nearly 0°·5.

§ 28. **Normal Acceleration.**—A particle is moving with constant speed in a circle. Its velocity is changing because the direction of motion is changing. The change of velocity is such that there is no tangential acceleration—that is, no change along the direction of motion. The change must therefore be at right angles to the direction of motion. In short, the *acceleration is wholly normal.* It is required to find the value of this normal acceleration. It is obvious that the quicker the change of direction the greater will be the acceleration. Let two points be moving with equal speeds in circles of different size, whose radii (say) are as 1 to 2. Then, the linear speeds being equal, the angular speeds of the respective radii will be inversely as the radii, or the rate of change of direction of motion in the smaller circle is *twice* that in the larger. The acceleration must consequently be greater. This simple consideration shows that the acceleration must depend on the radius of the path, being greater as the radius is smaller. The exact relation is easily found. Let PQ (fig. 17) be two successive positions of the particle, and let *op*, *oq* represent the velocities in these positions. Then *pq* is the total change of velocity; for this is the velocity which must be compounded with *op* to give *oq*. But evidently

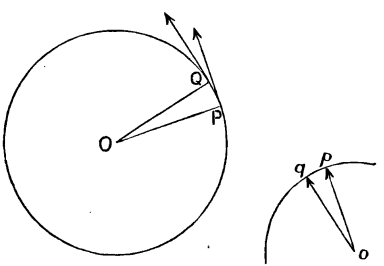


Fig. 17.

$$pq : op = \mathrm{PQ} : \mathrm{OP};$$

or, since PQ is the space described, we have

$$\frac{pq}{\text{speed}} = \frac{\text{speed} \times \text{time}}{\text{radius}}$$

or, $$\textit{normal acceleration} = \frac{pq}{\text{time}} = \frac{(\text{speed})^2}{\text{radius}}.$$

Thus, if v is the speed and r the radius, the expression v^2/r measures the normal acceleration of the moving particle.

When the particle is moving in any curved path with variable speed, the acceleration may be decomposed into a tangential component and a normal component. The former is measured by the rate of change of speed. The latter is measured by the ratio of the square of the speed to the radius of the circle which has at the moment the same curvature as the path. The demonstration is given at once by noticing that in the general problem the quantity corresponding to pq is always equal to the speed multiplied by the change of direction of motion, and can consequently be expressed at once in terms of the curvature. The complete demonstration is left as an exercise for the student.

As an example, let us calculate the normal acceleration of a particle revolving once a second in a circle of 3 feet radius. The circumference of the circle is 6π or 18·85 feet; hence the speed is 18·85 feet per second. Thus, the normal acceleration is $(18{\cdot}85)^2/3 = 118{\cdot}33$ feet-per-second per second.

§ 29. **Kinematics of Systems of Particles.**—It is evident that if we know how every particle moves we know how the whole system of particles is moving. The connection between the particles may be slight, as in the case of the particles of a gas; or it may be very close, as in the case of a solid body. Between these extremes we have an infinite number of gradations in the degree of connection. Two very important groups of cases fall to be considered. In the first place, if the connection is so close that the particles never alter their relative positions we have the case of a rigid body. Any ordinary solid under normal conditions is a sufficiently good realisation of a rigid body. In the second place, if the connection is such as to admit of changes in the relative positions of the particles, we have the various cases of **Strain.** The more important of these will be taken up in Chapter IX., and will be considered along with their associated forces or stresses.

In regard to the Kinematics of a rigid solid, a few remarks must here be made. Contrasting the rigid *extended* solid with the particle or material point, we see at once that the former has the power of rotating about an infinite number of axes passing through it. Thus, fixing one point of the solid does not reduce it to rest. It is still capable of rotating about axes passing through that point. On the other hand, the particle is capable only of translation. The motion of the rigid body will be fully specified if we know

the translational motion of any one point and the rotational motion of the rigid body about this point.

The velocity of a point is known if we know the speed and the direction of motion, and to specify a direction requires two numbers. Hence, to specify the velocity of the point requires *three* numbers. By exactly similar reasoning it follows that to specify the angular velocity of a rigid body, one point of which is fixed, requires *three* numbers also. For (1) there is the angular speed, which is *one number;* and (2) there is the direction of the axis about which the rotation is momentarily taking place, and, as before, this direction is specified by *two numbers.* Hence, to specify the free motion of a rigid body requires *six* numbers.

§ 30. **Degrees of Freedom and Constraint.**—A particle constrained to move along a given line has one degree of freedom corresponding to the one dimension of the space to which it is confined. If constrained to move on a given surface (e.g. a plane or a sphere), it has two degrees of freedom corresponding to the two dimensions of the space to which it is confined. If quite unconstrained, it has three degrees of freedom, which for definiteness may be supposed to be displacements along each of the three mutually perpendicular axes of reference.

Any extended solid will, so far as regards translation, have exactly the same number of degrees of freedom as a particle. But, because of its power of rotation, it will in its unconstrained condition possess three other degrees of freedom—namely, possibilities of rotation about each of the three axes of reference. Hence, a rigid body has at most six degrees of freedom, three of translation, and three of rotation. Let one point of the body be fixed: the translational constraint is complete, and only three degrees of freedom are left—those, namely, of rotation. Let a second point be fixed: the body can only now rotate about the line passing through the two fixed points, and there is but one degree of rotational freedom left. Fix a third point, not in the same line with the other two; the constraint is complete, and the body has no degrees of freedom. For every degree of constraint introduced one degree of freedom is lost. Thus, in the example just given, the fixing of the first point introduces three degrees of constraint; the fixing of the second point introduces two more degrees of constraint, making five in all, and leaving therefore only one degree of freedom; and, finally, the fixing of the third point introduces one more degree of constraint, making six in all. Otherwise, thus: To fix one point of a body requires three numbers. When this point is fixed every other point is confined to a definite spherical surface. Hence any

such other point has only two degrees of freedom, and is fixed by means of two numbers. But when two points are fixed every other point is confined to a circular path whose centre is in the line passing through the two fixed points. Hence to fix any such other point requires only one number.

A screw moving in its nut has only one degree of freedom, for, although its motion is partly translational and partly rotational, these component motions are not independent. There is a definite relation between them, so that every point of the screw can move along one definite line only.

The first leaf of a folding shutter has one degree of freedom—namely, rotation about the line through the hinges. The second leaf has two degrees of freedom, one of rotation like the first, but also one of translation, for each point in its line of hinges can move in a circular path. The third leaf has three, one of rotation, and two of translation, since in general each point of its line of hinges has freedom of motion in a plane. If there be a fourth leaf, it also has three degrees of freedom, two of translation, and one of rotation; and so on. In short, every leaf after the second has exactly the same number of degrees of freedom which it would have if it were quite disconnected, but constrained to remain in contact with and perpendicular to a given plane.

Each finger, with its successive joints, may be compared to the folding shutter. There is, however, an important difference in as much as the first joint has more degrees of freedom than the first leaf. The motion by which the fingers separate themselves is a motion in the plane perpendicular to the plane in which the finger bends. Thus the first joint has, relatively to the hand, two rotations instead of one, so that the end joint has four degrees of freedom instead of three. The arm, from the elbow to the wrist, is capable of three rotational motions, the joint at the elbow having all the characteristic properties of what is known as a universal joint. Each of these independent motions is effected by means of a special muscle, or set of muscles. Other examples might easily be deduced, such as the motion of the head, or the movement of the eye in its socket.

EXERCISES.

1. A postman walks at the rate of 4 miles an hour, but loses 1 minute in 5 in stoppages. Calculate his average speed in miles per hour, and in feet per second.

2. A ball set rolling along the ground comes to rest after describing 20 yards in 12 seconds. What were the initial speed and the acceleration (assumed uniform)?

3. A falling body describes the last 50 feet of its descent in $1\frac{1}{2}$ seconds. From what height did it fall, and with what speed does it reach the ground?

4. A point, moving with constant speed, takes 6 seconds to describe a circle of 3 yards circumference. Find the total change of velocity, both in magnitude and direction, at instants 1, 3, and 15 seconds after any chosen time.

5. Find the relative velocity of two vessels, one of which is sailing NNE. at 6 knots, and the other steaming WSW. at 12 knots.

6. Find the whole velocity of any point of a wheel rolling uniformly along the ground, the speed of the centre of the wheel being v and the radius r. Show that there is one point of the wheel momentarily at rest, and that the wheel is at the instant rotating about that point. [Note that the angular speed of every spoke is v/r.]

7. Calculate the average accelerations in the following cases:

a. A railway-train, travelling at 40 miles an hour, is brought to rest in a quarter of a mile.

b. A point moves once an hour round a circle of one foot radius.

c. A particle is attached to the free end of a string, 1 yard long, which rotates about its fixed end with angular speed 10.

SUMMARY.

§ 18. Speed.—Motion along a path.

§ 19. Average, Uniform, and Variable Speeds.—The Unit of Speed.

§ 20. Graphical Construction showing Relations among Space, Speed, and Time.

§ 21. Change of Speed.—Tangential Acceleration.

§ 22. Constant Tangential Acceleration.—Case of Gravity.

§ 23. Velocity.—Direction of Motion.

§ 24. Change of Velocity.

§ 25. Composition and Resolution of Velocities.

§ 26. Velocity resolved in a given direction.—Vectors.

§ 27. Change of Direction.—Angular Speed. Radian.

§ 28. Normal Acceleration.—Curvature.

§ 29. Kinematics of Systems of Particles.—Strains. Rigid Solids.

§ 30. Degrees of Freedom and Constraint.

CHAPTER IV.

MOMENTUM AND FORCE.

§ 31. **Inertia.**—In all cases in which the motion of a body is changing we usually recognise some reason for this change. It is invariably associated with the presence of other bodies. A ball set rolling along the ground has its speed gradually diminished until it comes to rest. If the ground is rough, or covered with long grass, the speed of the ball is very quickly reduced. On the other hand, if the ground is smooth, like a close-cut and well-rolled lawn or 'putting-green,' the loss of speed is much less rapid; while on a fine sheet of ice the ball rolls through a comparatively long distance before it is finally brought to rest. Now we know by experience that if we were to try to stop the motion of the ball at any instant we should have to exert ourselves; we should have to apply **force.** So far as the motion of the ball is concerned, the ultimate result in all these cases is the same; and this identity of effect is recognised in the usual phraseology—namely, that the diminution in the speed of the ball is due to force acting on it. The forces just considered are of the character of *resistances*—they reduce and finally destroy motion.

We may, however, by suitable effort, not only destroy the motion of the ball but give to it a velocity oppositely-directed to that originally possessed. To do this we must apply more force, as a cricketer does when driving a ball in the direction of the bowler. But the force of resistance due to the surface over which a ball is rolling cannot reverse the velocity of the ball. When the ball comes to rest it remains so until some quite different external agency acts upon it.

Let us then compare with this case the phenomenon

presented by a ball thrown vertically upwards. Here the upward speed is gradually diminished to zero, but immediately after begins to increase again *downwards* as the ball falls. Until the ball reaches its highest point it behaves in a general way exactly like the ball rolling along the ground; but thereafter all similarity ceases. The difference between the two cases may be stated thus:

In the one case the force involved depends upon the speed of the ball, always acts so as to resist the motion, and vanishes when the ball comes to rest; in the other case the main force involved is independent of the speed, and acts against the motion or with the motion according as the direction of the velocity is up or down. The former is a *resistance* only, and is called a *motional force*, in contrast with the latter, which is a *positional force*.

It is easy to convince ourselves that increase in the speed of a body requires the action of force. For example, no body will start from rest without the direct action upon it of some one or more other bodies. The stone falls with a steadily increasing speed because of the presence of the earth; the railway train gradually increases its speed under the influence of the force which is derived from the combustion of the fuel in the engine; the runner 'spurts' by a voluntary extra exertion; and so on in all similar cases. But it is not so demonstrably evident that a decrease in the speed of a body requires the action of force, and that the constancy of the velocity of a body means that there is no effective force acting. All this, however, is a legitimate inference from our experience. The more completely we get rid of all known resistances to motion the less evident is the diminution in the speed of the moving body. Moreover, in the motions of planets and satellites, astronomy gives us an object-lesson in which resistances due to motion play an altogether insignificant rôle. The forces which are all-important are those due to position. Their effects can be calculated long beforehand; and in the agreement between the observed and pre-

dicted configuration of the solar system at any instant we find the complete justification for the principle just stated.

By experiences similar to those described above we may further convince ourselves that to change the *direction* in which a body is moving also requires the action of force. And thus we arrive at the enunciation of what is known as **Newton's First Law of Motion,** namely,

Every body perseveres in its state of rest or of uniform motion in a straight line except in so far as it is made to change that state by impressed forces.

The property of matter involved in this enunciation is called **inertia.** That is to say, any portion of matter is inert, and cannot of itself change its state of motion. It continues to be as it is until some external agency acts upon it. Any change in its velocity either as regards magnitude or direction implies such an external agency, and is said to be (in somewhat metaphorical language) caused by a force. Force is thus conceived to be the cause of change of motion. The relation is expressed in **Newton's Second Law of Motion,** which is thus enunciated:

Change of motion (*motus*) is proportional to the impressed force, and takes place in the direction in which the force is impressed.

§ 32. **Momentum, Force.**—We must first consider carefully the precise meaning of Newton's word *motus*, here translated motion, and defined by Newton as being proportional to the velocity and the quantity of matter conjointly. For this purpose we appeal again to familiar experiences.

We find, for example, that some bodies yield easily to any slight exertion on our part; whereas it is utterly impossible to influence appreciably the motion of other bodies. In other words, the same force applied to different bodies may have very different effects upon their *velocities;* and yet the second law requires it to have the same effect upon the 'motions.' So long, in fact, as we are dealing with one body we may

assume the 'motion' to be proportional to the velocity; but as soon as we begin to compare the motions of two or more bodies we must assume 'motion' to involve a second factor, which has some simple relation to what we have called the inertia of the body—namely, that property in virtue of which it persists in its state of motion or rest. This factor we call *mass;* and the quantity represented by the product of the mass and the velocity we now call *momentum.* The second law of motion therefore asserts that the change of momentum is proportional to the impressed force.

Since mass, the one factor of momentum, does not change so long as the body to which it refers remains the same physical entity, it is clear that change of momentum is measured by the product of the mass into the change of velocity. This change of velocity takes place in a certain interval of time, and this is the interval of time during which the force is impressed. Thus, from the statement

$$\textit{Impressed Force} \propto \textit{change of momentum}$$

we deduce, after dividing both sides by the time involved, the relation

$$\begin{aligned} \textit{Average Force} &\propto \textit{average rate of change of momentum} \\ &\propto \textit{mass} \times \textit{average acceleration;} \end{aligned}$$

or, finally,

$$\textit{Force} \propto \textit{mass} \times \textit{acceleration.}$$

Force is, in short, a vector quantity parallel to the acceleration.

If we act on the same body with different forces we produce different accelerations. By measuring these accelerations we obtain a quantitative comparison of the corresponding forces.

Again, by acting on different bodies with the same force we produce, in general, different accelerations; and by measuring these we obtain a quantitative comparison of the *masses* of the different bodies.

§ 33. **Units of Mass and of Force.**—In addition to the units of space and time involved in the measurement of velocity and acceleration we must now introduce a third unit. This may either be the unit of force or of mass; but because of the constancy of the mass of any given body it is convenient to choose the latter for the fundamental unit. The unit of force then becomes a *derived* unit, and is defined by the equation which embodies Newton's Second Law, namely,

$$\text{Force} = \text{mass} \times \text{acceleration}.$$

When the unit mass is moving with unit acceleration it is being acted on by unit force; or unit force is that force which, acting on any mass for unit time, produces unit increase of momentum in the direction of its action.

If a mass of one pound has its speed increasing at the rate of one foot-per-second per second, then this pound is being acted upon by the unit of force in the foot-pound-second system of units. This unit force is called the **Poundal.** Similarly, if a mass of one gramme has its speed increasing at the rate of one centimetre-per-second per second, it is being acted upon by the unit of force in the centimetre-gramme-second system of units. This force is called the **Dyne.** Since a pound contains 453·59 grammes, and a foot contains 30·48 centimetres, we see that

1 poundal = 1 lb. × 1 foot-per-second per second.
= 453·59 grammes × 30·48 cms. per-second per second.
= 13,825 dynes.

The poundal and dyne are units of force which are practically little in vogue, but they are, scientifically, of great importance. Both are *absolute* units, being defined in terms of the fundamental units of length, mass, and time, and having, therefore, the same values everywhere.

§ 34. **The Force due to Gravity.**—Experiment shows (see § 42) that all bodies near the earth's surface in the mean latitude of Great Britain fall with an acceleration of 32·2 foot-second units or 981 centimetre-second units. Under the influence of the earth's attraction a pound is acted on by a force which produces an acceleration of 32·2. Hence the value of this force in poundals is 32·2, so that a poundal,

being a little less than $\frac{1}{32}$ of a pound weight, is equal to the weight of nearly half-an-ounce. Similarly, the value of the force that draws a mass of one gramme towards the earth is 981 dynes, so that a dyne is a little greater than the weight of a milligramme. These forces are conveniently called the weights of one pound and one gramme respectively. Thus, generally, if m be the measure of the mass of a body, g the measure of the acceleration due to gravity, the weight of the body is measured by

$$w = mg.$$

In words, the weight of m pounds is gm poundals, or the weight of m grammes is gm dynes. The quantity g is the same for all bodies at any one place (§§ 42, 43), so that the weight of a body is proportional to its mass. In practical life we use the weight of the pound as the unit of force. Being simply the attraction of the earth for the pound, it is a quantity whose value depends upon position. For instance, a body weighs more in higher latitudes than at the equator, partly because it is nearer to the earth's centre of attraction; hence, to specify without ambiguity the weight of a pound as a definite unit of force we must say where the pound is. This variability in the value of the weight of one pound at once condemns its use as a scientific unit of force, however convenient it may be as a practical unit.

The distinction between **mass** and **weight** is of great importance. The mass of a body is something which is invariable so long as the body remains unchanged in its inertia. It is, in fact, a measure of the inertia of the body, and is unaffected by position. A pound has the same mass wherever it may chance to be—whether at the earth's surface or anywhere else in the universe.

On the other hand, the weight of a body depends on its position relatively to the earth. It is the measure of the gravitation attraction between them. At any one place the weights of different bodies are proportional to their masses; and, practically, we compare masses by comparing weights.

Frequently, too, the words pound, ton, &c., are used in the sense of weight or force, especially by engineers; but the true significance of the word is evident from the context, and no real confusion results. It is advisable, however, to acquire the habit of speaking of pounds-weight, tons-weight, &c., when force is intended to be specified.

§ 35. **Tangential and Normal Forces.**—When a particle is moving in a path there is in every position a definite acceleration which it is convenient to decompose into tangential and normal components. The tangential acceleration is the rate of change of speed, and the normal acceleration the rate of change of velocity as regards direction. Now, the product of the mass and the total acceleration measures the force acting on the particle; and this force is also conveniently decomposed into tangential and normal components. The **tangential force** is conceived of as acting upon the particle in the direction of the motion, and its effect is to cause the speed to change at a certain rate. It is measured by the product of the mass into the rate of change of speed. The **normal force** is conceived of as acting upon the particle at right angles to the direction of motion, and its effect is to change the velocity as regards direction; it has no effect at all upon the speed. It is measured by the product of the mass, the square of the speed, and the curvature—in symbols mv^2/r, where r is the radius of curvature. The total force is the resultant of the tangential and normal forces.

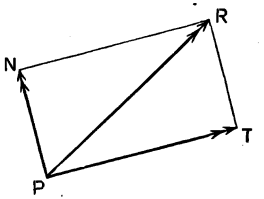


Fig. 18.

Let PT, PN (fig. 18) be vectors drawn to scale to represent the tangential and normal forces in direction and magnitude. Then PR, the diagonal of the parallelogram of which PT, PN are the sides, represents the resultant or total force acting on the particle at P.

Forces acting on a particle are resolved and compounded

exactly as accelerations and velocities are. This follows at once from the statement of the second law of motion.

It is important to notice that a force has no effect upon the speed with which a particle is moving at right angles to the line of action of the force. Thus, let PV (fig. 19) be the velocity of the particle, and let PF be the force acting on the particle.* Resolve PV along and perpendicular to PF. The one component PU remains unaltered because the force PF is normal to it. Whatever change occurs in the velocity must occur in the component PW. A very good illustration is afforded by the behaviour of a projectile.

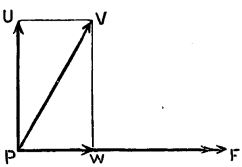


Fig. 19.

§ 36. **Projectiles.**—The simplest case is when the projectile is projected in a vertical line, either up or down. The force is then wholly tangential, and the projectile describes a straight vertical line with constant tangential acceleration. It is simply the case of falling bodies, already discussed in § 22. The transition from the kinematic equation to the dynamic is made by introducing the factor m on both sides. For example, the equation

$$\tfrac{1}{2}v^2 = gs$$

becomes

$$\tfrac{1}{2}mv^2 = mgs = fs,$$

when f is now the *force* acting on the mass m.

Slightly more complex is the case of a body projected horizontally with a given speed. Here the force due to gravity acts vertically and has no effect on the horizontal component of the velocity. Hence, the projectile persists moving with the same horizontal velocity until it is brought to rest by coming into contact with some obstacle. Under gravity the *vertical* fall of the body is exactly the same as it would have been had there been no horizontal velocity at all.

Let u (fig. 20) be the horizontal velocity of projection from

* Forces are distinguished, when necessary, from velocities or momenta by having two arrow-heads.

A, and let P be the position of the body at time t. Then clearly $AM = ut$, the horizontal distance travelled through in virtue of the constant speed u. But the body is all the time falling; the height fallen through is

$$MP = \tfrac{1}{2}gt^2.$$

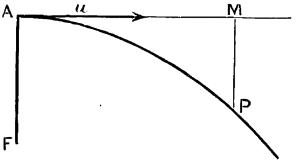


Fig. 20.

Thus we find

$$AM^2 = u^2t^2 = \frac{2u^2}{g}MP,$$

an equation which gives a property of the curve on which P lies. It represents a parabola whose axis is vertical and whose focus lies at the point F, where

$$AF = \frac{u^2}{2g}.$$

That is, AF is equal to the height to which the body would ascend if it were projected *upwards* with velocity u.

There is no difficulty extending the investigation to the more general case in which the projectile is projected at any angle to the horizontal with a given velocity U. This velocity U can be resolved into two components u and V, of which the former is

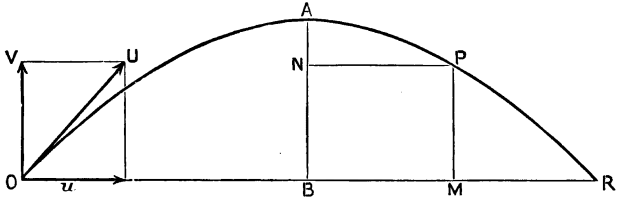


Fig. 21.

horizontal, and therefore unaffected by gravity. Thus, at time t the body has travelled through a horizontal distance $OM = ut$. Meanwhile, the vertical velocity has been changing. Its value at time t is

$$v = V - gt,$$

and the height of the body above O at this instant is

$$MP = Vt - \tfrac{1}{2}gt^2$$
$$= \frac{V}{u}OM - \frac{g}{2u^2}OM^2,$$

a well-known property of the parabola.

The range OR along the horizontal line through O is what OM becomes when PM vanishes. Thus we find at once

$$OR = \frac{2Vu}{g}.$$

If the speed of projection is given, V and u must satisfy the condition,

$$V^2 + u^2 = U^2, \text{ a constant.}$$

We may vary the ratio between V and u by varying the angle of projection; and to every angle of projection there will correspond a definite range OR. For what angle of projection is this range the greatest possible? Evidently when the product Vu has its greatest value. But $2Vu$ is never greater than $V^2 + u^2$, and is equal to it only when $V = u$. Hence the greatest value of OR is given when $V = u$—that is, when the angle of projection is 45° or half a right angle. Under these conditions

$$Vu = u^2 = V^2 = \tfrac{1}{2}U^2,$$

and

$$OR = \frac{U^2}{g}$$

is the greatest possible range for the speed of projection U.

For example, the greatest possible range for a speed of projection of 50 feet per second is $1600/32 = 50$. Again, the least speed of projection compatible with a range of 72 feet is

$$U = \sqrt{72 \times 32} = 12 \times 4 = 48.$$

Thus, with a given speed of projection the greatest horizontal range is attained when the angle of projection is 45°. If the angle of projection is greater or less than this, the projectile falls short of the greatest possible range. There are, however, two distinct paths by which any such shorter range can be covered. This is indicated by the form of expression $2Vu/g$, which measures the horizontal range not only when u is the horizontal component and V the vertical component of the velocity of projection, but also when V is the horizontal and u the vertical component. This will be clearly seen from fig. 22. Here OA the horizontal component of OC is equal, numerically, to OA′, the vertical component of OC′. Similarly OB, the vertical

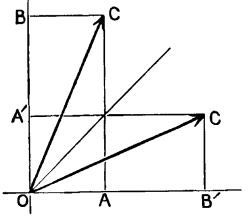


Fig. 22.

component of OC, is equal to OB′, the horizontal component of OC′. OC and OC′, as speeds, are equal; but as velocities they differ in direction. Clearly their directions are equally inclined to the line which bisects BOB′. Now, since

$$V_u = OA'.OB = OB'.OA,$$

the horizontal range is the same for both directions of projection.

§ 37. **Resistance of the Air to the Motion of a Projectile.**—The investigation just given assumes that a constant vertical force is the only force acting on the projectile. But, as a matter of fact, any body moving through the atmosphere is subject to another force which depends upon the form and size of the body. The following simple experiment is very instructive:

Cut out of fairly stiff paper a circular disc exactly the size of a penny or other coin, hold the coin horizontally between finger and thumb, set the paper disc on the top of it, and let both drop. They will fall to earth together. Then place the disc under the coin, and again let them fall. In this position also they will reach the earth together. But now hold them separate, one in each hand, and allow them to drop simultaneously. The coin will fall straight down as formerly; but the paper disc will settle down more slowly and with irregular fluttering.

The first two experiments show that under similar conditions the coin and paper fall with equal accelerations, so that weight is proportional to mass. The effective force acting on the two together is equal to the combined weight diminished by the resistance due to the air. This resistance depends on the surface, and is the same in both cases. But, in the third experiment, the effective forces acting on the coin and disc are different, being in each case the weight diminished by the resistance. The weight of the disc is much smaller than the weight of the coin, so that the resistance, being the same for both when the speeds and aspects are the same, is *relatively* of greater importance in the case of the paper disc. The acceleration of the lighter body is consequently appreciably smaller. If the coin and disc were dropped simultaneously and independently in a region of space altogether devoid of air, they would fall with exactly equal accelerations.

Scientific artillerymen have investigated with great care the resistance of the air to projectiles, and have found this

resistance to be more and more effective the greater the speed. So great is the resistance at high speeds that the velocity of projection, which may amount to 2000 feet per second, is rapidly cut down to a few hundreds before the shot has completed its range.

There is a certain speed for which each body experiences a resistance equal to its weight. If the body is moving straight downwards with a speed greater than this critical value, the resistance due to the air will exceed the weight of the body, and the downward speed will *diminish* and not increase. If the body is projected downwards with exactly the critical speed, the effects of the two forces will balance, and the body will continue to move with constant velocity. But if the body is projected downwards with a speed less than the critical speed, the weight will have the advantage, and the acceleration will be positive. In short, whatever be the downward speed of projection, whether great or small, the combined effect of gravity and atmospheric resistance is to cause the speed of the body to approximate gradually to the value of the speed for which the resistance is equal to the weight. This critical speed is called the **Terminal Velocity.** For projectiles of ordinary size the terminal velocity never exceeds a few hundred feet per second.

Suppose a meteoric stone travelling through space to be drawn down upon the earth's surface. It probably enters the higher regions of the atmosphere with a relative speed of several miles per second; but before it reaches the solid earth its horizontal velocity through the air will be almost entirely destroyed, while its vertical velocity will be cut down to very nearly the value of the terminal velocity. Thus the air acts as a buffer to meteoric bombardment, each stone being largely robbed of its original high momentum, and rendered comparatively harmless. The loss of motion is accompanied by a development of heat. The meteoric stone is consequently heated to incandescence, and is more or less burned away as it falls through the air.

When, in addition to its forward motion, the projectile is rotating about an axis inclined to the direction of flight, a

transverse force comes into play, causing the body to swerve from what would otherwise have been its path. Newton, writing in 1671, records his observation of the fact that a tennis-ball, struck with an oblique racket, described a curved line. 'For,' he comments, 'a circular as well as a progressive motion being communicated to it by that stroke, its parts, on that side where the motions conspire, must press and beat the contiguous air more violently than on the other; and there excite a reluctancy and re-action of the air proportionably greater' (see § 39). The direction towards which the rotating ball swerves is the direction of motion of the front of the ball. The same phenomenon may be frequently observed in the flight of a golf-ball. It is this effect, indeed, which, as shown by Tait, accounts for the long range of a well-driven golf-ball and the time it remains in the air. Such a ball is projected nearly horizontally with a rapid spin about a horizontal axis perpendicular to the direction of flight. In virtue of this *underspin*, as it is called, the front of the ball is moving upwards relatively to the centre. Hence an upward force is brought into play, the amount of which is approximately equal to the product of the speed and spin of the ball. Its effect is to counteract, and even to overcome, the downward acceleration. When the speed and the spin are both great enough, this force becomes greater than the weight of the ball, which accordingly describes a path concave upwards—a feature often observed during the earliest stages of the flight. If the axis of rotation be inclined also to the horizontal—as it frequently is in a badly-driven ball—the ball in its flight curves round to right or to left of the original direction of projection. The erratic behaviour of a toy-balloon projected with a spin about some axis illustrates the same phenomenon. By giving such a balloon a sharp undercut with a tennis-racket we can make it describe a looped path in the air, both the beginning and end being concave upwards.

§ 38. **The Balancing of Forces.**—The First Law tells us that any change of motion indicates the presence of force;

but it does not tell us that no change of motion indicates the absence of force. In the case of a body falling in air with its constant terminal velocity, we know that certain forces are acting on the body. But these forces balance. Their resultant is zero.

Another example is the case of a railway train travelling at a steady rate on the level. If steam is cut off, the train will soon come to rest because of the various frictional forces resisting its motion. This resistance is always acting when the train is in motion, and in the case of steady motion is exactly balanced by the driving force derived from the steam engine. These two forces balance, and hence the train moves with constant velocity as if no force at all were acting on it. To keep the train moving with constant speed downhill, the breaks must be applied. Here the impelling force is the resolved part of the weight of the train along the slope, and it is balanced by the friction, suitably increased by the action of the brakes.

Any body resting on a table is similarly under the action of a balancing system of forces. Its own weight acts downwards; but because of the impenetrability of matter, an equal and opposite force reacts upon it. This force is of the nature of a pressure. If we hold the body in our hand we feel its weight pulling downwards; but the body is kept at rest by the reactionary pressure exerted on the body by the hand. Or, again, suppose the body to be hung at the end of a string. The string is stretched; it is in a state of *tension*. And this tension acting upwards on the body balances the weight acting downwards. The obvious condition for the balancing of forces acting on a particle is that their resultant should be zero.

As an example of a case in which more than two forces balance, take the following: A body of known weight is attached to a point of a string whose ends are fixed. Let AOB (fig. 23) be the string; and O the point of attachment. The weight W of the body acts downwards through O, and is balanced by the tensions on the two parts of the string. As regards the point O, these

tensions act towards A and B respectively. If the length of the string and the position of the point of attachment are known, the directions OA and OB are known also. We may then find the values of the tensions in terms of the weight W by means of a simple geometrical construction. Let P, Q, be the values of these tensions. Then the forces P, Q, W must have a zero resultant. If any two are made the sides of a parallelogram, the third is the diagonal through their common point. Or, otherwise, lines equal and parallel to the vectors representing the forces form a triangle. The construction is as follows: Draw a vector (§ 26) to any arbitrary scale so as to represent the force W. Through the upper and lower ends draw lines parallel to the directions of the strings. These lines will meet at a point; and the remaining two sides of the triangle so obtained will represent to the same scale the tensions P and Q. The construction may be made on the original figure. Produce the line of action of W upwards. Lay off the value of W to the chosen scale. Through the upper end of this measured portion draw lines parallel to the strings. These will cut off from the strings lengths which to the same scale represent the tensions P and Q.

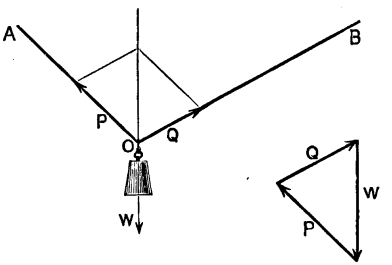


Fig. 23.

Let the end B be removed farther and farther from A (fig. 24). The parts of the string will become less and less inclined to the horizontal. If we construct the triangle of forces we see at once that P and Q become very large. The more nearly horizontal the string is drawn the greater become the tensions. Obviously it will be impossible to pull the string into the horizontal position, which indeed would imply infinite tensions. The practical limit is imposed by the strength of the string, which will snap when the tensions attain a certain value.

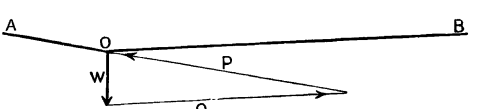


Fig. 24.

Observe that the tensions tend to pull A and B towards O, as

well as O towards A and B. And this leads us naturally to the consideration of the Third Law of Motion.

§ 39. **Newton's Third Law of Motion** is thus enunciated:

Action and Reaction are equal and opposite; or, to every action there is an equal and opposite reaction.

This law is necessary when we wish to pass from the dynamics of a particle to the dynamics of systems of particles which are connected together or mutually influence one another. The mutual action between any two bodies has indeed a double aspect. Corresponding to the force conceived as acting on the one body there is the equal but opposite force acting on the other. A stretched string pulls at both its ends, the pull on the one end being exactly equal and opposite to the pull on the other. A magnet attracts a piece of iron with a certain force; with an exactly equal but opposite force does the piece of iron attract the magnet. If both were free to move, and were influenced only by their mutual magnetic action, their momenta would change at the same rate. In like manner, the force which draws a stone to the earth is equal and opposite to the force which draws the earth to the stone. Their momenta change at the same rate; but because of the very much greater mass of the earth, its acceleration is infinitesimal compared to the acceleration experienced by the stone.

These various instances are all included under the following statement:

The total momentum of a system of particles—that is, the vector sum of all the individual momenta—is unaffected by the mutual actions of the particles.

For example, when a gun is fired, the ball is projected with a certain momentum forward. At the same time the gun 'recoils' with an equal momentum backward. For the sum of the momenta must remain zero. Thus, let a 14-lb. ball be projected with a speed of 1000 feet per second from a gun whose mass (including carriage)

is 1 ton. The principle just stated gives at once the following relation:

$$1 \text{ ton} \times \text{speed of recoil of gun} = 14 \text{ lb.} \times 1000.$$

Hence, $$\text{speed of recoil} = \frac{14000}{2240} = 6\tfrac{1}{4} \text{ feet per second.}$$

Both the initial speeds are, of course, rapidly cut down by frictional resistances.

§ 40. **Impact.**—In the **impact** or **collision** of bodies we find illustrations of the same principle. Many instructive experiments can be made with the form of apparatus first constructed by Wren, and described by Newton in the *Principia*. The apparatus consists essentially of two pendulums of equal length, so adjusted that the spherical bobs are just in contact when both are hanging vertically at rest. Let the one A (fig. 25) be drawn to one side through a certain arc A*a*, and then let go. It will reach its natural position of rest with a certain acquired momentum which would enable it to describe an equal arc on the other side were there nothing to stop it. In the present case, however, there is the bob B of the other pendulum. Accordingly, A strikes B and communicates to it part of its momentum. B is impelled forward through arc B*b*, while A describes a much smaller arc than that through which it swung before the impact. In the figure, A is represented as having followed on after B; but with suitable relations between the masses A may be reflected backwards. In all cases, however, it is found that the total momentum of the system is not changed by the impact. The momentum acquired by B and the momentum still possessed by A can be calculated from the positions reached by B and A.

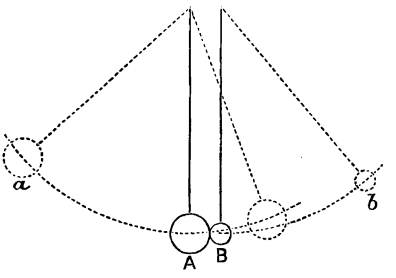


Fig. 25.

For the velocity in each case is that which corresponds to the height through which the corresponding mass is lifted; it is directly as the square root of the height ($v = \sqrt{2gh}$).

By attaching small pieces of putty to the contiguous sides of the two bobs we get them to stick together after the impact and to proceed as one body. Hence, the speed of the two together after impact will be less than the speed of the one before impact in the ratio of the mass of this one to the combined mass of the two. By such experiments we obtain interesting illustrations of the second law as well as of the third law of motion.

In these examples we have supposed the one body to be at rest; but exactly the same law holds when both are in motion. The momentum of the system is unaltered by the collision. If the two masses are moving in opposite directions before impact, then it is evident that the one which has the smaller momentum will have its motion reversed on impact; the other will follow on or be itself reversed according to circumstances. The 'driving' of a cricket-ball is a good illustration. In billiards, however, the spins or screws intentionally put upon the one ball greatly complicate the question, which can no longer be regarded as a case of collision of particles.

A little experience soon shows that balls of different substances behave very differently after collision. The total momentum of the pair of bodies is unchanged by the impact; but the manner in which the bodies separate after collision depends upon their form, mass, and elastic properties. By experiments with small *spheres* Newton established a very simple law which, with the momentum law, enables us fully to predict the effect of a particular collision, provided that the collision is not such as to produce great deformation or to alter permanently the form of the bodies. According to this law, the relative speed of the spheres after impact bears to their relative speed before impact a ratio which is constant for the same pair of bodies whatever be their initial speeds. This ratio is called the coefficient of restitution. It is always less than unity. According to Tait's recent experiments, the

average values of the coefficient of restitution between steel and the substances named are ·82 for native india-rubber, ·76 for vulcanised india-rubber, ·55 for cork, ·47 for steel. In the case of cork, the coefficient increased with successive impacts. The times of impact varied considerably among the different substances, but were of the order of ·01 and ·001 of a second, diminishing as the relative velocity of impact increased and increasing with the mass. Hertz was the first to work out theoretically the time of impact.

When a ball rebounds from a surface fixed to the earth we are not able, experimentally, to test the momentum law; but it is an easy matter to note the effect upon the *relative* speed of the ball and earth. Let the ball be dropped from a height H and rebound to a height h. Then, because of the relation $v^2 = 2gs$ (§§ 22, 36), the ratio of the heights h and H will be the ratio of the squares of the speeds immediately after and immediately before rebound. Hence, the constant ratio spoken of above will be $\sqrt{h/H}$. For example, a steel ball dropped from a height of 6 feet upon a steel slab will rebound to a height of

$$6 \times (\cdot 47)^2 = 1 \text{ ft. } 4 \text{ in. nearly.}$$

§ 41. **Centrifugal Force.**—A particle moving in a curved path is (§ 35) under the influence of a normal force, mv^2/r, where m is the mass of the particle, v its speed, and r the radius of curvature of the path. A body revolving in a circle at the end of a string and a railway train rounding a curve are familiar examples of the kind of motion here contemplated. These bodies are constrained to move in curved paths; and corresponding to the force acting on the body there is the equal and opposite reaction acting on the string and on the rails. The string is in tension, and if we hold the one end of the string we feel this tension as an *outward* pull on our hand. Similarly, in the case of the train there is an outward pressure upon the rails equal but opposite to the force that is compelling the train to deviate from its naturally straight path; to this reaction the name *Centrifugal Force* has been applied. The

term is unfortunate, for the force that acts on the body is not centrifugal, and the body itself has no tendency to *fly from the centre*. Its tendency is to proceed along the tangent to the path in which it is made to move because of the central or 'centripetal' force acting on it.

Every part of the earth's surface (with the exception of the poles themselves) is describing a circle about the polar axis of the earth. Consider for definiteness a body of mass m at the equator. It describes fully 25,000 miles in 24 hours, so that its speed is nearly 1540 feet per second. The value of the normal force corresponding is

$$\frac{m(1540)^2}{4000 \times 5280} = \cdot 112\, m.$$

The weight of the mass is mg, and hence the normal force is $\frac{1}{288}$ part of the weight of the mass. This fraction of the weight of each body is employed in compelling the body to move round the polar axis; consequently, the effective weight of a body at the equator is less than its true weight by $\frac{1}{288}$ part.

The question naturally suggests itself: at what rate must the earth spin in order that a body at the equator may appear to lose all its weight? The problem is answered by solving the equation

$$\frac{mv^2}{r} = mg, \text{ or } v^2 = gr,$$

where r is the earth's radius, and v is to be found. But the result is got at once by noting that the *square* of the speed is proportional to the force, other things being the same; hence, the speed is inversely as the square root of the force, and the period of rotation directly as the square root of the force. When the earth spins once a day the centrifugal force is $\frac{1}{288}$ of the weight; hence the centrifugal force will be equal to the weight when the earth spins $\sqrt{288}$ times per day—that is, almost exactly seventeen times per day. Consequently, if the earth were to spin round its axis once every 1 hour 25 minutes, objects apparently at rest at the equator would appear to have no weight at all.

The oblateness of the earth is generally regarded as an effect of centrifugal force, produced possibly when the earth was in a plastic condition, or, in more recent times, by denudation of the solid material down to the oblate ocean surface. A simple and instructive illustration is given

by the rotation of a flexible circular hoop about a diameter (fig. 26). The rotation causes the hoop to open out into an ellipse, with its shorter axis along the axis of rotation, as shown by the dotted line. Similarly, if an endless chain be attached by a string to the end of a vertical rod rotating about its own vertical axis, the chain will open out. The faster the rotation the more will the flexible hoop or chain bulge outwards. Each element of the hoop or link of the chain is kept in its circular path by the mutual action between it and its neighbouring elements or links. Hence, at every point of the material a tension exists, which increases with the square of the angular speed. This tension must not exceed a certain value, or the material will be ruptured. Consequently there is a certain rate of rotation up to which it is safe to go; but if a quicker rate be attempted the ring or chain will fly into pieces. Similarly, if a flywheel, a grindstone, the earth itself, were rotated fast enough, its particles could no longer be held together—disintegration would ensue. For small bodies like the grindstone, the forces that hold the particles together are inter-molecular. But in the case of a large body like the earth, it can be shown that the gravitation attraction between the two halves is 25,000 times greater than the molecular attraction between the opposing faces of these halves. In other words, gravitation is of more importance than cohesion in keeping the earth together.

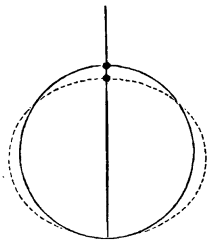


Fig. 26.

§ 42. **Conical Pendulum and Simple Pendulum.**—As a final example of great importance, let us consider the motion of the simple pendulum, either conically about a vertical axis, or with oscillations to and fro in one plane. The ideal simple pendulum is a particle hung at the end of a massless string. This can be approximately realised by suspending a small heavy sphere at the end of a thin wire.

Let us suppose the sphere A (fig. 27) to be moving in a horizontal circle with speed V. The radius of this circle is AM, the distance of A from the vertical axis through O the point of suspension. The only forces acting on the particle are the weight W (or mg) and the tension T of the string. Their resultant is the force that sustains the motion; it is therefore a force directed towards M, and having the value mv^2/AM. If we construct the triangle of forces we see at a glance that this force along AM is to W in the same ratio as AM to OM. Hence

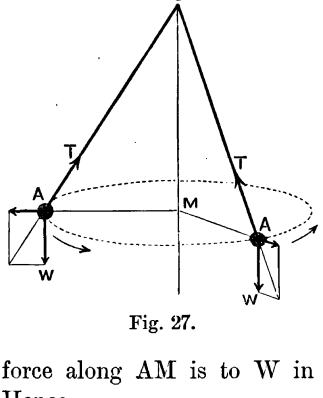


Fig. 27.

$$\frac{mv^2}{AM} : mg = AM : OM,$$

or

$$\left(\frac{v}{AM}\right)^2 = \frac{g}{OM}$$

Now, if T be the periodic time—that is, the time taken to make one complete revolution—we have

$$v = \frac{\text{circumference}}{T} = \frac{2\pi AM}{T}$$

where π is the number 3·14159..... ; and thus

$$\frac{v}{AM} = \frac{2\pi}{T}$$

Hence, finally, writing h for OM, the *vertical* distance between the particle and the point of suspension, we get

$$\frac{2\pi}{T} = \sqrt{\frac{g}{h}} \text{ or } T = 2\pi\sqrt{\frac{h}{g}}$$

The more quickly the pendulum is made to rotate about the vertical axis the higher does it rise. It is interesting to note

that, if pendulums of different lengths (as in fig. 28) are made to rotate with equal angular speeds about the same vertical axis, all the bobs will rise to the same height.

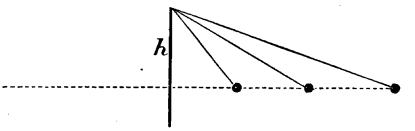


Fig. 28.

As the rate of rotation gets slower—that is, as the period gets longer—the length h will also increase. But h can never become longer than the length l of the pendulum. Hence the period of a conical pendulum can never exceed the value $2\pi\sqrt{l/g}$. This value is very closely approximated to when the pendulum bob describes a circle of comparatively small radius (as in fig. 29). Looked at from a little distance, this small conical motion is indistinguishable from a simple to-and-fro motion in a plane; and observation shows that the period of the conical pendulum of small range is equal to the period of the simple pendulum oscillating through small arcs in one plane.

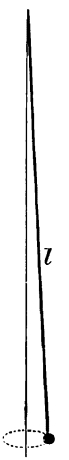


Fig. 29.

Hence for a Simple Pendulum oscillating in one plane the period for small arcs is $2\pi\sqrt{l/g}$. A seconds pendulum moves from one side to the other in one second, and consequently two seconds elapse between two successive occupations of the same end position. The complete period is, therefore, two seconds; and we have the relation

$$1 = \pi\sqrt{\frac{\mathrm{L}}{g}}$$

where L is the length of the seconds pendulum. Knowing g, we can calculate $\mathrm{L}(=g/\pi^2)$.

The pendulum has two very important applications. Its rhythmic motion is of value in all time measurements; for so long as the length remains unaltered the time of oscillation through a determinate arc is constant, the oscillations are isochronous. And even though the range or amplitude of oscillation should change, the period is unaffected if the oscillations are small. The pendulum

(or its dynamic equivalent, the balance-wheel) regulates the rate of clocks and chronometers. The principal source of disturbance to the isochronous oscillations of the pendulum is change of temperature, which is generally accompanied by change of length. The devices to correct for this are described in Chapter XIV.

The other important application of the pendulum is in the measurement of the quantity g, the acceleration due to gravity. We simply have to measure the period of oscillation and the length of the pendulum, and we find g from the equation given above, put in the form

$$g = \frac{4\pi^2 l}{T^2}.$$

The period is measured by counting the number of oscillations in a particular interval of time. By filling a hollow spherical bob with different materials, Newton proved that all kinds of matter experience the same acceleration at a given locality, and that consequently weight is proportional to mass.

§ 43. **Variation of Weight with Latitude.**—When experiments are made with the same pendulum at different points on the earth's surface, it is found that g varies with the latitude. The apparent weight of any mass diminishes by about 5 in 1000 as it is taken from either pole to the equator. It was shown above that the mere rotation of the earth accounts for a diminution of 1 in 288, or nearly 3·5 in 1000. The residue of 1·5 per 1000 is to be explained as due to the oblateness of the earth and to irregularities in the distribution of matter within it. The mean value of g in foot-second units for latitude l at the sea-level is given by the formula

$$g = 32{\cdot}09 + {\cdot}165 \sin.^2 l.$$

This gives 32·20 for the latitude of Edinburgh.

EXERCISES.

1. Compare the momentum of 1 ton moving with a speed of 1 inch per second with that of 2 ounces moving with a speed of 100 feet per second; and calculate the respective forces required to bring them to rest in half a minute. Express these forces in pounds' weight.

2. A train of 40 tons is travelling at the rate of 30 miles an hour on the level; find the force in tons' weight which brings it to rest in 20 seconds. How much sooner would it come to rest if it

were ascending an incline of 1 in 100? [Note that, in addition to the friction of the brakes, gravity must also be overcome.]

3. A mass is projected horizontally from the top of a tower 64 feet high, and falls to earth 15 feet from the base of the tower. What was the speed of projection?

4. A projectile has an initial speed of 50 feet per second in a direction making 30° with the horizontal. Find the height attained and the range on the level.

5. A mass of 1 pound hangs at the end of a cord which, passing over a smooth pulley, is attached to a mass of 2 pounds resting on a smooth horizontal table. Discuss the motion of the connected masses. [The force acting is the weight of 1 lb.; the mass moved is 3 lb.]

6. Two equal masses M, M are attached to the ends of a light string which hangs over a smooth peg. A small mass m is placed on one of the two masses. Find the acceleration of the system. After t seconds m is removed. What is the subsequent motion? [This kind of problem is commonly referred to as the problem of *Atwood's Machine.*]

7. In the last question, calculate the tension of the string joining the masses M and $M+m$.

8. One particle, moving with a speed of 20, overtakes another moving in the same direction with speed 10. After collision they move on as one with speed 12. Compare their masses.

9. A man is standing on a lift which begins to descend with an acceleration of 5 feet-per-second per second. What is the pressure between the man and the lift?

10. A mass of 3 lb. is whirling in a horizontal plane once a second at the end of a string 3 feet long, the other end of the string being fixed. Calculate the tension in the string both in poundals and in pounds-weight.

11. With what angular speed must the string in the last example be whirled in a vertical plane so that the tension may be zero as the mass passes through the highest point?

12. Calculate the length of the pendulum which oscillates four times in 3 seconds at a place where the acceleration due to gravity is 32 foot-second units.

13. A mass of 10 lb. hung at the end of a string is pulled aside by means of a second string, which is always kept horizontal. Find the tensions in the strings when the angle between them is 120°.

14. Particles starting from rest simultaneously from the top of a sphere slide down smooth cords under the action of gravity.

Prove that they all reach the circumference of the sphere at the same instant.

15. Show that the result of question 14 enables us to find by a simple geometrical construction the straight path down which a particle will slide from a given point to a given line in the shortest time. Such a path of shortest time is called a *brachistochrone.*

16. In the arrangement shown in figure 25, page 55, the mass A is 3 lb. and is allowed to fall from a position level with the point of suspension. To what height will B, whose mass is 1 lb., be driven after impact, the coefficient of restitution being ·5, and the length of each pendulum 3 feet? Also, how high will A rise after the impact?

17. If, in example 9, the lift were to begin to descend with an acceleration of 35 feet-per-second per second, what would be the sensation experienced by the man?

SUMMARY.

§ 31. Inertia.—Force. Resistance. Motional and Positional Forces. Newton's First Law of Motion. Newton's Second Law of Motion.

§ 32. Momentum.—Force. Interpretation of the Second Law. Comparison of masses and of forces.

§ 33. Units of mass and of force.—The *Poundal* and *Dyne.*

§ 34. Force due to Gravity.—Distinction between mass and weight.

§ 35. Tangential and Normal Forces.

§ 36. Projectiles.—Parabola. Greatest Range for given speed of projection.

§ 37. Resistance of Air to Motion of a Projectile.—Terminal Velocity. Meteoric bombardment. Flight of rotating sphere—tennis-ball, golf-ball.

§ 38. The Balancing of Forces.—Important statical problems.

§ 39. Newton's Third Law of Motion.—Constancy of Momentum.

§ 40. Impact.—Newton's laws of impact.

§ 41. Centrifugal Force.—True weight of body on earth's surface diminished because of rotation. Oblateness of earth. Cohesion of flywheel, grindstone, &c.

§ 42. Conical Pendulum and Simple Pendulum.—Importance of pendulum (1) in time measurements, (2) in measurement of acceleration due to gravity.

§ 43. Variation of Weight with Latitude.

CHAPTER V.

WORK AND ENERGY.

§ 44. **Work done against a Force.**—In raising a body through a certain height we exert ourselves and experience fatigue. A double exertion is required to raise the same body through twice the height, a threefold exertion through three times the height, and so on. Then, again, the heavier the body the greater must the exertion be in raising it through a given height. It is easily recognised by trial that it is the *raising* of the body which demands the exertion. A slight push will turn an ordinary door upon its hinges; whereas a horizontal trap-door of the same weight requires a great exertion to open it. In the one case we are simply overcoming inertia and such frictional forces as may exist at the hinges; but in the other case we are acting against the *weight* of the body; we are overcoming the steadily acting force of attraction between the body and the earth. In thus moving the body against the force that is acting upon it we are said to **do work against the force.** As hinted above, the work done is proportional to the force and to the distance through which it is overcome.

In lifting three pounds through 10 feet we do thirty times the amount of work required to raise one pound through one foot. This latter amount of work is conveniently taken as a practical unit of work and is called the Foot-Pound.

Definition. The foot-pound is the amount of work done in raising one pound through one foot at the earth's surface.

This, however, is not the scientific unit of work, which is the amount of work done in overcoming the *unit force*

through unit distance. Since the weight of one pound is g times the unit force, we see at once that one foot-pound is equal to g times the absolute unit of work—that is,

$$1 \text{ foot-pound} = 32{\cdot}2 \text{ foot-poundals.}$$

In the centimetre-gramme-second (or, briefly, C.G.S.) system of units, a definite name has been given to the derived unit of work. It is called the **erg.** Thus an erg is the work done in overcoming one dyne through one centimetre. Corresponding to the *foot-pound* in the English units we have the *kilogrammetre* in the metric units. It is the work done in raising a kilogram through one metre.

Thus,

1 kilogrammetre = work done in raising 1000 grammes through 100 centimetres,
= 100,000 centimetre-grammes,
= 98,100,000 ergs = $9{\cdot}81 \times 10^7$ ergs,

since one gramme weight is g dynes, and g is equal to 981 C.G.S. units. A similar calculation gives us,

1 foot-pound = 13,825 centimetre-grammes,
= $1{\cdot}356 \times 10^7$ ergs.

Consider now the case of a body which has been projected vertically upwards. It is moving against the force acting upon it, and work is therefore being done against this force. But if work is being done, there must be an equivalent loss somewhere. The equivalent loss we find in the lessening power which the moving body itself has of doing work. For it is abundantly evident that a body in motion has a power of *doing*. By impact it can set another body in motion or it can shatter it. Every effect the moving body so produces is at the expense of its motion; and when it ceases to move its power of energetic action is gone.

The power which a body or system of bodies has of doing work—that is, of being active—is called its **Energy.** In the case under discussion this energy is due to motion—and hence we call it **Kinetic Energy.** It is in virtue of its kinetic energy, originally acquired at projection—that is, imparted to it by some other energetic agent—that the body, so to speak, lifts itself against gravity. As it ascends its speed diminishes,

and this implies a corresponding diminution in the kinetic energy. The kinetic energy is, in fact, spent in doing work against the force that draws the body to the earth. Finally, when the body comes momentarily to rest, the original kinetic energy has been wholly expended, and if left to itself the body will begin to fall back again to earth.

The work done against gravity during the ascent of the body is measured by the product of the weight of the body into the height through which it is raised. If m is the mass, the weight is mg; hence the work done in raising m through the height h is measured by the quantity mgh. But if v is the speed of projection, we have (§ 22)

$$\tfrac{1}{2}v^2 = gh.$$

Hence,

$$\tfrac{1}{2}mv^2 = mgh.$$

This quantity $\frac{1}{2}mv^2$ is the equivalent of the work done in raising the body through the height h. It is therefore the measure of the kinetic energy of the body whose mass is m and whose speed is v.

§ 45. **Work done by a Force.**—As the body descends under the action of its weight, it moves in the direction of the force. In this case we speak of the force doing work on the body; and the *work done by the force* has its equivalent in the kinetic energy gained by the body. The equation is the converse of that given above, namely,

$$mg \cdot h = \tfrac{1}{2}mv^2,$$

or the work done by the force mg in acting through the distance h is equal to the kinetic energy gained by the body. Compare with this the previous interpretation of the equation —namely, the work done against the force mg through the distance h is equal to the kinetic energy lost by the body.

Work is done by or against a force only when there is displacement along the line of action of the force. When a particle describes a circle with uniform speed, the force is wholly normal—that is, it is always at right angles to the

direction of motion. Hence there is no displacement in the line of action of the force, no work is done either by or against the force, and there is no change in the kinetic energy of the particle.

In general, when a body acted on by the force F (fig. 30) is displaced to B, the work done against the force is measured by the product of F into Ab, which is the component of the displacement along the line of action of the force. If the body move to B′, the work done by the force is similarly measured by the product F.Ab'.

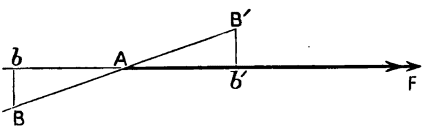


Fig. 30.

When a particle is describing a path under the action of a given force, it is the tangential component only which is involved in the gain or loss of the kinetic energy of the moving particle.

§ 46. **Potential Energy or Energy of Position.**—Kinetic energy has been defined as the power of doing work which a body possesses in virtue of its motion. This is the energy whose expenditure enables the body to lift itself against the force of gravity. But it is at once evident that a body raised to a certain height also possesses energy. By suitable connections we can make it draw another body up. This it does by itself descending. Thus we recognise that a body may have the power of doing work in virtue of its position. There is energy, but it is *potential*, not active. The higher the body is, the greater is this potential energy; for more work can be done by it as it is allowed to descend. If the body be allowed simply to fall, the potential energy diminishes as the kinetic energy increases; and we may regard this potential energy as having been originally imparted to it by the work done in raising it to the given height. We thus arrive at a new interpretation of the equation

$$\tfrac{1}{2}mv^2 = mgh$$

—namely, that the potential energy gained is equal to the kinetic energy lost, or *vice versâ*.

A more instructive way of putting this principle is as follows: Let H, h be successive positions, measured as heights above the earth's surface, of the mass m, and let V, v be the corresponding speeds. In the one position the kinetic energy is $\frac{1}{2}m\mathrm{V}^2$, and the potential energy is $mg\mathrm{H}$. Hence the total energy is

$$\tfrac{1}{2}m\mathrm{V}^2 + mg\mathrm{H}.$$

Similarly, the total energy in the other position is

$$\tfrac{1}{2}mv^2 + mgh.$$

But we know that (§ 22)

$$v^2 - \mathrm{V}^2 = 2g\,(\mathrm{H} - h),$$

for $(\mathrm{H} - h)$ is the distance through which the body has fallen as its speed has increased from V to v. Multiplying by $\frac{1}{2}m$, and rearranging, we get

$$\tfrac{1}{2}mv^2 + mgh = \tfrac{1}{2}m\mathrm{V}^2 + mg\mathrm{H},$$

or *the sum of the kinetic and potential energies is the same for all positions.* In other words, the total energy remains constant.

§ 47. **The Conservation and Transformation of Energy.**—The above is a simple example of the Conservation of Energy, the most important principle of physical science. Its complete significance cannot be grasped until we have studied the laws of heat, until we know, in fact, what becomes of the energy which is apparently lost to a material system. For instance, in the case just discussed, the interchange or **transformation** of potential into kinetic energy, or of kinetic into potential energy, does not as a matter of fact go on indefinitely. The mass will be arrested by the earth, and, even though it may rebound several times, will finally come to rest—the kinetic energy being altogether destroyed and there being no equivalent potential energy in its place; but it can be proved that the energy, though non-existent in either

of the dynamical forms named, still exists in the universe in a form more difficult to recognise, but none the less energy.

As a further illustration, consider the rhythmic motion of a simple pendulum. At the extremities of its range the pendulum possesses potential energy only; in intermediate positions part of the energy is kinetic. The kinetic energy attains its maximum when the pendulum is in its lowest position—that is, when the potential energy has its least possible value. As the pendulum oscillates to and fro there is a continual transformation going on of the one kind of energy into the other kind: as the one increases the other diminishes, and *vice versâ*. If it were not for the action of frictional forces in gradually destroying the motion, the pendulum would continue to oscillate to and fro indefinitely, its total energy remaining unchanged. By proper precautions we can approximate to this ideal case, and are always able to recognise the presence of a factor which prevents its complete realisation. As in the case of the falling body, so here—the energy that appears to be dynamically lost to the system is transformed into other less evident forms.

It is to be noted that in starting the pendulum we impart energy to it. We then consider it as forming with the earth a dynamical system which if left to itself will continue to act in the way described. So long as energy is neither added to nor taken from this system of earth and pendulum, so long will the energy of the system remain unchanged and the mutual transformations of kinetic and potential energies will continue indefinitely. The practical difficulty is to isolate the system from the rest of the universe. It is not merely difficult; it is impossible. Consequently, energy passes between the limited system contemplated and the environment, with which it unites to form a still larger system. Within this extended system, the energy, which in the more limited system has been apparently lost, is still present, though possibly not in a dynamical form. The energy is still conserved, but it has been transformed partly into a form which is no longer directly

available for producing mechanical effect. Within no *finite* system which can be experimentally or observationally studied by us can we expect to find the energy rigorously conserved. Nevertheless, we are able in many cases to elucidate the properties of the real system by reference to a similar ideal system complete in itself. To this ideal system we apply the principle of the Conservation of Energy, and the results obtained we find to be more or less faithfully reproduced in the corresponding real system.

The loss of energy in a form available for useful transformations is invariably associated with every dynamical transformation of energy that occurs. The 'energy' is, no doubt, conserved; but it has lost, in a measure, its *motivity* or availability for useful transformation. It has, in a sense, been degraded. The principle of the **Degradation** or **Dissipation** of **Energy**, first clearly stated by Kelvin, will be considered more fully under Heat.

§ 48. **The Potential Energy of a Body of Finite Size.**—The height to which a body is raised is one of the factors in the expression for the potential energy. If we experiment with a ball, or a cube, or a body of regular form, we easily discover how to measure this height; but should the body be of irregular shape, and should it be turned round in the act of being raised, how are we to measure accurately the height through which the body has been lifted? Generally speaking, different parts of the body will be lifted through different heights. Imagine the body cut up into small cubes or constituents, each one of which we may treat as a particle occupying its geometric centre. The work done in lifting any one constituent is measured by the product of the weight of the constituent and the vertical height between its centres. Form the products for all the particles and add them together. The sum will be the work done in lifting all the particles to their respective positions—that is, the work done in lifting the body as a whole into the position occupied by it. The ratio of this work done to the weight of the body will be the true height

through which the body is raised. Now it can be shown that there is a definite point in the body which is always raised through exactly this height. This point is popularly known as the Centre of Gravity; the more appropriate name is the Centre of Mass.

§ 49. **The Centre of Mass** or **Centre of Inertia** (known also as the **Centre of Gravity**).—Many of the properties of the centre of mass are familiar to all. For example, it is the point through which the whole weight of the body may be supposed to act. Hence, if a body be suspended by a string, the centre of mass will tend to take up a position vertically below the point of suspension (see § 52). Again, if a body be supported on an axis which passes through the centre of mass, any rotational motion of the body about that axis will not involve any work done against gravity; thus, a man may set in motion a huge flywheel which ten men could not lift. In so doing he overcomes only the inertia of the wheel. Whatever work is done against gravity in lifting certain parts of the wheel is balanced by the work gained because of the descent of other parts of the wheel, so that on the whole no work is done. This consideration, which is obviously in perfect accord with the statements of last paragraph, leads at once to the method for calculating the position of the centre of mass of any given distribution of matter. For simplicity, consider first the case of two particles which we may imagine to be rigidly connected by a rod of no weight; in other words, their distance apart is invariable. The problem is to find the position of their centre of mass. Let the masses be A and B (fig. 31). Now, if G be the centre of mass, any rotation of the system about G must involve no work done. If A descend through a distance a, and B rise through a distance b, then the

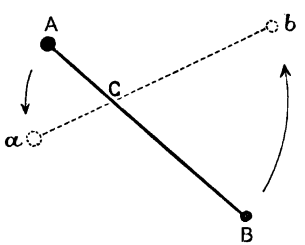


Fig. 31.

products Aa and Bb must be equal; for these quantities are proportional to the amounts of work done by and against gravity respectively. But clearly the vertical distances a and b are proportional to the radii of the arcs described by A and B in their motion about G. Hence, the position of G must be such that its distances from A and B are *inversely* as the masses. Thus, if A and B are as 2 to 1, the distances AG and BG are as 1 to 2—that is, AG is one-third of the distance AB. The general rule is obvious and may be thus enunciated:

The centre of mass of two particles is in the straight line joining them and divides this line into segments which are inversely as the corresponding masses.

The distance of either particle from the centre of mass bears to the distance between the particles the same ratio as the mass of the *other* particle bears to the sum of the masses.

By continuation of this process we can find the centre of mass of any number of particles. For any two we may substitute the combined mass at the centre of mass; and whatever be the number of particles we can always find a definite equivalent system of particles whose number is one less but whose total mass is the same. Thus, step by step, we can reduce the system to one single particle of mass equal to the whole, and this particle is situated at the centre of mass of the original system.

When the particles are so close as to form a practically continuous or apparently continuous body of finite size, an application of the same principle enables us to find the position of the centre of mass. The most important cases are those of uniform distribution of density. For example, a uniform rod has evidently its centre of mass at the middle point. A triangular plate, which may be regarded as built up of a succession of thin rods parallel to any side, must have its centre of mass in the line joining the middle point of this side to the opposite angle. But, similarly, the centre of mass will lie in the like lines which join the middle points of the other sides to the opposite angles; hence the centre of mass of a triangle is the meeting-point of these three *medians*, as they are called—a point which is known to be one of the points of trisection of all three.

§ 50. **Energy of Rotation of a Rigid Body.**—The kinetic energy of a particle of mass m and speed v is measured by the quantity $\frac{1}{2}mv^2$. A body of finite size so moving that every point of it has the same velocity has its kinetic energy measured in exactly the same way. But, in general, all the points of a moving body have not the same velocity. For example, in the case of a carriage-wheel, the uppermost portion of the wheel has the greatest speed; while the part that is in contact with the ground is momentarily at rest. There is in fact a rotational motion along with the translational motion.

A somewhat simpler case is the flywheel of a stationary engine. The wheel as a whole is rotating about a fixed axis. Every part of the wheel has a definite speed, which is proportional to its distance from the axis. Let ω be the speed of any point at unit distance (one foot) from the axis; or, as it is usually expressed, let ω be the angular speed. Then $r\omega$ will be the speed of the part distant r from the axis. The wheel may be regarded as built up of a great many small elements. Then if m_1 be the mass of one of these elements at distance r_1, its kinetic energy is

$$\tfrac{1}{2}m_1 (r_1\omega)^2 = \tfrac{1}{2}m_1 r_1^2 . \omega^2 .$$

The quantity ω belongs to the wheel, and has the same value whatever part be considered. Thus the kinetic energy of the element m_2 whose distance is r_2 is $\frac{1}{2}m_2 r_2^2 . \omega^2$; and so on for all the elements which build up the wheel. The kinetic energy of the whole wheel is the sum of the kinetic energies of all the constituent elements, or

$$\text{K.E.} = \tfrac{1}{2}\omega^2 (m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \ldots . + m_n r_n^2 + \ldots)$$

The quantity in the brackets measures what is called the **Moment of Inertia** of the body about the given axis. Its value depends not merely on the mass of the body but on the distribution of that mass relatively to the chosen axis. The farther removed the substance of a body is from the

axis of rotation, the greater is the moment of inertia, and the greater will be its kinetic energy for a given rate of rotation.

The following apparatus, designed by Smeaton the engineer, illustrates very simply the dynamical significance of moment of inertia. A vertical axis, working freely on pivots above and below, is provided with a horizontal cross-piece AB (fig. 32), on which two equal lead weights, M, M, can be fixed at various distances from the axis. Suppose for simplicity that the lead weights are first fixed at the extreme ends of the cross arms, and that in this configuration a certain quantity of rotational energy (E) is communicated to the system. Most of this will be possessed by the lead masses, the kinetic energy possessed by the light wooden framework being negligible by comparison. If a is the length of each arm, and ω the angular speed, we may write

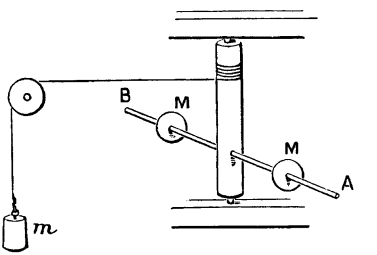


Fig. 32.

$$E=\tfrac{1}{2}(M+M)\,a^2\omega^2=Ma^2\omega^2.$$

Now, place the lead masses at the middle points of the arms, and let the same quantity of energy be again communicated to the system. If ω' is the angular speed in this configuration, then (to the same approximation)

$$E=M\left(\frac{a}{2}\right)^2\omega'^2$$

$$\text{Hence } \omega'^2=4\omega^2 \text{ or } \omega'=2\omega.$$

In the second configuration the system will rotate twice as fast as in the first configuration.

The most effective way of communicating a definite amount of energy to the system is by means of a falling body. A small mass (m) hangs at one end of a string, which passes over a pulley as shown in the figure. The other end of the string catches, by means of a loop, a small peg on the axis. By suitable rotation of the axis a certain length of string is wound upon it, and the small weight raised through a height equal to this length. The system is then left to itself. The weight descends and sets the apparatus

in rotation until all the string is unwound and the loop slips off the peg. The weight falls freely to the earth, and the apparatus is left rotating at a definite rate. The energy of rotation so acquired is the equivalent of a definite fraction of the potential energy lost by the weight during its fall through a distance equal to the length of the string initially wound on the axis. Another but much smaller fraction of the potential energy lost has its equivalent in the kinetic energy of the descending mass, and part is spent in overcoming frictional forces. Now, it is a simple matter to use always the same length of string in starting the rotation. Under these conditions, whatever be the configuration of the lead masses, very nearly the same fraction of the potential energy of the falling mass will be communicated to the system. With the two configurations already described we should find by direct measurement of the rates of rotation that this rate was very approximately doubled when the masses were placed at half their original distance. In the apparatus as constructed by Smeaton the measurement of the rate of rotation is facilitated by means of a gearing of toothed wheels which set an index or pointer rotating above a graduated dial.

§ 51. **Activity, Power, or Rate at which Work is being done.**—It is important to distinguish carefully between work done and rate at which work is done—between, for example, a foot-pound and a horse-power. The total amount of work done in raising a mass through a given height is always the same whether it be done in a short or in a long time. But in many practical cases it is important to have the work done quickly. A small force acting through a long distance will do as much work as a large force acting through a short distance. But if, as is generally the case, they take different intervals of time to do this work, they are working at different rates. The practical value of any machine or engine depends in great measure upon the rapidity with which it can do a certain amount of work. A sewing-machine works much more rapidly than the most accomplished seamstress; its *activity* or *power* is greater. In like manner the steam-engine can accomplish far more foot-pounds of work per second than the most powerful horse.

Before the steam-engine was invented the horse was the

most important agent for doing heavy work. Hence the unit of power adopted by engineers was defined in terms of the work a strong horse could do in one minute. This was believed to be about 33,000 foot-pounds;* and the **Horse Power** is now accurately defined as being equivalent to 33,000 foot-pounds per minute or 550 foot-pounds per second. This is the unit used in measuring the power of all engines. Thus, an engine of 500 horse-power is an engine which can work at the rate of 550 × 500 or 275,000 foot-pounds per second, or nearly 1230 foot-tons per second. In one second such an engine could raise 1 ton through 1230 feet (nearly a quarter of a mile).

In a scholium to the Third Law of Motion, Newton gives a second interpretation of the law that Action and Reaction are equal and opposite. The literal translation is as follows: 'If the action of an agent be measured by its force and its velocity conjointly; and if, similarly, the reaction of the resistance be measured by the velocities of its several parts and their several forces conjointly, whether these arise from friction, cohesion, weight, or acceleration, action and reaction in all combinations of machines are always equal and opposite.' The product of the force and the velocity of an agent is what we now call the rate at which the agent is doing work. The full significance of Newton's second interpretation, which strictly speaking is a Fourth Law, seems to have escaped notice till 1863, when Thomson and Tait showed that it was equivalent to the following statement: Work done on any system of bodies has its equivalent in work done against friction, molecular forces, or gravity, if there be no acceleration; but if there be acceleration, part of the work is expended in overcoming the resistance to acceleration, and the additional kinetic energy developed is equivalent to the work so spent. The work done against molecular forces, as in bending a bow, or against gravity, as in raising a weight, can be recovered at some future time; but the work done against friction is practically lost to the system. As we now know, it has been transformed into heat, and in this form the energy is no longer so available for useful dynamical transformations.

* This, however, was an overestimate: 22,000 foot-pounds per minute is a better approximation to the power of a horse.

EXERCISES.

1. A railway-train of 50 tons has its speed cut down from 60 miles per hour to 10 miles per hour in a space of 2 furlongs. Find the average resistance in tons-weight, and express the work done in foot-poundals.

2. Compare the momenta and kinetic energies of an ounce bullet moving at 500 feet per second and a 14-lb. ball moving at 50 feet per second.

3. If two masses have equal momenta, show that the smaller mass must have the greater kinetic energy.

4. A rope, passing over a smooth pulley, supports a man weighing 10 stone at the one end and at the other a mass of 1 cwt. If the man draws himself up 50 feet by pulling on the other part of the rope, find the work done against gravity.

5. A flywheel of mass 200 lb. and effective radius 3 feet is rotating five times per second. Find the work done in bringing it to rest. If this is accomplished in 2 seconds, what is the horse-power of the agent?

6. The moment of inertia about the axis of a uniform cylinder of radius a and mass m is $\frac{1}{2}ma^2$. What is the whole kinetic energy of a cylinder which is rolling along a horizontal surface at the rate of 6 miles per hour, the mass of the cylinder being 5 lb. and its radius 2 feet? [The energy consists of two parts, translational and rotational. The former is that of the whole mass moving with the speed of the centre of mass; and the latter is the rotational energy about the axis through the centre of mass.]

SUMMARY.

§ 44. Work done against a Force.—Foot-pound; foot-poundal; kilogrammetre; erg. Energy. Kinetic Energy.

§ 45. Work done by a force.

§ 46. Potential Energy or Energy of Position.—Constancy of sum of potential and kinetic energies.

§ 47. The Conservation of Energy.—Degradation of Energy.

§ 48. Potential Energy of a body of finite size.

§ 49. Centre of Mass (of Inertia or of Gravity).—Case of two particles. Extension to any number. Solid bodies.

§ 50. Energy of Rotation of a rigid solid.—Moment of Inertia. Smeaton's Machine.

§ 51. Activity or Rate at which Work is being done.—Horse-power. Newton's Second Interpretation of the Third Law.

CHAPTER VI.

STATICS—EQUILIBRIUM.

§ 52. A body at rest is said to be in equilibrium under the action of the various forces acting upon it. The most important force with which we have practically to deal is the *weight* of the body; and this, we have seen, acts through its centre of mass. Accordingly, in questions relating to the equilibrium of bodies at the earth's surface, the Centre of Mass plays an all-important rôle. In the simplest cases of equilibrium, in which motion of the body as a whole is considered, we may treat the body as a particle situated at the centre of mass; for all the forces the weight and the various pressures or tensions which may be present may be reduced to forces acting through this point and having zero resultant.

For example, a body suspended by a string is held in equilibrium, the weight of the body acting downwards being balanced by the equal and opposite tension of the string acting upwards. Imagine the string continued downwards. This continuation will pass through the centre of mass (fig. 33). And the centre of mass will always lie in the same line with the supporting string whatever be its point of attachment to

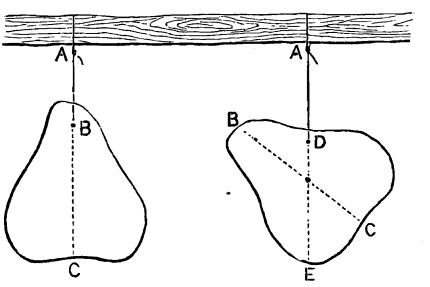


Fig. 33.

the body. This principle enables us readily to find the position of the centre of mass of a body of any shape. We simply have to suspend it first by one point (B) and then by another (D), and consider where the vertical line (DE) through the latter intersects the line (BC) which was the vertical line through the point of suspension in the first position.

In the case of a body resting on a surface, the reactionary pressure must balance the weight. This reactionary pressure acts through the common surface or point of contact. If there are several distinct points of contact, as in the case of a sphere resting in a V-groove (fig. 34), the several pressures acting at these points will have a resultant which is equal and opposite to the weight. For a sphere resting on a horizontal plane there is but one point of contact; and the case is exactly similar to the sphere hanging by a string. Such a sphere or ball if uniform will rest in any position on the horizontal plane. However it may be placed, the pressure will act upwards through the point of support and therefore through the centre of mass. Hence the weight and pressure will always balance.

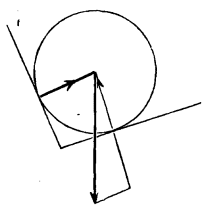


Fig. 34.

When a body is placed upon an inclined plane, the weight has a component acting down the surface. To keep the body in equilibrium a force must be applied acting up the plane. There are then three distinct forces acting on the body—namely, the weight acting vertically down, the reactionary pressure of the inclined plane acting perpendicular to the plane, and the applied force acting up the plane. The condition of equilibrium requires that these three forces can be arranged as the sides of a triangle. The construction, which is left as an exercise, is simple and gives at once the relations among the three forces. We may, however, arrive still more simply at the relation between the applied force and the weight by use of the principle of work. For clearly

(see fig. 35) the work done by the force P in pulling W upwards along the inclined plane (assumed to be smooth) is equal to the work done in lifting the body against its weight. Hence, if l is the length of the inclined plane and h its height, we get the relation

$$Pl = Wh.$$

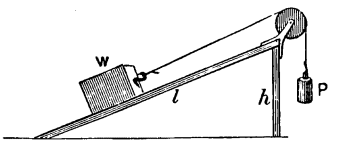


Fig. 35.

P is smaller than W in the ratio of the height to the length of the plane. It is thus possible to raise a very heavy load by application of a comparatively small force. The smaller the angle of inclination of the inclined plane, the smaller the force required to raise the body to a given height.

§ 53. **Stable, Unstable, and Neutral Equilibrium.**—Suppose, now, that a sphere is placed on a concave surface, say, inside a bowl. If left otherwise free, it will come to rest in the lowest possible position. In this position A (fig. 36) the upward pressure will as in the former case balance the weight. But now displace the sphere to position B. In this position the sphere cannot rest. The reactionary pressure through B cannot possibly balance the weight acting at the centre of mass C; for two forces have zero resultant only when they are equal and oppositely directed *in the same line*—clearly impossible in this case, since the point B through which the one force must act lies quite outside the line of action of the other force.

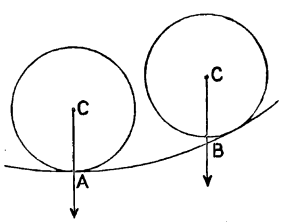


Fig. 36.

Contrast with this the case of the same sphere placed on a convex surface. Clearly there is here also one position A (fig. 37) in which the point of contact through which the reactionary pressure must act lies in the vertical line through the centre of mass. Yet we know by experience that the

sphere will not rest permanently in this position. In any other position, such as B, the forces cannot balance, just as in the case of the concave surface. But there is an important difference between the two cases, which is readily understood when we consider how the weight acts upon the body with reference to the point of contact. Under the action of its own weight the tendency of the body is to fall; and the effect of the weight is to produce in the body that motion which is associated with the descent of the centre of mass. In both cases the sphere descends. In both cases it moves towards that side of the point of contact on which the vertical line through the centre of mass lies. In the case of the concave surface the sphere is brought back to the position in which weight and pressure are equal and opposite. In the case of the convex surface the sphere falls farther and farther away from this position. In both positions the condition of equilibrium is satisfied; the forces have zero resultant. A very slight displacement destroys this equilibrium. In the one case the combination of forces brought into play resists the displacement and brings the sphere back to its position of equilibrium. This kind of equilibrium is distinguished as *Stable Equilibrium.* It is the truly practical kind. In the other case, however, the combination of forces brought into play by a slight displacement acts with the displacement and increases it, so that the sphere falls away from its position of equilibrium. This kind of equilibrium is called *Unstable Equilibrium.* When the sphere is resting on a horizontal plane the equilibrium is said to be *Neutral.* A slight displacement calls into play no unbalanced force.

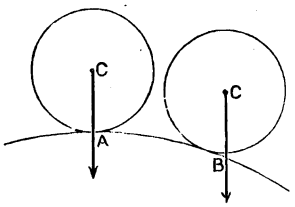


Fig. 37.

The distinction between these kinds of equilibrium may be otherwise stated thus: In stable equilibrium any slight displacement involves work done against gravity—the sphere is

raised—the potential energy is increased. In neutral equilibrium no work is done during a displacement either against or by gravity—the sphere is neither raised nor lowered—the potential energy is unchanged. In unstable equilibrium any slight displacement involves work done by gravity—the sphere is lowered—the potential energy is diminished. Note carefully that equilibrium to be truly stable must be so for all possible displacements. If there exists a single displacement which involves a diminution of potential energy, the configuration is one of instability. For example, a sphere placed on a saddle-back is in unstable equilibrium, although for certain displacements the potential energy is increased.

Briefly put, the condition for stable equilibrium is that **the potential energy be a minimum**; and in this form the principle is applicable to complex systems of bodies. If a change of configuration is possible which involves a decrease of potential energy, that change will occur. The body or system of bodies will always tend to assume a configuration of minimum potential energy.

When the vertical line through the centre of mass of a body falls outside the base on which the body rests, the body is not in equilibrium. It will topple under the influence of the moment of its weight about the point of the base which is nearest to the vertical line passing through the centre of mass. In such a position the centre of mass cannot move without descending. In all cases stability requires that the vertical projection of the centre of mass lie within the base, or, if the body be resting on three points, within the area of the triangle formed by lines joining these points.

§ 54. **Equilibrium as regards Rotation.**—On trying the experiment of the ball displaced from its position of rest in the bowl we find that the ball in general *rolls* back to its position of stable equilibrium. It is necessary, therefore, to consider the rotational as well as the translational equilibrium of a body.

To simplify the problem, let us consider the centre of

mass of the body to be fixed, and let forces act at given points of the body. We shall confine our attention mainly to forces in one plane; but it is not difficult to extend the investigation to the more general case of forces in space.

And, first, we define what is meant by the **equivalence** of two forces or of two systems of forces. Two systems of forces are said to be equivalent to one another when, under the prescribed conditions, the work done by or against the one system of forces during any small displacement is equal to the work done by or against the other system under exactly similar circumstances.* If then, as in the case under discussion, the body is in equilibrium under the action of a given number of forces, the equivalent of these forces may be taken to be zero. Hence the work done during any small displacement is zero also.

Thus, let O (fig. 38) be the fixed point, and let A, B be the points at which the forces P, Q are acting. For any small rotation about O, A moves to A′ and B to B′, and the angles AOA′, BOB′ are equal. Hence

$$AA' : OA = BB' : OB.$$

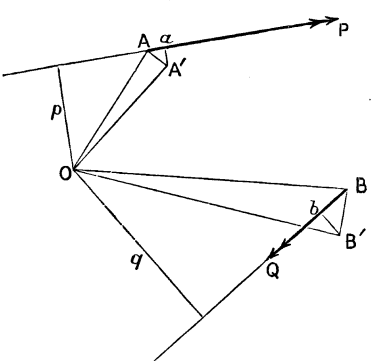


Fig 38.

The work done by P during the displacement is P.Aa, where Aa is the resolved part of AA′ parallel to P. For the same reason Q.Bb is the work done by Q. Hence, by the principle enunciated above, we must have

$$P.Aa + Q.Bb = 0.$$

But if p, q be the perpendiculars from O on the directions

* A little consideration will show that this principle includes the case of forces acting on a particle. The *resultant* of any number of forces acting at a point is the *equivalent* of these forces.

of P and Q, it is easily proved by similar triangles that for very small displacements

$$AA' : OA = Aa : p$$

and $$BB' : OB = Bb : q.$$

Hence also $$Aa : p = Bb : q.$$

In the work equation we may put for Aa, Bb any quantities to which they are proportional. Hence, finally,

$$Pp + Qq = 0.$$

If there is a third force R acting at distance r, then the condition for equilibrium is

$$Pp + Qq + Rr = 0,$$

and so on for any number of coplanar forces.

Note particularly that in this equation the positions of the points A and B originally chosen are of no importance. Any point in the line of action of P can be substituted for A. What is of importance is the distance of this line of action from the fixed point.

The product of a force into its distance from a point is called the **Moment of the Force.** It is positive or negative according as the force tends to rotate the body in the one sense or in the other. If there is equilibrium, the forces cannot all tend to rotate the body in the same sense. In the case of two forces, one must have a negative moment and the other a positive moment. This is shown at once by the equation given above—namely, $Pp + Qq = 0$, or $+Pp = -Qq$.

§ 55. **General Conditions of Equilibrium.**—When a body is in equilibrium under the action of a given system of forces, the equilibrium is not destroyed *if any point be fixed.* But when any one point is fixed, the condition for equilibrium is that the sum of the moments of all the forces about that point vanishes. Hence we conclude that a system of forces will keep the body in equilibrium when the sum of the moments about *any point whatever* vanishes.

As an example of the application of this important principle, let us find the condition of equilibrium for a number of parallel

forces acting at given points along a rod. Let there be four forces P, Q, R, S acting at A, B, C, D (fig. 39). Take any point O

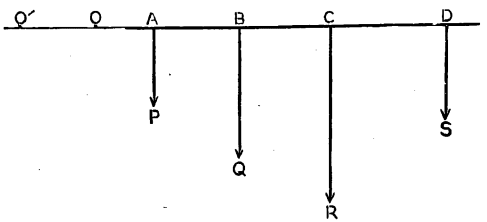


Fig. 39.

in the line of the rod. Then for equilibrium we have

$$P.OA+Q.OB+R.OC+S.OD=0.$$

Similarly, if O′ be any other point, we get

$$P.O'A+Q.O'B+R.O'C+S.O'D=0.$$

Subtracting the first from the second we find

$$P.O'O+Q.O'O+R.O'O+S.O'O=0,$$

or

$$P+Q+R+S=0.$$

In words, the algebraic sum of the forces is equal to zero—a condition which obviously must exist to prevent translational motion. The other condition, that the sum of the moments about any one point be equal to zero, is necessary if there is to be no rotational motion.

Since any one of these forces balances the others, it follows that these others have a resultant equal and opposite to the one which they unitedly balance. Hence follows at once the rule for finding the resultant of a system of parallel forces. For example, let P, Q (fig. 40) be two parallel forces acting at the points A,B. Their resultant (R) must be such that, when reversed, it balances P and Q—that is,

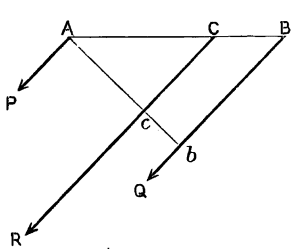


Fig. 40.

$$P+Q-R=0 \text{ or } R=P+Q.$$

Then again the moment of R about any point is equal to

the sum of the moments of P and Q about the same point. Let A be the point about which moments are to be taken. Draw Ab perpendicular to Q. Then if the resultant R passes through the points C, c, in AB, Ab respectively, we have by the equation of moments

$$R.Ac = P \times 0 + Q.Ab = Q.Ab$$

or, (since $Ac/Ab = AC/AB$)

$$(P+Q)\ AC = Q.AB,$$

which gives at once

$$AC = \frac{Q}{P+Q}.AB,$$

and, after a simple transformation,

$$P.AC = Q\ (AB - AC) = Q.BC.$$

Either of these relations enables us at once to find the point C through which the resultant force must act. It is usual to call C the centre of the parallel forces acting at A and B. The weights of all the elementary masses which may be supposed to build up a body at the earth's surface form a set of parallel forces, proportional to the masses of the corresponding elements, and having their centre coincident with the centre of mass of the body. At this centre, accordingly, the resultant or total weight acts.

§ 56. **The Couple.**—A very curious case arises when the system of forces is reducible to two equal and opposite forces acting in parallel lines. Let P, – P (fig. 41) be two such forces. The resultant R = P – P = 0. The moment about any point O is

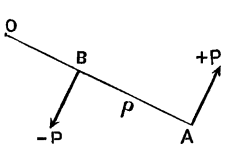


Fig. 41.

$$K = OA.P - OB.P$$
$$= AB.P = pP,$$

where OBA is the common perpendicular from O to the forces, and p is the distance between them. The value of K is independent altogether of the position of O. This peculiar system has the same moment about every point in its plane and indeed about every point in space. Such a combination

of forces is called a **Couple.** Couples differ as regards the planes in which they act and as regards their moments. The characteristic of a Couple is in fact its moment, which can be expressed as the product of a force and a distance. The couple may be represented in an infinite number of ways by means of a pair of equal and opposite forces at the suitable distance apart. Thus we can put

$$K = Pp = Qq = Rr = \ldots\ldots \&c.,$$

where *pqr* are distances separating pairs of equal and opposite forces, $\pm P, \pm Q, \pm R$—the distances being inversely proportional to the corresponding forces (fig. 42). In the

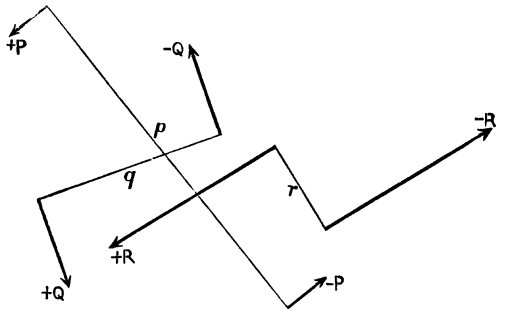


Fig. 42.

figure each combination represents the same couple. A couple can be balanced only by an equal and opposite couple.

§ 57. **Friction.**—To produce relative sliding motion of two bodies in contact requires the application of a finite force in the direction in which the sliding is to take place. Such a force acts in a direction tangential to the surfaces which are in contact. The particular value of force necessary to produce sliding depends on the character of the surfaces and on the pressure with which the surfaces bear upon one another. If a force smaller than this be applied there will be no sliding, the one body will be at rest relatively to the other. Hence there must have come into existence an equal and opposite force

acting tangentially to the surfaces in contact. This reaction is known as the **force of friction.** So long as there is no motion the force of friction called into exercise is just sufficient to balance the applied force. Up to the particular value already referred to, the reaction due to friction grows with the force applied. But when this value is exceeded the force due to friction ceases to increase with the applied force. There is an excess of force in one direction and the body moves.

In the present chapter we shall confine our attention mainly to friction as preventing motion; in other words, to **Statical Friction.**

Let us suppose a block of wood to be resting on a table, and a horizontal force to be applied to it by means of a string which passes over a pulley and has a load attached to the other end (fig. 43). This load is adjusted so that a slight increase will make the block begin to slide on the table. The load so obtained is a measure of the maximum force of friction between the block and the table. If Q is the weight of the block, and P the maximum load that can be supported without the block beginning to move, then the ratio P/Q is the coefficient of friction. Within considerable ranges this coefficient is found to be constant. Thus, if we put an extra mass on the top of the block so as to increase the pressure between the block and the table, we are able to add to the load P a proportional increment. By experiments of this kind we establish the law of statical friction—namely:

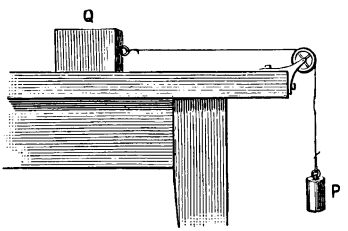


Fig. 43.

The maximum force of friction between two given surfaces bears a constant ratio to the pressure between the surfaces. This ratio is called the Coefficient of Friction.

Another and very instructive mode of considering the problem is the following: Set the block on a horizontal board, and then gradually incline the board more and more by raising one end. The block remains at rest until a particular inclination is reached, when it begins to slide down the inclined plane. Let α be this critical inclination, and let h and l be the height and length respectively of the inclined plane (fig. 44). Resolve the weight W into two forces: P, normal to the surfaces in contact, and Q, tangential to these surfaces. Then clearly the force of friction, F, must be equal and opposite to the tangential force Q. Hence (compare § 52)

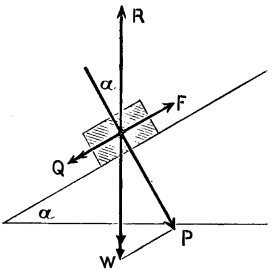


Fig. 44.

$$\frac{F}{W} = \frac{Q}{W} = \frac{h}{l}$$ by similar triangles = sin. α, by trigonometry.

By similar reasoning we find $\frac{F}{P} = \text{tan. } \alpha$,

so that tan. α is the measure of the coefficient of friction, being the ratio of the tangential to the normal force. Otherwise thus: In order that the block may rest in equilibrium the total reaction (R) of the plane must be equal and opposite to the weight. This reaction may be decomposed into a normal component $(-P)$ and a tangential component (F or $-Q$). When the maximum friction has been developed, the ratio of F to $-P$ measures μ, the coefficient of friction, and the reaction R makes with the normal to the surfaces an angle whose tangent is μ. This angle is called the **angle of friction.** It is known when the coefficient of friction is given, and, when it is known, it becomes a simple question of geometry to draw the limiting directions within which the reaction between two surfaces must lie.

For example, let the problem be to discuss the limiting position of equilibrium of a ladder resting against a wall in the usual way,

the coefficients of friction between the ladder and ground and between the ladder and wall being given. First calculate the angles of friction corresponding, and suppose the ladder just to be on the point of slipping, so that the friction is fully developed at both ends. The reactions must act so as to resist the tendency to motion. Through A (fig. 45), the lower end of the ladder, draw a line making the appropriate angle (the angle of friction between ladder and ground) with the perpendicular to the ground—that is, with the vertical; and through B draw a line making the proper angle (the angle of friction between ladder and wall) with the perpendicular to the wall—that is, with the horizontal. These lines will meet in some point S. Then AS, BS give the *directions* of the reactions when the ladder is just on the point of slipping. The condition of equilibrium requires that the weight, W, of the ladder must be balanced by these two reactions. Hence all three must meet in a point; and G, the centre of mass, must be vertically below the point S.

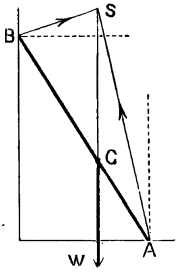


Fig. 45.

If G lies to the left—that is, too high—the three forces will form a triangle and will produce a couple tending to rotate the ladder counter-clockwise. The ladder can evidently yield to this couple by slipping down. On the other hand, if G lies to the right—that is, lower on the ladder—the moment of the triangle of forces so formed will act clockwise; but to this the ladder cannot yield without moving up. Now, an *upward* motion is out of the question, for frictional forces never acquire greater values than are just necessary for equilibrium. In fact, if G is moved to the right the point S must follow, which simply means that one or both of the reactions act at angles *less* than the limiting angles of friction. The obvious conclusion is that a ladder in a given position will be less liable to slip the lower the centre of mass is. A ladder resting against a wall will therefore be rendered more stable when a man stands on it at any point lower than its centre of mass; but, as the man ascends, the stability diminishes because the centre of mass of the whole system rises, and if the inclination is originally too near the limiting position the ladder will slip down when the man has reached a certain height on it.

Friction is a property of matter of the greatest practical importance. Without it many of our commonest actions, such

as walking, running, pushing, &c., would be impossible. It is the frictional grip of wheel on rail which enables our locomotive engines to move at all; it is friction which makes the nail so thorough a fastener, and which, in general, gives stability to objects resting on approximately horizontal surfaces.

A very common method of transferring rotational motion from one set of wheels and axle to another is a gearing of belts or ropes (see § 60). One condition that makes this kind of gearing possible is the great friction that exists between any curved surface and a rope or belt stretched tightly over it. The tension of the rope determines the pressure between it and the surface, and the maximum force of friction that can be brought into play to resist slipping is equal to the coefficient of friction multiplied by the pressure. Thus it is possible for one man holding the one end of a hawser which has been coiled once or twice round a post or 'belaying-pin' to balance the pull exerted on the other end by a ship in motion. The difference of the tensions on the two parts of the rope is balanced by the frictional force acting tangentially all round the line of contact of the rope and post. For every additional coil the maximum pull that can be in this way balanced increases in a definite ratio. For example, if one coil enables the man pulling with a force F to balance a pull of 10 F on the other part of the rope, two coils will enable him to balance 100 F, three coils 1000 F, and so on. On the same principle, a few turns of the cord round a peg prevents the descent of a heavy Venetian blind.

EXERCISES.

1. A mass of 10 lb. suspended at the end of a string is pulled aside by means of a second string, which is always kept horizontal, until the first string makes an angle of 45° with the vertical. Find the tensions on the strings.

2. A uniform rod, 2 feet long, weighs 5 lb. A string is tied to it at a distance of 5 inches from the middle. Where must a weight

of 4 lb. be made to act so that the rod, when suspended by the string, is horizontal?

3. The same rod is set on two supports distant 7 inches and 9 inches from the middle point which lies between them. Weights of 2 and 3 lb. act at the ends of the rod. Find the pressures on the supports.

4. A cylindrical column, 6 inches high and 3 inches in diameter, rests on an inclined plane, which is gradually tilted until the column topples. What do we infer as to the value of the angle of friction?

5. A body weighing 3 lb. slides down a plane, which is inclined 30° to the horizontal, with an acceleration of 2 feet-per-second per second. Find the frictional force between it and the plane; and thence calculate the work done in pulling the body up the plane to a height of 10 feet.

6. A rope passing over a horizontal cylinder is just on the point of slipping when a mass of 10 lb. is attached to the one end and a mass of 15 lb. to the other. Assuming the friction to be constant, find the acceleration of the system when the masses on the two sides are 10 lb. and 20 lb.

7. Find the limiting positions of equilibrium when a body, resting on a rough inclined plane, is attached by an elastic string to a point in the plane.

8. Find the size of a nut which will just escape from a given pair of crackers.

SUMMARY.

§ 52. Equilibrium of a Body. Reactionary Tensions and Pressures. The Inclined Plane.

§ 53. Stable, Unstable, and Neutral Equilibrium. Energy condition for Stable Equilibrium.

§ 54. Equilibrium as regards Rotation. Equivalence of systems of Forces. The Moment of a Force.

§ 55. General Conditions of Equilibrium.

§ 56. The Couple.

§ 57. Friction. Static and Kinetic. Coefficient of friction. Angle of friction. Case of ladder resting against wall. Great importance of friction. Belts and ropes coiled round cylinders. Belay.

CHAPTER VII.

DYNAMICAL ILLUSTRATIONS.

To emphasise the importance of the dynamical principles that have just been established, it is convenient to consider some familiar applications. As a matter of fact all forms of machine give endless illustrations, and none more forcibly than the very simplest of these. The **Inclined Plane** has been discussed already in §§ 52, 57.

As a further example, consider the lever.

§ 58. **The Lever**, in its many important forms, consists essentially of a rigid rod supported at some point on a prop called the fulcrum. At some other point a force acts so as to balance or overcome a second force acting at a third point.

In the first place, suppose the fulcrum to be at the centre of mass of the rod; then, clearly, for the balance of the two forces, it is necessary that their moments about the fulcrum be equal and opposite. The two forces are generally known as the **Power** and the **Resistance** (or Weight), the Power being the force that is applied by the agent, the Resistance being the force that is balanced or overcome.

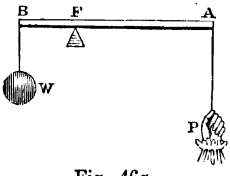


Fig. 46*a*.

It is usual to distinguish three different kinds of levers according to the position of the fulcrum relatively to the power and resistance; but the classification is misleading, since all forms of levers are not included.

The typical form of straight lever is shown in fig. 46*a*, in which the fulcrum, F, lies between the points of application of the

power and resistance (P and W). The see-saw, the crowbar in some applications, various forms of balance and steelyard, the spade in digging earth, a pair of scissors—all are of this type. Then, in figs. 46*b* and 46*c*, we have the power and resistance on the same side of the fulcrum. Since the moments about the fulcrum must be equal and opposite, the forces themselves must in these cases act in opposite directions. To the second type here figured belong the crowbar in some of its uses, boat-oars (the blade in the water being the fulcrum), wheelbarrows, nut-crackers, and certain forms of pump-handle. The power is always less than the resistance, since it is farther from the fulcrum.

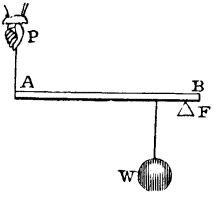


Fig. 46*b*.

In the third type, to which such machines as fishing-rods and coal- and sugar-tongs belong, the power is necessarily greater than the resistance; but the advantage of such an arrangement is that the resistance is overcome through a distance greater than that through which the power acts. The importance of this is shown by the range through which our arms work. The power that works the forearm is supplied by the biceps muscle, which is attached to the bone a little in front of the elbow-joint. From the shoulder to the tips of the fingers, the limb is a series of levers of approximately the same type. The nearer the joint is to the free extremity, the less is its mass, so that its motions are more easily effected and more quickly controlled.

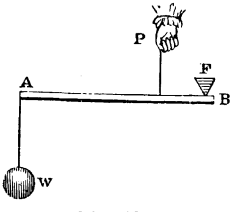


Fig. 46*c*.

As an instance of a lever that is included in none of these typical kinds take a hammer when used to extract nails (fig. 47). The power is applied near the end of the handle in an approximately horizontal direction. The resistance overcome acts downwards;

the fulcrum, being where the head of the hammer rests on the board, remains practically at a constant distance from the power, but changes its position relatively to the resistance as the nail is being drawn out. When the power is just on the point of overcoming the resistance, the moments of the two about the instantaneous fulcrum are—as in the previous cases—equal and opposite.

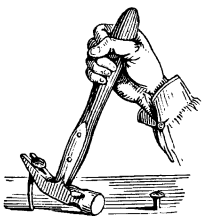


Fig. 47.

So far we have supposed the fulcrum to be at the centre of mass, or at least so situated that the weight of the lever itself had no moment about the fulcrum. Even though either of these conditions be not fulfilled, the weight of the lever may be, and often is, negligible in comparison with the power and resistance. In many cases, however, it is necessary to consider the weight of the lever as a third force acting at the centre of gravity. This is particularly the case in all forms of balance.

§ 59. **The Common Balance.**—Let us consider as a type the common balance (fig. 48) of equal arms. At the extremities of the beam AB, which rests by a knife-edge on a surface at C, scalepans are hung. In the position of equilibrium, before any masses are placed in the scalepans, the centre of mass of the beam will be vertically below C, say at the point G. Any tilting of the beam will displace the point G out of the vertical line through C, and hence the weight acting at G will have an appreciable moment about the fulcrum. Let P, Q be the weights of the masses placed in the scalepans, and let us suppose that Q is the greater. Then, if D be the distance of each from the fulcrum C, and if d be the distance of G′

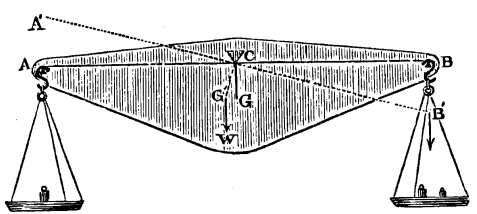


Fig. 48.

from the vertical line through C, the principle of moments gives

$$PD + Wd = QD,$$

or

$$Q - P = Wd/D.$$

Hence $Q = P$ only when d vanishes—that is, when G lies in the vertical line through the fulcrum.

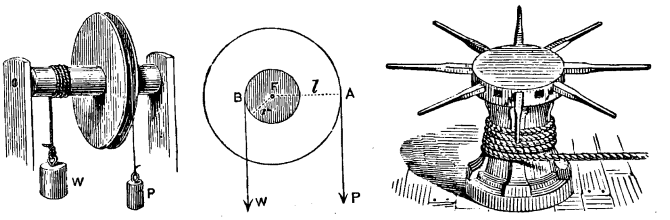
A balance, in addition to its truth or accuracy, has two important qualities that merit special notice—namely, *sensibility* or *sensitiveness* and *stability*. The balance is sensitive when it responds to a very slight excess in weight of one mass over the other. By means of a pointer moving in front of a scale, the eye is able to appreciate a certain minimum deviation of the beam from horizontality. For a given value of CG, there will be a definite minimum value of d which is appreciable. Hence, that $(Q - P)$ may be as small as possible, we must have D as great as possible. Thus, other things being equal, *the longer the arms of a balance the greater is the sensitiveness.* But again, if the lengths of the arms be kept constant, $(Q - P)$ will be smaller—that is, the sensitiveness will be greater—the smaller W is or the smaller d is for a given tilt. Hence, *the lighter the beam the greater the sensitiveness;* and *the nearer the centre of mass to the fulcrum the greater the sensitiveness.*

Stability means the promptitude with which the balance, after any disturbance, returns to its position of equilibrium. Upon this quality depends the rapidity with which the operation of weighing can be accomplished. If the weights P and Q are very nearly equal, it is the weight W which is mainly instrumental in bringing back the beam to its position of equilibrium. And the effect of W will be greater (1) according as W is itself greater, (2) according as the distance CG is greater. These conditions are quite opposed to the conditions for sensibility. Again, the longer the arms the greater will be the moment of inertia of the masses about the

fulcrum, and the more slowly will the balance respond to the couple acting on it in virtue of the displacement of G from the position of equilibrium. Hence stability is increased by shortening the arms, other things being equal.

Thus it is impossible to combine great sensitiveness with great stability. In the Chemical Balance the desideratum is sensitiveness; and with it accurate weighing is a long and laborious process. In the approved modern form of Chemical Balance the arms are made short, thereby securing fair rapidity of oscillation; while sufficient sensitiveness is obtained by adjusting the position of the centre of mass.

§ 60. **The Wheel and Axle** is a modification of the lever, in which the forces are applied at the circumferences of wheels or cylinders having a common axis. Simple forms are shown in fig. 49. Fig. *b* is a diagrammatic sketch of



a *b* *c*

Fig. 49.

fig. *a*, and shows at a glance that P is to W as r to l, that is, inversely as the radii of their respective circles. Fig. *c* represents the capstan, the power being applied manually at the extremities of the radiating arms, while the weight acts along the rope.

In many machines rapid rotatory motion is obtained by means of belts which pass round the large driving-wheel and a comparatively small wheel fixed to a cylindrical shaft. The rotational energy of this shaft, which is virtually a wheel and axle, can then be utilised in a variety of ways. Wheel and axle combinations also occur in which the power is

applied by means of cogs or toothed wheels. These are well seen in common clocks and watches.

§ 61. **The Pulley** consists essentially of a wheel grooved on the circumference and movable on an axis. In practical forms of pulley the wheel or **sheave**, as it is called, is contained in a hollow case or **block**. Usually there are two or more sheaves in the same block, the combination forming what is known as a 'purchase-block.' A 'single block' contains only one sheave, round which a rope or chain passes; and its function is simply to change the direction of pull along this rope or chain. By a suitable combination of single pulleys, or more generally by the use of purchase-blocks, the apparatus fulfils an important mechanical purpose, in virtue of which it is possible to sustain or lift a heavy mass by the action of a comparatively small force.

A usual type of purchase-block is shown in fig. 50*a*. Here there are two blocks with four and three sheaves respectively. A continuous cord, with its one end fixed to the top of the lower block, passes round the sheaves of the upper and lower blocks alternately, and to the other end a force P is applied. To the lower block a weight W is attached; and this weight will be distributed uniformly over the seven supporting stretches of the cord. Hence, if P is $\frac{1}{7}$ of W, there is equilibrium. Practically because of friction a force less than $\frac{1}{7}$W will sustain W. But, for the same reason, if W is being raised, a tension appreciably greater than $\frac{1}{7}$W must be applied along the free stretch of the rope. Nevertheless, even with the loss of energy due to imperfect flexibility and

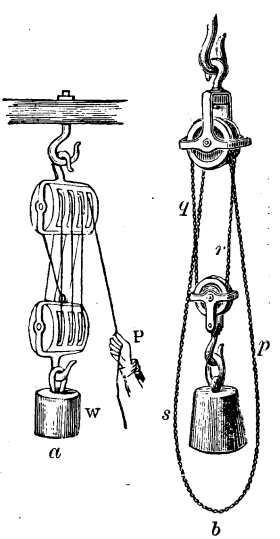


Fig. 50.

friction, there still remains in all practical cases a distinct mechanical advantage, so that one man using the blocks is able to lift a mass which several men could not move.

Another important type is the **Differential Pulley,** so called because its action depends upon the difference of the radii of the two unequal sheaves included in the same block, or, to express it more accurately, the difference of the radii of the two grooves of the double sheave which constitutes the apparatus. It is shown in fig. 50*b*. The differential block is the upper one; and a continuous endless chain passes round both the upper and lower blocks, as indicated in the figure. To prevent slipping, the grooves of the sheaves are furnished with teeth, cogs, or ridges, which grip the links of the chain. The friction is so great that, as shown in the figure, a large weight can be supported without any counterpoise or fixing of the loose loop of the chain. Following the course of the chain $p\ q\ r\ s$, we find that it passes in succession round the larger sheave of the differential block, the lower pulley, and the smaller sheave of the differential block. Let there be m ridges on the larger sheave and n on the smaller. Then when p is pulled down so that m links are uncoiled, only n links of r will be uncoiled. That is, every point of q will rise m inches for every n inches fall of r. Hence, since q and r remain stretched, the lower pulley with the attached weight must rise half the difference of m and n. Thus, if the hand which grasps the chain p moves down m inches the weight is raised $\frac{1}{2}(m-n)$ inches; and (neglecting friction) the force required to be exerted is to the weight in the ratio of $m-n$ to $2m$. In order to lower the weight, a pull must be applied to the chain s, that part of the loose loop which leaves the smaller sheave. To take a numerical example, let there be 8 and 9 ridges respectively on the two sheaves. Then a pull of 10 lb. weight on the chain will raise a weight of 10×18 or 180 lb.

§ 62. **The Screw.**—A simple instrument which has many important applications is the screw. In Chapter II. its

value in effecting geometrical subdivision is referred to. It remains now to consider it as a machine for doing useful work. In one of its most usual applications the screw is employed to produce and sustain a great pressure. To effect this the screw works in a fixed nut, and its extremity bears upon the substance which is to be subjected to the pressure. A simple form is shown in fig. 51. For every complete revolution of the handle above, the screw advances or recedes through a distance equal to the thread of the screw. Suppose the thread to be 1 inch and the handle to be 2 feet long. Then, if a power P be applied at the handle, it will be capable of overcoming (friction being meanwhile neglected) a weight W which is greater than P in the ratio of the circumference of the circle traced out by either end of the handle to the corresponding distance moved through by the screw—that is, as $3\frac{1}{7} \times 24$ to 1, or $75\frac{3}{7}$ to 1. This ratio is the mechanical advantage of the screw-press. If we represent it by m we have generally $W = mP$.

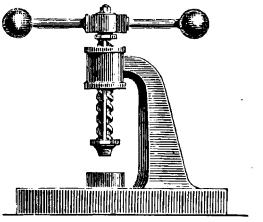


Fig. 51.

When the end of the screw is pressing down hard upon the substance, the upper faces of the threads of the screw which are for the moment within the nut are pressing hard upwards upon the corresponding faces of the screw cut out in the nut. This gives rise to considerable friction, so that practically a much greater power must be applied to produce a pressure W than is given by the expression W/m. In other words, because of friction, the practical mechanical advantage is considerably less than the theoretical mechanical advantage above defined. In short, as a machine for doing useful work, the *efficiency* of the screw—that is, the ratio of the useful work done to the energy supplied—is very small.

When, after producing a considerable pressure, the power is removed, the pressure is still sustained by the frictional forces

on the threads, which, however, act now in a direction opposite to that in which they were acting when the power was being sustained. In the ideal frictionless screw and in screws of very large pitch the action may be reversed—that is, instead of applying a couple to produce a push or pressure, we may apply a pressure and produce a couple. But in ordinary screws, because of the great frictional forces brought into play, this is impossible. A push or pull applied to the screw in the direction of its axis brings into play frictional forces sufficient to balance the force applied. The action, in short, cannot be reversed. This is the reason why a screw-press can sustain a pressure for an indefinite time, and why a screw-nail is so effective a fastener.

One very important application of the screw is to navigation. The blades of a ship's screw are parts of the threads of a screw having a definite pitch. When the screw is rotated, the blades press with their broad surfaces upon the water, just as the threads of an ordinary screw press upon the nut. The reactionary pressure causes the screw to advance or retreat through the water according to the direction of rotation. Because of what is technically called *slip*, the screw does not advance during one rotation by an amount equal to the pitch. The real advance through the yielding liquid is much less than this; and, because of the slip and the accompanying commotion in the water, a considerable loss of energy takes place, and only a fraction of the energy supplied is transformed into the kinetic energy of the moving vessel.

§ 63. **Kinetic Stability,** or the stability due to motion, is a physical principle of great importance. Here we can do little more than indicate its significance by means of familiar illustrations derived chiefly from bodies in rotation. A tee-totum or top must be set spinning about its axis before it will stand on its point or pivot. Once the rotation ceases, the top falls over on its side and assumes its position of statical equilibrium. Similarly a penny is unstable if set on its edge; but if spun about an axis in its plane it will remain standing on its edge so long as sufficient rotation remains. In like manner a bicycle must be in motion—that is, its wheels must

be rotating—if it is to preserve its upright position. A circular disc or wheel will roll for a considerable distance on its edge; but as soon as the rolling ceases it falls flat.

In all these cases rotation about an axis is the common feature. The general dynamical principle is that a rotating body tends to preserve its axis of rotation unchanged in direction. It is a case of inertia; and its most important illustration is the daily rotation of the earth, which gives us our unit of time (§ 9). Then, again, horizontal axes about which wheels generally rotate tend to remain horizontal; hence the disc continues to roll, the bicycle tends to remain upright. Of course, in certain cases, as in the spinning-top, the dynamics is complicated by the action of gravity. For if, say, the rolling disc lurches a little to one side, the weight acting through the centre of gravity and the reaction through the point of support form a couple tending to produce an angular acceleration about an axis perpendicular to the original axis of rotation. Now, just as a force acting perpendicular to the velocity of a particle changes the direction of motion without affecting the speed, so a couple whose axis is perpendicular to the axis of rotation changes the direction of the axis of rotation without changing the angular speed about this axis. Thus, the bicyclist steers his machine without affecting the speed of progression. The ordinary spinning-top, when it becomes deflected from the perfectly upright position, moves so that its axis describes a cone in space. This effect of gravity may be prevented by pivoting the top at its centre of gravity. Such a top, known as Troughton's Top, will preserve its axis of rotation unchanged in direction. On board a ship the top will appear to be in constant oscillation; but in reality it is the ship which oscillates, not the top.

Very remarkable effects can be studied by means of a **gyrostat,** which is simply a top surrounded by a case within which the top is set in rapid rotation. When the top is spinning, the gyrostat will rest on the edge of the case, just like the rolling wheel, but without its forward motion. If

the case be grasped in the hand, and an attempt be made to rotate the gyrostat about any axis not coincident with the axis of rotation, the attempt will be frustrated in an extraordinary manner. The sensation felt is as if a living creature with a powerful will were residing in the gyrostat, and were determined to resist all such attempts. The gyrostat may be made to take many positions which are obviously unstable from a statical point of view, but which are kinetically stable.

Similar effects are obtained with the scientific toy known as the **gyroscope** (fig. 52). A wheel with large moment of

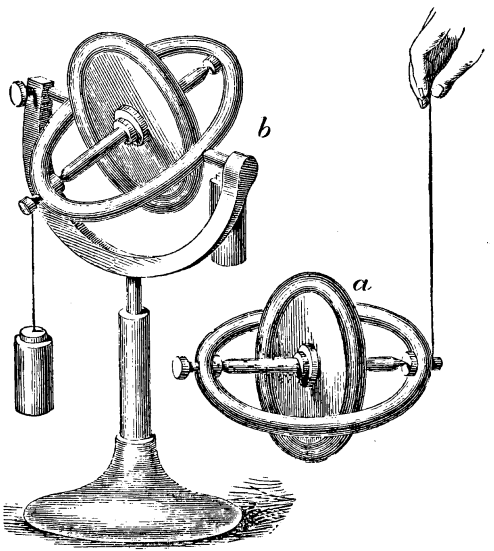


Fig. 52.

inertia is set in rapid rotation about an axis pivoted in a circular hoop or framework. With this framework hung at the end of a string (fig. 52*a*), the axis of the rotating wheel may be set horizontal or at any oblique angle with the vertical. The action of gravity is unable, so long as the rotatory motion continues, to make the gyroscope fall into

the familiar position of statical equilibrium. The only effect of the gravitational couple is to cause the gyroscope as a whole to rotate round the vertical axis through the point of support. The direction of rotation depends upon the direction of spin of the gyroscope wheel. Again, if the spinning gyroscope is placed on a suitable stand, as shown in fig. 52*b*, the effect of hanging a weight on one side is to cause the gyroscope not to tilt over, but to rotate about the vertical axis through the point of support. The phenomena form instructive illustrations of the composition of rotations.

EXERCISES.

1. Show by means of the principle of work done that the mechanical advantage of a series of straight levers placed end to end is equal to the product of the mechanical advantages of the successive levers.

2. A uniform bar, of mass 3 lb., is employed as a lever to move 1 cwt., the fulcrum being $\frac{1}{8}$ of the length of the lever from the one end. What force must be applied at the other end?

3. A man is raising a weight of 100 lb. The rope that supports the weight is coiled round an axle whose diameter is 6 inches, and the man applies the power at a handle which is fixed to the axle by a crank 15 inches long. What force must he apply, and at what rate does he work, when he rotates the handle once every 2 seconds?

4. A mass is to be weighed by means of a balance whose arms are of unequal length. When placed in the one scalepan, the mass has an apparent weight P; and when placed in the other, an apparent weight Q. Show that the true weight is $\sqrt{PQ}$.

5. Two spherical shells, one of gold and the other of silver, have the same mass, size, and surface. When allowed to roll down an inclined plane the silver shell moves the faster. Why?

SUMMARY.

§ 58. The Lever. Various types. Arm, fingers, &c. Hammer extracting nails.

§ 59. The Balance. Sensibility and Stability.

§ 60. Wheel and Axle.

§ 61. Pulley. Purchase-Block. Differential Pulley.

§ 62. Screw. Condition of Reversibility. Ship's Screw.

§ 63. Kinetic Stability. Troughton's Top. Gyrostat and Gyroscope.

CHAPTER VIII.

GRAVITATION.

§ 64. **Every particle in the universe attracts every other particle with a force whose direction is in the line joining the two, and whose magnitude is directly as the product of the masses, and inversely as the square of their distance from each other.**

This is the enunciation of the universal law of gravitation established by Sir Isaac Newton. Kepler, after prodigious arithmetical labour, had established three empirical laws of planetary motion. These were:

1. Every planet describes an ellipse of which the sun occupies one focus.

2. The radius-vector of each planet describes equal areas in equal times.

3. The square of the periodic time is proportional to the cube of the major axis.

Newton's deductions from these were somewhat as follows:

The Second Law means that the moment of the velocity about the focus is constant, and that consequently the acceleration must have no moment, that is, must be along the radius-vector. Hence, the force acting on each planet must be directed towards the sun. Such a force is called a central force, because it acts towards a definite centre.

Elliptic motion about a centre of force which occupies the focus leads mathematically to the conclusion that the force varies inversely as the square of the distance from its centre.

The Third Law leads to the same conclusion, with the additional result that it is the *same* gravitation, diminishing as the square of the distance increases, which acts on each

one of the planets. The planetary orbits are ellipses of small eccentricity. On the supposition that they are circular, we get the result very simply thus: Let T be the periodic time, R the radius (or semi-major axis). Then, by Kepler's III. Law,

$$\frac{T^2}{R^3} = \text{constant.}$$

But, if v is the speed, $2\pi R = vT$.
Hence, substituting for T^2 we get

$$\frac{T^2}{R^3} \propto \frac{R^2}{v^2R^3} \propto \frac{1}{v^2R}.$$

Hence $$v^2R = \text{constant},$$

or $$\frac{v^2}{R} \propto \frac{1}{R^2}.$$

But v^2/R is the normal acceleration, in this case proportional to the central force, which force accordingly varies inversely as the square of the distance.

The Earth and Moon form a system in themselves comparable to the system formed by any planet and the sun. The moon describes an ellipse round the earth. To a first approximation we may regard this ellipse as a circle whose radius (r) is nearly sixty times the earth's own radius (R). The mean speed of the moon in its orbit is

$$v = \frac{2\pi r}{(\text{periodic time})} = \frac{2\pi \times 60R}{(27\frac{1}{3} \text{ days})}.$$

Hence the acceleration is approximately

$$\frac{v^2}{60R} = \frac{4\pi^2 \times 60R}{(27\frac{1}{3} \times 24 \times 60 \times 60)^2}$$

$$= \frac{40 \times 4000 \times 5280}{60^3 \times (27\frac{1}{3} \times 24)^2} = \frac{1}{60^2} \times 32{\cdot}8 \text{ (ft.-sec. units).}$$

Thus the acceleration acting on the moon is less than the acceleration acting on a particle at the earth's surface in the ratio of 1 to 3600. This result is in full accord with the law

of gravitation, if we can regard the earth's gravitation attraction as being directed towards its centre. But Newton proved in a simple manner that, the inverse square law being assumed to hold between particles, any uniform spherical shell will attract external particles as if its whole mass were condensed in its centre. What holds true for spherical shells will hold true for any solid sphere built up of concentric uniform shells. Thus we are fully warranted in taking the centres of the nearly spherical bodies which constitute our solar system as true **Centres of Gravity,** points towards which the attractions of the respective masses may be supposed to act.

The part of the law of gravitation which refers to the masses is a generalisation from the truth already referred to (§ 34) that weight is proportional to mass. The weight of a body at the earth's surface is simply the attraction between it and the earth. Let the body be carried away to a distance equal to that of the moon; its weight will be diminished in the ratio of 1 to 3600. Increase the mass of the body; its weight will be increased in the same ratio.

§ 65. **Kepler's Laws only Approximate.**—Although primarily a deduction from Kepler's Laws, the Law of Gravitation shows that these laws are only approximate. For the law of universal gravitation implies that each planet exerts a force of attraction upon every other planet, so that the resultant force acting on any one planet cannot accurately pass through the sun's centre of gravity. But the mass of the sun is so great compared to even the combined mass of all the planets that the mutual actions of the planets lead to comparatively small effects. Then the Newtonian Law implies that the two mutually-attracting bodies move round their common centre of mass, so that, for example, the sun moves round the earth just as truly as the earth moves round the sun. Here again, however, the sun's mass is so great compared to the total planetary mass that the centre of mass of the solar system lies within the body of the sun.

§ 66. **Comparison of Masses of Earth and Sun.**—Let E be the earth's mass and S the sun's mass; R the earth's radius and r the mean distance of the earth from the sun. Let T be the periodic time of the earth—namely, one year. The acceleration due to the sun at the earth's distance is

$$\frac{(\text{orbital speed of earth})^2}{r} = \frac{4\pi^2 r}{T^2}.$$

The acceleration due to the earth at the *same* distance r is

$$g\frac{R^2}{r^2}.$$

But these accelerations must be proportional to the respective masses, or

$$\frac{S}{E} = \frac{4\pi^2 r^3}{gR^2T^2},$$

in which of course every quantity must be expressed in the same units. If we take g equal to 32·2 we must measure r and R in feet and T in seconds. The calculation is easily made, and shows that the sun's mass is 324,000 times the mass of the earth.

By an exactly similar process the mass of a planet which has one or more satellites can be compared with that of the earth; and also the mass of a Double Star when the period of revolution of the components around one another and their distance apart are known.

Thus the ordinary data of astronomy enable us, with great ease, to compare the masses of the earth, most of the planets, the sun, and certain double stars; and processes of a more abstruse nature, but all direct deductions from the grand law of gravitation, lead to an estimation (in terms of the earth's mass) of the masses of the moon and those planets which have no satellites. Not only so, but from observed peculiarities in the motion of the planet Uranus, otherwise inexplicable, Adams and Leverrier in 1847 independently calculated the mass and position of the planet Neptune,

till then unknown. To express the masses of sun, planets, &c., in absolute units we must first find the earth's mass in pounds or grammes.

§ 67. **The Mass and Mean Density of the Earth.**—One obvious process for finding the earth's mass in pounds is to compare its gravitation attraction for a given mass with the gravitation attraction for the same mass of a body whose mass is known. This is the process used in two of the methods which have been carried out. Of these the Cavendish Experiment (devised by Michell, but first carried out by Cavendish) is the more trustworthy.

Two small balls, *ab* (fig. 53), are fixed to the ends of a horizontal rod, which is suspended at its middle point by a long fine wire. The whole of this part of the apparatus must be enclosed in a case so as to minimise the effects of currents of air. A small vertical mirror is attached to the middle of the rod; and, by means of a scale reflected in the mirror and viewed through a telescope at some distance, any slight rotational motion of the rod and balls about the vertical axis can be seen and measured. The motion to be measured is so very small that the most elaborate precautions must be taken to prevent any disturbances such as easily arise from slight differences of temperature and slight earth tremors.

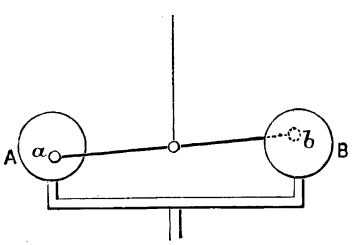


Fig. 53.

Outside the case are placed two equal massive spheres of lead, A, B, which can be adjusted very close to the small balls; one on one side and the other on the other side of the rod. In the position shown in fig. 53 A will attract *a* with a certain force, and B will attract *b*. The result will be a couple acting on the rod *ab*, rotating it through a small angle until it is balanced by the force of torsion produced in the

wire. The scale as seen in the telescope is then read. The system A, B is now gently rotated round a vertical axis until A comes close to *b* and B close to *a*, but on the sides opposite to those occupied by B and A in the first position. The couple now acts in the opposite sense, and the rod *ab* takes a new position of equilibrium, which is measured on the telescopic scale. Finally, the balls A, B are moved back to their first position, and the scale read a third time. The difference between the second reading and the mean of the first and third gives the angle through which the rod has been rotated because of the shifting of the massive spheres. The torsional rigidity of the wire being known, the couple corresponding to this angular displacement can be calculated in absolute units; and from this the force of attraction between each large sphere and the small ball at the given distance can be calculated. Hence we can estimate, by use of the law of the inverse square of the distance, the force which the sphere would exert upon unit mass placed at a distance from its centre equal to the earth's radius. But the weight of unit mass is the gravitation force due to the earth at this same distance. Therefore the ratio of these forces gives the ratio of the masses of the sphere and the earth. Thus we can measure the earth's mass in pounds or grammes. If we divide the mass of the earth in grammes by its volume in cubic centimetres we get a quantity which is known as the **Mean Density of the Earth.** Newton, in the *Principia*, made the following remarkable statement: 'Since the common matter of our earth, on the surface thereof, is about twice as heavy as water, and a little lower, in mines, is found about three, four, or even five times more heavy, it is probable that the quantity of the whole matter of the earth may be five or six times greater than if it consisted all of water.' The earlier experiments agreed in assigning for the mean density of the earth values lying between 5·4 and 5·7.

Boys, by use of a quartz suspension, has greatly reduced

the dimensions of the Cavendish apparatus, and almost entirely eliminated disturbances due to differences of temperature within the case. His value for the mean density is 5·527. Poynting has, with success, substituted a delicate balance for the torsion wire—the massive sphere being placed below each of the equal-balanced masses in alternation.

A determination of the earth's mean density has also been made by comparing its attraction as a whole with the local attraction of a mountain. This, generally known as the Schiehallion Experiment from the name of the Perthshire mountain which was first utilised for the purpose, is not so satisfactory as the Cavendish Experiment, because of the difficulty of estimating the real mass of the mountain.

The same source of error is present in the third method, that known as the Harton Experiment. In this experiment the different values of g at the surface and at the bottom of the Harton Coal Pit were measured by pendulum observations. Now, Newton proved that a uniform spherical shell has no attraction on any particle in its interior. Hence, on the assumption that the earth's crust is fairly homogeneous, the value of g at the bottom of the mine will depend on the amount of matter included in the spherical surface which is concentric with the earth's surface and passes through the bottom point of the mine. In other words, the mass which acts gravitationally upon the pendulum when it is at a given depth is less than the whole mass of the earth by the mass of crust whose thickness is this depth. This crust is accessible so that its mass may be estimated, and, in conjunction with the observed value of g above and below, gives data sufficient to calculate the earth's mass. The value of the mean density obtained by Sir George Airy from the Harton Experiment was nearly one-fifth too large.

§ 68. **Gravitational Energy.**—The great simplicity and universality of the Law of Gravitation have led some to think that, in stating the Law, Newton had said the last word on the subject. But this was not the view of Newton

himself, who writes, in his *Letters to Bentley*, 'The cause of gravity is what I do not pretend to know, and therefore would take more time to consider of it. . . . That gravity should be innate, inherent, and essential to matter, so that one body may act upon another at a distance through a *vacuum*, without the mediation of anything else, by and through which their action and force may be conveyed from one to another, is to me so great an absurdity that I believe no man who has in philosophical matters a competent faculty of thinking can ever fall into it.'

Many attempts have been made to find a mechanical explanation of the law of gravitation. A medium or mechanism of some sort filling all intermediate space is clearly demanded; and then the gravitation attraction between two particles will be explained as a push or pull through this medium, or, by virtue of this mechanism, a push or pull which is conditioned by the presence of the particles themselves. Newton himself speculated on the possible existence of a medium whose density increased with distance from material particles; others have suggested tensions and pressures in fluids at rest or in motion; while in Le Sage's famous hypothesis, small so-called ultramundane corpuscles are imagined which, during their rapid motions in all directions, impinge upon ordinary matter and by these impacts produce the effects of gravitation.* None of these hypotheses have withstood satisfactorily the scientific criticism to which they have been subjected. What is really required is a plausible explanation of the potential energy that exists between two particles in virtue of their gravitation.

If the law of gravitation be put in the form

$$\text{Attraction} = k\frac{mm'}{r^2},$$

where k is a constant depending on the units of mass, length, and time which are employed, then it can be shown that

* For a detailed account and criticism of Le Sage's hypothesis, see Clerk Maxwell's article 'Atom' in the *Encyclopædia Britannica*; also chap. vii. in Tait's *Properties of Matter*.

the expression for the mutual potential energy of the masses mm' distant r apart is

$$W - k\frac{mm'}{r}$$

where W is the potential energy when the particles are an infinite distance apart.

The quantity k is easily estimated in terms of the earth's mass and the weight of the unit mass. Let m be the earth's mass, and m' any mass at the earth's surface. Then the force acting on m' is

$$m'g = k\frac{mm'}{R^2}$$

where R is the earth's radius. Hence

$$k = \frac{gR^2}{m}.$$

The quantity k is called the gravitation constant. Its value in the C.G.S. system of units is $\frac{1}{1{\cdot}543 \times 10^7}$.

The work done in carrying the mass m' from a position at distance R to a position at distance r from the earth's centre is

$$kmm'\left(\frac{1}{R} - \frac{1}{r}\right) = m'gR^2\left(\frac{1}{R} - \frac{1}{r}\right).$$

When r is great compared to R, its reciprocal may be neglected. Hence the expression $m'g$R measures the work done in carrying the mass m' from the earth's surface to a practically infinite distance.

If, therefore, a pound of matter attracted towards the earth from a great distance falls upon its surface, it brings with it an amount of energy equal to gR, approximately 4000×5280 foot-pounds. This will be in the kinetic form, the final speed, as the mass falls upon the earth's surface, being 36,800 feet per second, or $8\frac{3}{4}$ miles per second. After the collision the energy will be ultimately transformed into heat. A similar though more complex calculation may

be made to determine the loss of potential energy by the matter of the earth as it consolidates to the present size, from an infinitely scattered distribution, in virtue of the mutual gravitation of its parts. The potential energy so lost becomes transformed ultimately into heat. This is Von Helmholtz's explanation of the internal heat of the earth, and also of the high temperature of sun and stars.

By far the greatest amount of energy that is used by us comes from the sun in the form of light and radiant heat; and this is in great part if not wholly due to the gradual shrinking of the sun because of the mutual gravitation of the matter composing it. It is therefore ultimately on gravitation that the varied activities of mundane life depend. No one has as yet devised a method for utilising the energy of tidal motion, the larger part of which is lunar and not solar. But the whole of it is obviously gravitational in its origin. Again, the comparatively small amount of energy which may be traced to the internal heat of the earth (hot springs, volcanoes, &c.) is also referable to a gravitation source; and the same might be said of the feeble stellar radiation which comes in upon us from all sides. And thus we recognise in gravitation the most important mechanical source of the energy of the physical universe.

SUMMARY.

§ 64. The Law of Universal Gravitation. Kepler's Laws and Newton's deductions. Case of Earth and Moon.

§ 65. Kepler's Laws only approximate.

§ 66. Comparison of Masses of Earth and Sun. Masses of Planets and Double Stars.

§ 67. Mass and Mean Density of the Earth. Cavendish Experiment. Schiehallion. Harton.

§ 68. Gravitational Energy. Source of Terrestrial and Solar Heat.

CHAPTER IX.

EQUILIBRIUM AND MOTION OF FLUIDS.

§ 69. Ordinary matter exists either as solid or fluid. The solid has a form which is with more or less difficulty altered; and this property leads to the conception of the ideal rigid solid, which is the subject-matter of what is commonly called Rigid Dynamics. On the other hand, the fluid has no power of permanently retaining any definite shape. It takes the form of the mould which contains it. If the fluid is a liquid, any disturbance of its surface rapidly disappears as soon as the disturbing agent is withdrawn. It is by the possibility of having a free surface that the liquid fluid is distinguished from the gas or gaseous fluid; for the gas at once fills the whole closed region in which it may be confined.

The characteristic property of fluidity or tendency to flow is supposed to be possessed in perfection by the ideal perfect fluid. No real fluids are in this sense perfect, for all such have, in their **Viscosity,** or **Fluid Friction,** a hindrance to the relative motion of contiguous parts. When one solid surface slides along another the force of friction exists as a force acting against the motion *tangential* to the surface. The behaviour of fluids in motion discloses a similar tangential force acting between two contiguous elements of fluid which are moving parallel to one another with different speeds. In the ideal Perfect Fluid no such tangential forces are supposed to exist. In other words, any mutual action that may exist between contiguous elements must be at right angles to the interface.

If, in any natural fluid, a tangential force is applied, the

fluid is set in motion. Hence, in the natural fluid at rest, the mutual action that may exist between contiguous elements will be normal to the interface, exactly as in the perfect fluid. Consequently, the abstract theorems of Hydrostatics, that branch of Hydrodynamics which treats of the equilibrium of the perfect fluid, apply at once to natural fluids in equilibrium. It is quite otherwise, however, in regard to Hydrokinetics, the branch of Hydrodynamics which treats of the motion of fluids. The theorems established for the perfect fluid have only a qualified application to the motion of real fluids, and in many cases do not apply at all.

Between the two types of fluid with which we are familiar—namely, liquid and gas—a very marked difference generally exists. A given mass of gas can be made to assume any volume within very wide limits. The changes of volume which a given mass of liquid can undergo are comparatively small, especially if the effect of heating or cooling be excluded. Thus a liquid like water suggests an *incompressible fluid*, whose density remains unchanged under all circumstances; whereas a gas like air suggests a highly *compressible fluid*, whose density depends upon surrounding conditions. The incompressible fluid will first engage our attention.

§ 70. **Pressure in a Fluid.**—Let a cylindrical vessel with flat bottom be filled with liquid. The total weight of this liquid presses down upon the bottom, and the bottom presses upwards upon the lowest layer of liquid. This total pressure is obviously distributed uniformly over the surface. Dividing the total pressure by the area of the surface, we get the *pressure per unit area*, or simply the *pressure*. Now, just as the solid bottom has to support the whole weight of liquid, so, if we divide the liquid column into two parts, an upper and a lower, the lower part must support the weight of the upper. Hence over any horizontal surface drawn in the fluid a definite pressure must exist. If, then, a small horizontal surface be imagined within the fluid, the contiguous elements

separated by this surface must exert a pressure upon one another, a pressure which is at right angles to the interface.

In cases of equilibrium we may introduce further constraints without destroying the equilibrium. Suppose then that any portion of the fluid becomes solidified, but so as to be unaltered as regards mass and form. It will be in equilibrium under the action of its weight and the pressures upon its surfaces. Let now the solidified part be taken smaller and smaller. The mass will diminish as the volume—that is, as the *cube* of the linear dimensions; whereas the total pressures on the sides will diminish as the areas—that is, the *square* of the linear dimensions. Hence the weight will diminish more rapidly than the total pressures; and ultimately a small volume element taken round a point will be in equilibrium under the action of surface pressures alone.

Suppose the element to be hemispherical. Then the total pressures on the elements of the curved surface, being normal to the surface, will all pass through the centre of the sphere; hence their resultant will also pass through the centre. But this resultant must be balanced by the resultant normal pressure on the plane base of the hemisphere. Consequently the resultant pressure on any small plane element passes through the centre or mean point of the element (fig. 54*a*).

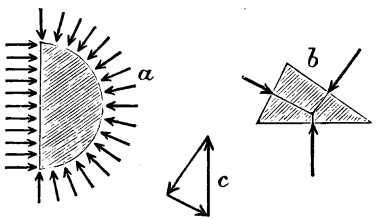


Fig. 54.

Now, let the small volume element be prismatic with a triangular section. The total pressures on the sides will bear normally on the centres of the sides, and will therefore meet in a point (fig. 54*b*). But equilibrium requires not only that these three forces meet in a point, but that they can form the sides of a triangle. Evidently, since each force

is perpendicular to the sides of the triangle of section of the prism, the triangle of forces (*c*) will be similar to the triangle of section (*b*). Hence the total pressures are proportional to the corresponding sides of the prism; and the pressures per unit area are equal on the three sides. The same theorem will hold for any form of prism. Thus we are led to the following very important conclusion:

If any small plane area be taken about a point in a fluid, the total pressure between the elements separated by this area or the total normal stress across this area is ultimately the same as the area is taken smaller and smaller, whatever the aspect of the area may be.

The ratio of the total pressure to the area is the average pressure over the area, and is ultimately the pressure per unit area at any point of the area when the area is taken vanishingly small. The theorem just stated may be more simply (but somewhat elliptically) expressed in these words:

The pressure at a point in a fluid is the same in all directions.

It should be borne in mind that this is a rigorous deduction from the condition that the pressure on any surface is normal to that surface.

§ 71. **Liquid under Gravity.**—In a liquid under the action of gravity the pressure increases with the depth, for with the greater depth there is the greater height of column of fluid to support. Any finite volume of the liquid is in equilibrium under the action of its weight and the pressures acting on its surfaces. Consider for definiteness a vertical uniform column of water whose section is one square inch. The pressures on its vertical sides, being of course horizontal, do nothing towards supporting the column. Hence the upward pressure on the lower end must balance the downward pressure on the upper end, together with the downward acting weight of the column. Now, one cubic foot of water contains $62\frac{1}{3}$ lb. or nearly 1000 ounces. Hence a column of

water one foot high will exert a pressure of $62\frac{1}{2}$ lbs.-weight per square foot, or ·433 lbs.-weight per square inch. Thus with a column of water x feet high, the difference of pressures on its lower and upper ends will be $x \times$ ·433 lbs.-weight per square inch.

The calculation is simpler if we use the centimetre-gramme system of units. In this case a cubic centimetre contains one gramme. Hence the difference of pressures on the ends of a column x centimetres high is x grammes-weight per square centimetre.

Let the pressure at the surface of a liquid be P, then the pressure p at any depth h is such that

$p - \mathrm{P} =$ weight of column of height h and cross section unity
$= h\rho g$

in absolute units, where ρ is the mass per unit volume, and g is the measure of the weight of unit mass.

Mercury is a liquid whose density (or mass per unit volume) is 13·6 times the density of water. Hence the pressure due to a foot of mercury is 13·6 × ·433 or nearly 5·9 lbs.-weight per square inch. Otherwise the pressure due to a column of mercury one metre high is

$$100 \times 13{\cdot}6 \times 981 \text{ dynes per square centimetre}$$
$$= 1{\cdot}36 \text{ kilogrammes-weight per square centimetre.}$$

Thus the pressure at any point in a liquid depends simply on the depth. Since every point of the surface is exposed to the same pressure (that due to the atmosphere) it follows that the pressure is the same at all points at the same depths. All points at which the pressure is the same will lie on a surface parallel to the free level surface of the liquid. Such points are said to be themselves at the same level, so that surfaces of equal level are also surfaces of equal pressure.

When any number of communicating vessels are filled with the same liquid, the level of the free surface of the liquid is the same in all (see fig. 55). It matters not what shape the different vessels may have, for the pressure must be constant over every horizontal surface that can be drawn through the

liquid; and hence the free surfaces must be parts of the same horizontal surface.

Should the communicating vessels contain different fluids

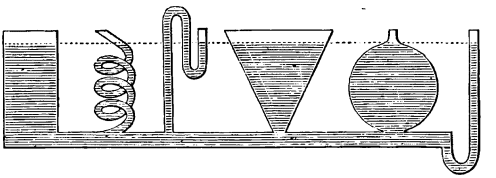


Fig. 55.

which do not mix, the levels will in general be different because of the different densities of the fluids.

An example of this is shown in fig. 56. The bend of the tube is filled with a dense liquid AC, and in the one leg a much lighter liquid is poured in. The pressure at A must be the same as the pressure at a point at the same level in the other leg. Hence the pressure due to height h' of the heavier liquid is equal to the height h (= AB) of the lighter liquid. These heights must be inversely as the densities. If the liquids were mercury and water, the height h would be 13·6 times the height h'.

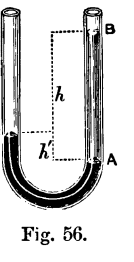


Fig. 56.

§ 72. **The Atmospheric Pressure.**—An important instance of the same phenomenon is given by the experiment which measures the atmospheric pressure. A glass tube about a yard long is closed at one end and filled with mercury. The open end is momentarily stopped, say with the finger, and is dipped into a dish of mercury or other liquid until the end is covered, when the finger is removed, and the tube fixed in a vertical position. The mercury will then be found standing in the tube to a height of about thirty inches, so that only six inches of the tube will be empty of mercury. The final condition is shown in fig. 57. The pressure at any point in the tube will be due to the weight of the superincumbent

column of mercury. Hence the pressure on the free surface outside the tube, being equal to the pressure at the same level inside the tube, must be that due to a column of mercury about thirty inches or $2\frac{1}{2}$ feet high. The mass of this column is $2\frac{1}{2} \times 5{\cdot}9$ or 14·7 lb., and its weight is balanced by the pressure on the free surface due to the atmosphere. Thus we prove that the atmospheric pressure at the sea-level is 14·7 lb.-weight per square inch.

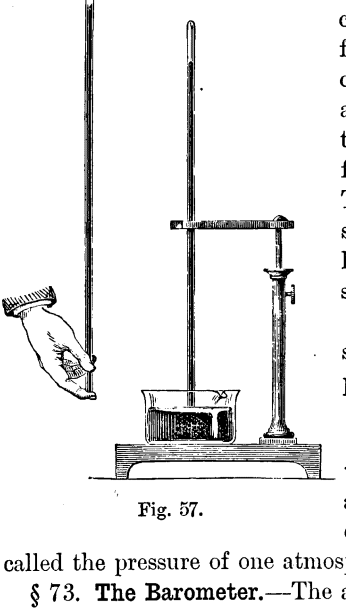


Fig. 57.

Expressed in dynes per square centimetre, the same pressure has the value,

$$76 \times 13{\cdot}6 \times 981 = 1{\cdot}014 \times 10^6,$$

where 76 centimetres is taken as the height of the mercury column. This pressure is called the pressure of one atmosphere, or one *atmo.*

§ 73. **The Barometer.**—The apparatus just described, when carefully constructed, forms the instrument called the mercurial barometer. Its function is to measure the pressure of the atmosphere in terms of the height of a column of mercury. Its practical importance arises from the fact that the manner in which the pressure of the atmosphere may be at any instant changing is closely connected with the character of the weather. Thus the barometer is frequently called the weather-glass. If the atmospheric pressure increases, a longer column of mercury is required to balance the pressure, and the 'glass rises.' On the other hand, a 'falling barometer' means diminishing pressure; and this, in the majority of cases, is followed by influx of wind, condensation of

water-vapour, and precipitation of rain. Broadly speaking, a falling barometer foretells wet weather, a rising barometer fine weather. The average height of the barometric mercurial column at the sea-level is about 30 inches. In the space above the column there should be no air, for a very small quantity of air will make the indications of the barometer quite untrustworthy. A slight quantity of mercury vapour must, however, be present in the otherwise empty space, which forms what is known as the **Torricellian* Vacuum.**

A barometer might be constructed with other liquids besides mercury; but a much longer column would in general be required. For example, since mercury is 13·6 times denser than water, the water column corresponding to 30 inches of mercury would be 30 × 13·6 inches or thirty-four feet. This is the maximum height to which water can theoretically be raised in a tube by producing a vacuum over it. Upon this principle depends the action of the siphon and suction-pump.

§ 74. **The Siphon.**—In its simplest form the siphon consists of a bent tube filled with liquid, which forms the communicating channel between two vessels containing the same liquid (fig. 58). Because the vessels are in communication the liquids will tend to come to the same level. If the liquid in the one vessel stands at a higher level than the liquid in the other, a flow will take place along the siphon from the former to the latter until either the levels are made the same or the level in the one vessel becomes lower than the end of the siphon. The height of the upper bend of the siphon above either surface must not be greater than the barometric height of the liquid that is being siphoned. For if it were so, a Torricellian vacuum would be produced, separating the upper

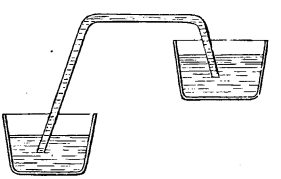


Fig. 58.

* Torricelli (1608-47) was the first to construct the mercurial barometer, and thereby demonstrate that the air had weight.

ends of the liquid columns in the two legs, and the two quantities of liquid would cease to be in communication with each other. A ten-foot siphon will operate with water, but will be quite useless with mercury.

§ 75. **The Suction-pump.**—The principle of the suction-pump is illustrated in the familiar operation of suction. Thus water can be drawn up through a tube (e.g. a straw) to the mouth, because of the partial vacuum produced above the liquid surface in the tube. The pressure in this partial vacuum is less than the atmospheric pressure which acts on the liquid surface outside the tube. This excess of pressure outside forces the water up inside the tube; and the water will rise until the pressure within the tube at the same level as the outside surface is equal to the atmospheric pressure.

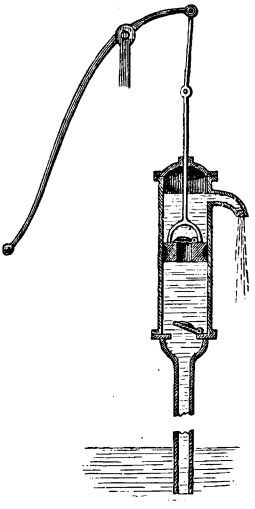


Fig. 59.

In the suction-pump (fig. 59) the partial vacuum is produced in the tube by a simple mechanism of piston and valves. There are two valves, one on the piston and the other at the base of the tube in which the piston works up and down. Both valves open upwards, but at once become closed when any downward pressure acts upon them. At every down-stroke of the piston the bottom valve becomes closed, while the piston valve opens and allows the upward escape of any air or water which the diminished space in the tube below the piston can no longer contain. At the succeeding up-stroke of the piston the piston valve closes, and prevents the re-entrance of air or water into the space below the piston. But this space is now increasing, and can be filled again only by way of the bottom valve, which, opening up-

wards, allows water to be pushed into it because of the excess of atmospheric pressure on the water surface outside the tube. Obviously water cannot be raised by suction from a greater depth than 33 or 34 feet. Similarly it is impossible to support by suction a column of mercury longer than 30 inches.

§ 76. **Buoyancy.**—When a solid is immersed in a liquid it appears to lose in weight. This loss in weight is quite appreciable to the senses, as may be easily tested by lifting or trying to lift a stone out of water; for so long as the stone is in the water it is much more easily moved than when it is out of the water. A perfectly scientific demonstration may be given thus: Hang two masses to the ends of a stick, which is supported by a string so that a true balance is obtained, and then allow one of the masses to dip into water. As soon as this is done the stick tilts down towards the mass that is not immersed (fig. 60).

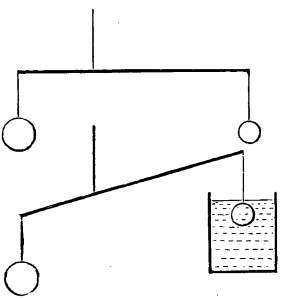


Fig. 60.

It may be shown by experiment, or it may be proved from hydrostatic principles, that the loss in weight of a substance immersed in liquid is equal to the weight of the liquid displaced.

The simplest proof is as follows: Imagine any portion A

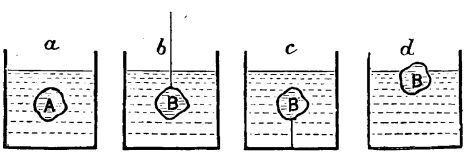


Fig. 61.

(fig. 61*a*) of the liquid to become solid, but to be otherwise unchanged; it will still be in equilibrium. Hence its weight, acting downwards, must be balanced by an equal

resultant total pressure acting upwards. But the pressures act on the surface parts only. Let, then, any other substance B of the same bulk and shape be substituted for A. The surface pressures will remain unchanged. Hence B will be buoyed up by a resultant total pressure equal to the weight of the same volume of liquid. If the weight of B is greater than the weight of the displaced liquid, B (fig. 61*b*) will still have a tendency downwards, and the string by which it hangs will support a tension equal to difference of the weights of B and A. But if the weight of B is less than the weight of the displaced liquid, it will be necessary, for complete immersion, to keep B forcibly under the liquid, say by fastening it by means of a string to the bottom of the vessel (fig. 61*c*). This string is under a tension equal to the difference of the weights of A and B. When the string is cut, B will rise and float on the liquid (fig. 61*d*), part of it being above the surface and part below. In this condition the weight of B will be balanced by an upward total pressure equal to the weight of the displaced water.

§ 77. **Specific Gravity : Density.**—Density has already been defined as the mass in unit volume. If a substance is denser than water, it will also be heavier, bulk for bulk. It will sink when immersed in water, and its apparent weight will be diminished by the weight of an equal bulk of the liquid.

The ratio of the weight of the substance to the weight of an equal bulk of water is called the specific gravity of the substance.

Because of the proportionality of mass and weight, densities are proportional to specific gravities. And if, as is the case in the C.G.S. system of units, water be chosen as the substance of unit density, the numbers which measure specific gravities also measure densities.

The readiest mode of measuring specific gravities of substances denser than water is suggested by the experiment just described. First weigh the substance in air and then weigh

it when immersed in water. If W, W′ be the weights in these two conditions, evidently the difference, W − W′, will be the weight of the displaced water. Hence the specific gravity of the substance is

$$s = \frac{W}{W - W'}.$$

For a substance lighter than water this method is inapplicable. But the difficulty is readily surmounted by attaching the substance (a piece of wood say) to a piece of heavier material sufficiently heavy to sink both together. Let w be the weight in water of the sinker alone, and W′ the weight in water of the two together. Because of the buoyancy of the substance, W′ is *less* than w, and the *negative* difference $W' - w$ corresponds to the quantity W′ in the expression given above. It is the negative weight of the substance in water. Hence the expression for the specific gravity becomes

$$s = \frac{W}{W - W' + w}.$$

In the determination of specific gravities in this way, great care must be taken that no air-bubbles cling to the substance when it is being weighed in water. Of course the method is inapplicable to substances which dissolve in water.

When a substance floats on water, the fraction of its volume which is submerged measures its specific gravity; for if the total volume be V, and if v be the submerged part, and therefore the volume of the displaced water, we have at once, s being the specific gravity of the substance,

$$Vs = v \times 1, \quad \text{or} \quad s = v/V.$$

For example, an iceberg, whose density is about ·9, has $\frac{9}{10}$ of its volume submerged, so that the whole volume of the iceberg is ten times greater than what is visible above water.

Having measured the specific gravity of a given solid, we may measure the specific gravity of any liquid that is less dense by substituting this liquid for the water. Practically, however, more convenient methods are in use for this purpose.

For very accurate determinations of liquid densities, the specific-gravity bottle is universally employed. This is simply a flask, generally of some definite content (50 cubic cm., 100 cubic cm., and so on), which is filled with the liquid whose density is to be measured. The combined mass of liquid and flask is determined by delicate weighing, and from this the known mass of the flask is subtracted. This gives the mass of liquid which completely fills the bottle. If the volume is known, then the ratio of the mass to the volume gives the density at the temperature of observation. If the volume is not known, then distilled water is substituted for the liquid in the bottle; and a comparison of the masses of the equal volumes of liquid and water gives the density in terms of the density of water at the temperature of observation.

For practical purposes the **Hydrometer** is the most convenient instrument for measuring the specific gravities of liquids. The common form of hydrometer (fig. 62*a*) consists of a stem loaded below so that it floats vertically with a certain length of the stem projecting above the liquid. The denser the liquid the farther the stem projects; the lighter the liquid the lower the stem sinks. The stem is graduated so that the mark at which the water surface stands gives at once the specific gravity of the liquid.

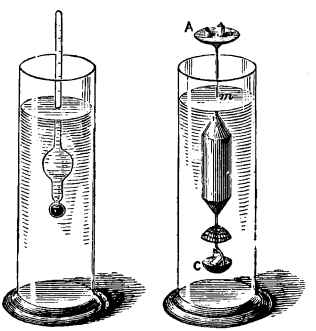


Fig. 62.

A more elaborate form is Nicholson's Hydrometer (fig. 62*b*), which is made to sink to a certain mark *m* by means of extra weights put on the tray A. The volume of liquid displaced is therefore always the same; and it is the weight of this displaced liquid which balances the known combined weight of the hydrometer and the extra standard masses placed on the tray. Knowing the extra weight required to sink the hydrometer to the mark in water of unit density, we find by a simple calculation the density of the liquid under investigation.

Nicholson's hydrometer may also be used for determining the specific gravities of solids. The weight of the solid must be less than the extra weight required to sink the hydrometer to the fixed mark. The solid is then placed on the tray A together with whatever weights may be found necessary to produce the usual amount

of sinking. Knowing the total weight required to do this, we find by a simple subtraction the weight of the solid in air. The solid is now transferred to the cage C, so as to become immersed in water, and weights are added above until again the hydrometer is immersed to the mark *m*. This gives the means of calculating the weight of the solid in water; and the specific gravity is found by the usual calculation. The specific gravities of solids less dense than water are found by this method quite as simply and directly as the specific gravities of solids denser than water.

The weight of a body in air is not its real weight; for the air, being a fluid, has a certain buoyancy. In short, the ordinary *apparent* weight of a body is less than its real weight by an amount equal to the weight of the displaced air. The specific gravity of air (see below) is nearly $\frac{13}{10000}$, so that the loss of weight of a quantity of water is fully $\frac{1}{8}$ per cent., a loss which is practically negligible.

§ 78. **Densities of Various Substances.**—The following table will give a general idea of the range of densities met with in the physical universe:

Substance.	Density (approximate).	Substance.	Density (approximate).
Iridium	24·0	Water	1·0
Platinum	21·2	Alcohol	·81
Mercury	13·6	Compressed Oxygen	·71
Lead	11·4	Lithium	·59
Copper	8·9	Oxygen	·00143
Tin	7·3	Air	·00129
Bromine	3·2	Steam	·0008
Aluminium	2·7	Hydrogen	·00009

The range from Iridium to Hydrogen in the ordinary condition is about 250,000 to 1. But we can easily reduce hydrogen to $\frac{1}{100000}$ of its ordinary density. Hence we can exhibit side by side portions of matter whose densities are as 25 thousand millions to 1.

The densities just given are approximate only, for the values for any one kind of substance vary within certain limits according to the mode of preparation or the subsequent treatment. As a general rule, alloys formed of different metals

have their densities intermediate to those of the component substances; but the density of the alloy is not always what would result if the substances were simply mixed together. In a simple mixture, the volume of the mixture should equal the sum of the volumes of the components. But in the case of bronzes or alloys of copper and tin, the volume of the alloy is less than the sum of the volumes of the components by as much (in some cases) as 7 per cent. In other words, the density of the alloy is greater than the calculated mean. On the other hand, certain alloys of lead and tin have densities less than the calculated mean—there is an increase of volume as the alloy is forming; and the same is true of the alloys of copper, zinc, and tin, known as German silver.

Similar peculiarities are met with in the mixing of liquids or solutions. For example, the volume of a mixture of approximately equal volumes of water and alcohol is about 3·6 per cent. less than the sum of the volumes.

In these cases of contraction other effects may generally be observed. Thus, as the alcohol and water become mixed, there is an appreciable rise in temperature.

§ 79. **The Hydraulic Press.**—In any fluid at rest under the influence of gravity the pressure is the same at all points at the same level. Suppose we have two vessels, one wide and the other narrow, filled with water and communicating with each other by means of a channel, as shown in fig. 63. Let also pistons, fitting tightly in their respective vessels, rest on the surfaces of the liquid. Suppose for definiteness that the area of cross-section of the narrow vessel (B) is 1 square inch, and that of the wide vessel (A) 100 square inches. Then whatever pressure P be exerted over B, the same pressure per unit area will be exerted over A. Hence the total thrust acting over the surface of the large piston

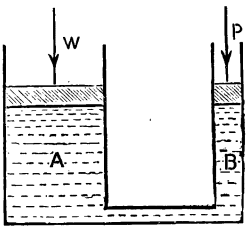


Fig. 63.

will be 100 P. The same conclusion is readily arrived at by application of the principle of work. Let the areas of A and B be as n to 1. Then, if the piston in B moves down through a small distance x, the piston in A must rise through a distance x/n, for the action means the transference of a small volume of liquid from B to A. Neglecting the work done in transferring this liquid, we find that the work done by P acting through x is spent in raising W through x/n, and that consequently $Pn = W$.

This is the principle of action of the hydraulic press, one form of which, known as the Bramah Press, is shown in section in fig. 64.

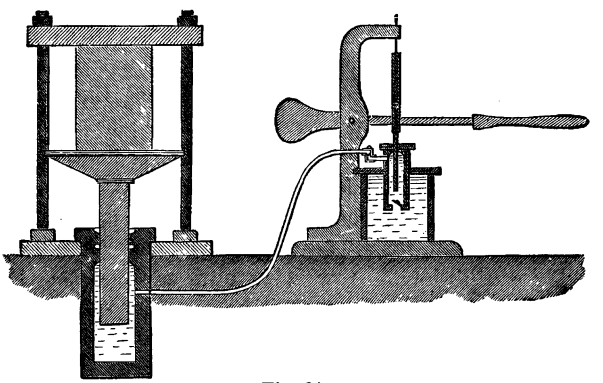


Fig. 64.

There are two chambers filled with water and communicating by a pipe. The chambers have very different cross-sections, and each is provided with a plunger or piston which can nearly fill it. The small plunger is worked up and down by a lever. At every down-stroke so much water is forced from the small chamber into the large one, and the large piston is consequently forced up through a certain distance. At every up-stroke a valve in the connecting tube closes and prevents any back-rush of water, while a valve in the bottom of the small chamber opens and allows water to flow in from a surrounding vessel so as to supply the deficiency. Thus, at every complete stroke, the large piston moves up through a certain small distance. When finally, as shown in the figure, it begins to press upon some substance placed between it and the

fixed platform above it, the total pressure brought to bear upon the substance will be much greater than the force applied at the handle of the lever. For example, let the lever have a mechanical advantage of 6, so that a force of one pound-weight applied at the handle will produce a force of 6 lb.-weight on the small piston. If then the areas of the pistons are as 1 to 100, the total pressure exerted on the large piston will be 600 lb.-weight, and this will be the total pressure applied to the substance that is being squeezed. By the exertion of a comparatively small force we are able by mere manual labour to squeeze and permanently distort wood, lead, silver, gold, and many other solid substances. The principle is of very wide application in hydraulic machinery.

§ 80. **Flotation.**—Any body will float which can displace its own weight of liquid. But, unless the body be perfectly symmetrical or be wholly immersed, it will not float indiscriminately in any position. A long cylinder like a pencil or wine-cork cannot float with axis vertical; nor can a flat disc float edge up. On the attempt being made the former will fall on its side, the latter on its flat end. A uniform sphere will float in any position.

In general, then, for all floating bodies there is a position of stable equilibrium. Now it is obvious that the forces acting on the body in any position are the weight of the body, which acts downwards through its centre of mass, and the resultant fluid pressure, which acts upwards through the point called the *Centre of Buoyancy*. This point would be the centre of mass of the liquid, which would refill the region of the displaced water. That there may be equilibrium, stable or unstable, these two equal and opposite forces must act in the same line, so that the centre of buoyancy must lie vertically above or below the centre of mass. When any slight displacement is given, these forces will in general act in parallel lines and form a couple. If this couple act against the displacement the equilibrium will be stable, if with it the equilibrium will be unstable.

For example, contrast the two following cases (fig. 65). In both G represents the centre of gravity of the floating body,

C the position of the centre of buoyancy when the body is in equilibrium, C′ the position of the centre of buoyancy for a slight displacement. Now, it is evident at a glance that for the case *a* the couple formed by the two parallel forces through G and C′ acts clockwise—that is, with the displacement. On the other hand, the similar couple for the case *b* acts counter-clockwise. Hence the former is a case of instability, the latter a case of stability.

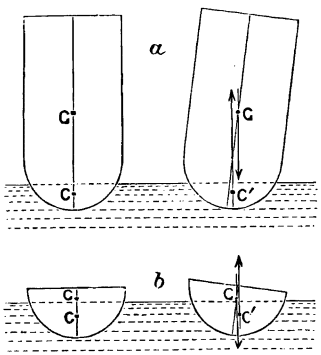


Fig. 65.

Produce the line of buoyancy (the vertical line through C′) till it meets the line through G which was vertical in the position of equilibrium—that is, the line GC regarded as being fixed in the body. The point of meeting in the case *a* lies *below* G, in the case of *b* *above* G. A little consideration will show that the condition of equilibrium may be expressed in terms of the position of this point of intersection or *Metacentre.*

Definition: The Metacentre is the point where the line in the body which coincides with the equilibrium position of the line of buoyancy is cut by the line of buoyancy for a position slightly disturbed from that of equilibrium.

The equilibrium is stable when the metacentre is above the centre of gravity of the body; neutral when it coincides with the centre of gravity; and unstable when it lies below the centre of gravity. The height of the metacentre above the centre of gravity measures the *stability* of the floating body. A large displacement of the floating body from its position of equilibrium is in general accompanied by a change in the position of the metacentre. For practical stability in a ship, the metacentre should rise as the vessel rolls. A rapid rise

indicates a 'stiff' ship with large initial stability; while a slight rise makes an 'easy' boat that rolls much but slowly.

§ 81. **Head of Water.**—Let there be a tank filled with water whose upper surface is exposed to the pressure of the atmosphere, and let a small orifice be opened anywhere in the wall of the tank. The water element behind this orifice will be exposed on the outside to the atmospheric pressure; but on the inside it will be exposed to a pressure which is greater than the atmospheric pressure by the pressure due to the height of the water-column from the orifice to the free surface. Hence the water will be forced out through the orifice. As may be easily shown experimentally, the water is driven out more briskly when the orifice is nearer the bottom of the tank —that is, when the pressure is greater.

If the tank is connected by means of a pipe with a second tank, we know that the water will fill the second tank to the level of the water in the first—that is, the pressure due to a given height of water-column can force water up to the same height. If a pipe from the bottom of the tank is bent up so that the water will issue from the orifice in a vertical direction (fig. 66), the same thing will hold. The pressure due to the water in the tank will be sufficient to force water through the orifice to the height of the free water surface. This is the principle of all fountains.

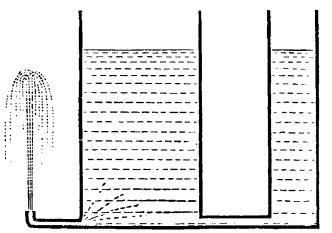


Fig. 66.

Thus each particle of water is projected with a speed which is able to lift it to the height of the column that produces the pressure. If this height is h, and the speed of efflux v, the relation between them is evidently

$$v^2 = 2gh,$$

a formula first established in this connection by Torricelli.

The pressure that acts so as to produce this speed v is $g\rho h$, where ρ is the mass of unit volume, and thus

$$\tfrac{1}{2}\rho v^2 = g\rho h = p.$$

This pressure is called the *head of water*; and $\frac{1}{2}\rho v^2$ measures the kinetic energy of the mass ρ—that is, of unit volume of the fluid. Before the orifice is opened each unit of volume possesses potential energy equal to the head of water; and this becomes transformed into kinetic energy as the efflux is allowed to take place.

Practically, the water of a fountain falls short of the level of the water which supplies the head, because there is loss of energy due to friction as the water escapes through the orifice. The amount of loss is found to depend upon the size and form of the orifice and the material of which it is made.

Since the speed of efflux is inversely as the square root of the depth of the orifice below the free surface, it is evident that the quantity of fluid that passes through a given orifice in a particular time will also be inversely as the square root of the depth. Thus, with two equal orifices in a tank at depths of one foot and four feet respectively below the free surface, it will be found that the discharge through the lower orifice in any time is twice the discharge through the upper orifice.

§ 82. **Pressure in Flowing Liquid.**—When water is flowing through a pipe its energy is partly due to its motion, partly to its head. If P be the head in pounds-weight per square inch, the corresponding height in feet is measured by the quantity $P/g\rho$. If V be the speed, the corresponding height is $V^2/2g$. Hence the whole energy per unit volume of the liquid is that due to a height

$$\frac{P}{g\rho} + \frac{V^2}{2g}.$$

In steady motion this remains constant (neglecting loss due to friction, &c.). Hence if V be made to change, P

will change correspondingly, increasing if V diminishes, decreasing if V increases.

Froude has illustrated this truth by the following simple experiments, in which a steady stream is kept up through pipes of variable section. In fig. 67 are shown two tubes A and B of variable section. At different points small tubes are inserted, through which water rises to certain heights depending on the pressures in the liquid flowing through the pipes. It will be found that the column rises higher in the small tube which is at the wider part of the pipe; and it is just at the wider part that the speed of the stream is less.

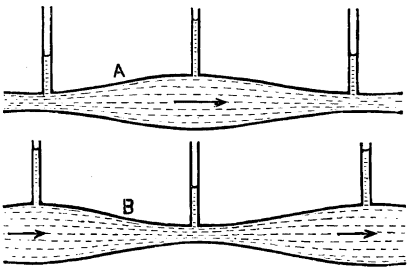


Fig. 67.

In the flow of liquids through pipes there are various resistances which cause loss of energy. The most important of these is that due to the surface friction of the pipe. This resistance is found to be directly as the length of the pipe, and inversely as its diameter. Were there no such frictional resistance, the amount of liquid that would with a given speed flow across any section of a uniform pipe would be directly as that section, so that a 2-inch pipe would discharge four times as much liquid as a 1-inch pipe. But because of the resistance just mentioned, the discharge through the smaller pipe suffers a greater proportional diminution than the discharge through the larger pipe. If a single pipe bifurcates, then the *kinematic* condition for a steady full flow is that the area of section of the single pipe shall equal the sum of the sections of the branches. But since this condition requires each branch to be necessarily of a smaller diameter than the single pipe, the resistance to flow in it will be greater. Hence it is *mechanically* impossible for two such branches to give a discharge equal to that of the single pipe. To utilise the single pipe to the full, the sections of the branches must be increased.

In the case of tubes of very narrow bore—that is, capillary tubes—the discharge is as the fourth power of the radius, an important increase to the resistance being caused by the **Viscosity** of the fluid (see next article).

§ 83. **Viscosity or Fluid Friction.**—When a vessel partially filled with liquid is slightly tilted the liquid surface will move so as to become horizontal again. Some liquids settle down to their new position of equilibrium after a succession of rapid oscillations like a pendulum; while others do not oscillate, but gradually creep to their final configuration. The latter are said to be more viscous than the former. When the *viscosity* is small the oscillations will continue for a considerable time, and the *mobility* of the liquid is great. Sulphuric ether has great mobility and small viscosity; treacle has small mobility and great viscosity. Water is fairly mobile, and resembles sulphuric ether rather than treacle.

Again, let the water in a bowl or pail be set in rapid rotation and then left to itself. Gradually the rate of rotation decays until the water comes to rest. Energy is lost in overcoming some kind of resistance to motion. The water in contact with the surface of the vessel is retarded by tangential forces due to friction. Thus the outer ring of fluid is rotating more slowly than the ring of fluid immediately inside it. Between these two contiguous portions of fluid moving at different speeds a tangential stress acts, tending to retard the more quickly moving portion and to accelerate the more slowly moving portion. Because of the constantly acting retardation on the outermost elements of fluid, the final effect is a retardation on all the elements. For slow motions it is probable that the liquid in contact with the retaining wall does not slide past the wall, but is held at rest against it. In this case the retarding action is due entirely to fluid friction between the contiguous elements of fluid.

Another illustration of the effects of fluid friction is shown in the manner of flow of water in a channel. By observing the behaviour of fine particles suspended in the water, we note that the upper parts shoot ahead of the lower parts of the fluid, and the central parts gain upon the parts near the margin. A column of fluid originally vertical will become more and more inclined forwards as it flows on with the

stream. The vertical column A becomes the inclined column A′ (fig. 68); a change which is produced by the sliding of one layer over another. There is friction between the lowest layer and the bottom of the channel; and there is friction between every successive pair of layers.

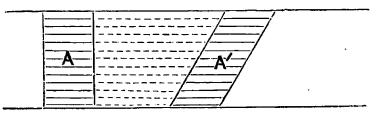


Fig. 68.

In the decay of all kinds of relative motion in fluids—ocean waves, eddies, currents, winds—viscosity is the prime factor. It is the same property which brings about the resistance experienced by a solid to motion through a fluid. For such motion necessarily sets up motion in the contiguous parts of the fluid. The solid imparts energy to the fluid; and the elements of fluid so set in motion quickly come to rest as they mingle with the rest of the fluid.

Finally, if it were not for viscosity it would be impossible to set up continuous relative motion in a fluid. An oar drawn through a perfect frictionless fluid would experience no resistance at all; or, if it did experience a slight loss of energy because of the skin of fluid which might be supposed to adhere to it, it would experience no more, for between the fluid skin adhering to it and the rest of the fluid there is, by hypothesis, no tangential stress.

§ 84. **Vortex Motion.**—A spoon or oar drawn half-immersed along the surface of a liquid is followed by two eddies or whirls which retain their identity for a considerable time. These eddies are not independent, but are the ends of a bent column of water rotating about its axis, the shape of the column being roughly similar to the edge of the spoon or oar whose motion generated it. The same rotational motion is often shown by smoke rings issuing from a funnel. As the smoke leaves the funnel the outer parts are retarded while a strong blast is blown through the heart of the ring. This sets up a rotation about the axis, so that, as the ring moves on as a whole, every section of the ring is itself a whirl or eddy.

Vortex motion, of which these are illustrations, may be defined as fluid motion in which the elements are rotating, so that if any small element of the fluid became solidified it would be found to be rotating with a definite moment of momentum. If vortex motion exist in a perfect fluid, it can be proved, as was first done by Von Helmholtz, that the rotating elements will preserve their rotational characteristic for ever. It will be impossible to destroy this characteristic in the vortex element; and it will be as impossible to produce vortex motion in an element which is non-rotational in its motion. Again, those vortex elements in the perfect fluid are necessarily arranged in continuous, endless ring-shaped filaments, or they come to an end at the boundary of the fluid. Viscosity alone can destroy vortex motion; and it is in virtue of viscosity that vortices can be generated in ordinary fluids.

Von Helmholtz's theory of vortex motion suggested to Lord Kelvin the theory of the vortex atom. The hypothesis is that the ultimate indestructible atoms of matter are vortex rings in a perfect fluid which fills all space. Since any two originally distinct vortices remain always distinct and cannot mix, impenetrability appears as an essential character of the vortex atom.

EXERCISES.

1. When a plane surface, inclined to the horizontal, is immersed in an incompressible fluid, the pressure will not be the same at every point. Show that the *average pressure* is equal to the pressure at the centre of inertia of the plane area, and that the total pressure on the surface is equal to the average pressure multiplied by the area.

2. Calculate the total pressures in pounds-weight exerted on the bottom and walls of a cubical vessel, 1 cubic foot in volume, which is half full of mercury and half full of water.

3. The bend of a U-tube, whose section is $\frac{1}{2}$ square inch, contains mercury to a height of 6 inches in each leg. How much water must be poured into one of the legs so that the free water surface may be 6 inches higher than the mercury surface in the other leg?

4. A solid weighs 35 ounces in air, and 5 ounces in water. When immersed in another liquid it weighs 14 ounces. Calculate the specific gravities of the solid and liquid.

5. A balance is arranged under water, and a mass of iron ore of density 7 in one of the pans is counterpoised by 3 kilogrammes of lead of density 11 in the other. What is the mass of the iron ore?

6. A body floats with one-tenth of its volume above the surface of pure water. What fraction of its volume would project above the surface if it were floating in liquid of density 1·25?

7. An alloy of density 8·74 is formed by combining 80 parts by weight of copper and 20 parts by weight of tin. The densities of copper and tin are 8·9 and 7·3 respectively. By what percentage does the volume of the alloy differ from the sum of the volumes of the copper and tin?

8. By means of a Bramah Press, a total pressure of 1 ton weight is produced by a force of 5 lb.-weight acting at the extremity of the lever. The diameters of the pistons are as 8 to 1. Find the ratio of the arms of the lever.

9. Calculate the speed of the efflux of water through a small hole in the bottom of a full tank 6 feet high.

SUMMARY.

§ 69. The ideal Fluid. Fluid Friction or Viscosity. Hydrostatics and Hydrokinetics. Compressible and Incompressible Fluids.

§ 70. Pressure in a Fluid. Pressure at a point.

§ 71. Liquid under Gravity. Pressures per square inch due to Water and Mercury Columns.

§ 72. The Atmospheric Pressure.

§ 73. The Barometer.

§ 74. The Siphon.

§ 75. The Suction-pump.

§ 76. Buoyancy.

§ 77. Specific Gravity. Density. Determined by weighing. Specific-gravity Bottle. Hydrometer. Apparent Weight in Air.

§ 78. Densities of Various Substances. Alloys and Solutions.

§ 79. The Hydraulic Press. Bramah Press.

§ 80. Flotation. Centre and Line of Buoyancy. Metacentre.

§ 81. Head of Water. Torricelli's formula. Speed of efflux of water from tank.

§ 82. Pressure in Flowing Liquid. Froude's Experiments. Flow in Pipes. Flow in Capillary Tubes.

§ 83. Viscosity or Fluid Friction.

§ 84. Vortex Motion. Kelvin's Vortex Atom.

CHAPTER X.

ELASTICITY OF FLUIDS.

§ 85. **Elasticity.**—Any body which has had its original form or bulk changed by application of suitable forces, and which requires the continued application of the forces to prevent entire or partial recovery, is said to be elastic. If the forces are removed the recovery takes place, complete or partial according to circumstances. The ordinary rubber substance called *Elastic* is elastic not because it can be easily stretched, but because it can recover itself when the stretching-force ceases to act. It is well known, however, that under repeated or long-continued stretchings this substance loses the power of complete recovery, so that its elasticity is really imperfect.

As mentioned above, the change may be in form or in volume. Such a change is called a **strain,** and the combination of forces which produce it or sustain it is called the corresponding **stress.** The theory of elasticity is the theory of the relation of stress and strain.

Fluids and solids differ markedly as regards their elastic properties; for in fluids there is no elasticity of form. A fluid may be readily deformed, but there is no tendency to recovery; whereas solids, in general, when bent or twisted or otherwise strained have a tendency to recover. Continued application of a form-changing stress on a fluid is accompanied by a continual change of form. In the case of the solid a similar continued application of stress simply sustains the strain at first produced. Fluids, then, have no elasticity of form, but have only elasticity of bulk. In such a case the strain is a change of density, and the corresponding stress a hydrostatic pressure.

§ 86. **Compressibility.**—In all known cases the effect of increased pressure is to compress the substance—that is, diminish its volume, increase its density. The strain is measured by the **compression** or change produced in unit volume; the corresponding stress is the **pressure** estimated per unit surface. The ratio of the compression to the pressure is called the **compressibility.** In gases, which yield readily to change of pressure, the compressibility is comparatively large. In liquids, which yield very slightly to even large pressure, the compressibility is small.

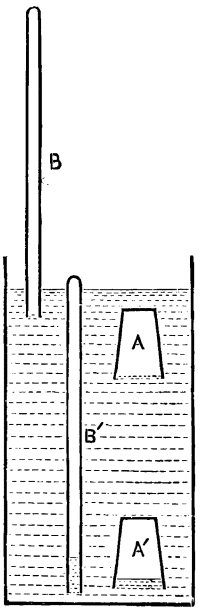


Fig. 69.

§ 87. **Compression of a Gas.**—Invert a long tumbler on water which fills a tall glass jar, and push it down to the bottom (fig. 69). At first the tumbler appears full of air (A), but as it descends the water rises within the tumbler, rising higher as the tumbler gets lower (A′). This is because the air in the tumbler is compressed more and more by the increasing pressure at the greater depths. A better way of studying the same effect is to take a long, narrow tube closed at the upper end, as shown at B in the figure. The compression of the air will be more apparent than with the tumbler.

§ 88. **Boyle's Law.**—It is necessary to measure the relation between the change of volume and the pressure producing the change. This was first done by Boyle,* and his form of apparatus is still recognised as by far the best for the purpose.

A long glass tube (the longer the better) is bent round near one end so as to have the shape of

* Boyle published his *Defence of the Doctrine of the Spring and Weight in Air* in 1662; Mariotte, after whom the law is generally named on the Continent, published his *Discours de la Nature de l'Air* in 1676.

a J. The shorter leg is closed above, and mercury is introduced into the bend so as to enclose a definite quantity of air in the shorter leg. Care should be taken to have the mercury surfaces in the two legs at exactly the same level (fig. 70*a*). Then we know that the air in the shorter leg is subjected to a pressure of one atmosphere. Now, pour mercury into the longer leg until the air has been reduced to half its original volume (*b*). It will be found that the column of mercury in the open leg is longer than the column in the closed leg by an amount which is almost exactly the height of the barometer column, so that the pressure on the enclosed air is two atmospheres. Pour in more mercury (if the tube be long enough), and, when the air is compressed to a third of its original volume, the mercury column in the open leg will be found to be twice the height of the barometer column (*c*). That is, under three atmospheres of pressure, the volume is reduced to one-third of its original volume.

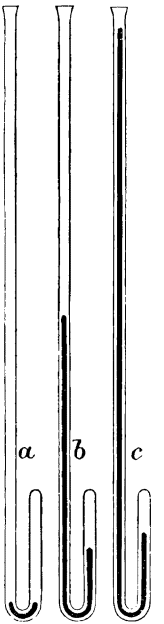


Fig. 70.

These particular increments of pressure have been chosen so as to bring out the great simplicity of the law established by Boyle; but many other intermediate values of the pressure might be used. Whatever be the length of the mercury column between the two surfaces we add to it the length of the barometer column, and so obtain an estimate of the pressure in terms of height of mercury column. We then measure, what is sufficiently accurate in such a rough experiment, the length of the air-space in the shorter tube. The product of these two numbers, which are proportional respectively to the pressure and volume, will always be the same. In symbols

$$PV = \text{constant}$$

where P is the pressure and V the volume of a given quantity of air. In words:

The volume of a gas is inversely proportional to the pressure; or, the density of a gas is directly proportional to the pressure.

The apparatus just described is specially suited for working with pressures greater than one atmosphere. To establish the law for smaller pressures, Boyle used two tubes, one slipping freely within the other. The arrangement is shown in fig. 71.

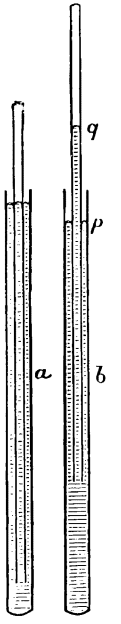


Fig. 71.

A little air is introduced into the inner tube in such a way that when the inner tube is well pushed down mercury fills both tubes to the same level (position *a*). The air enclosed above the mercury in the inner tube is therefore at a pressure of one atmosphere, say 30 inches of mercury. To fix our ideas, let the length of the air-space be 6 inches. Now, if the inner tube be drawn up out of the mercury, the level of the mercury in the outside tube will sink more quickly than the level in the inside tube. Suppose the inner tube to be drawn up till 6 inches of mercury column separate the two levels p and q; then it will be found that the air-space has increased in length to $7\frac{1}{2}$ inches. But

$$7\tfrac{1}{2} : 6 = 30 : 24,$$

and $30 - 24 = 6$, the height of the column pq. Similarly, if pq is equal to 12 inches, the air-space will expand to length 10 inches; for

$$10 : 6 = 30 : 18$$

and

$$18 + 12 = 30.$$

In short, the pressure in the air-space at the top of the inner tube is equal to the pressure of the atmosphere diminished by the pressure due to the difference of level of the mercury surfaces.

It should be noted, as the discoverer himself pointed out, that Boyle's Law holds only when the temperature of the gas is kept constant throughout a series of measurements. The effects of heat will be studied later (§ 123).

§ 89. **Boyle's Law not accurate.**—The careful investigations of Regnault, Natterer, Andrews, Amagat, and others have shown that in no case does any gas accurately obey Boyle's Law. For air, nitrogen, oxygen, hydrogen, and marsh-gas the deviation is small at moderate pressures; for vapours like steam or carbonic acid gas, which are near their liquefying points, the deviations become considerable.

The most recent are Amagat's measurements, made with a Boylean apparatus 1000 feet long, so that the pressure was accurately known up to 400 atmospheres. A few of his results will show how far air deviates from correspondence with Boyle's Law:

Pressure in Atmospheres.	PV at 0° C.
1	1·0000
100	·9730
200	1·0100
300	1·0975
500	1·3400
700	1·6020
1000	1·9920

At a pressure of 152·3 atmospheres (about one ton weight per square inch) air fulfils accurately Boyle's Law. For smaller pressures the Law assigns too high a volume and at higher pressures too small a volume. In other words, air is at first more compressible, but at high pressures less compressible than a gas which obeys Boyle's Law. Nitrogen and oxygen are similar to air; but hydrogen is less compressible than the Law requires at all pressures.

The elasticity of gases is perfect. After compression the gas will on being released recover exactly the original volume. The same perfect elasticity is also possessed by liquids.

§ 90. **The Compressibility of the Boylean Gas.**—Compressibility is the ratio of the change of unit volume to the pressure producing it. Let V be the volume at pressure P, and v the volume at pressure p. By Boyle's Law,

$$PV = pv,$$

also,

$$pV = pV.$$

Subtracting, we get $V(p - P) = p(V - v)$

or

$$\frac{1}{p} = \frac{V - v}{V} \frac{1}{p - P}.$$

But the quantity $(V - v)/V$ is the compression under the increase of pressure $p - P$, so that their ratio measures the average compressibility between the pressures p and P. Hence, by considering a very small increase of pressure and a correspondingly small diminution of volume, we find that the compressibility of a gas at a given pressure p and *at a constant temperature* is measured by the reciprocal of the pressure $(1/p)$. The elasticity of bulk or resistance to compression is therefore measured by the pressure.

§ 91. **Rarefaction and Condensation of Gases.**—Boyle's second form of experiment illustrates the expansive power of air, in virtue of which it completely fills the closed region containing it, however large that region may be made. The more the air expands the less is the density, the smaller also the pressure. A very small density may be obtained by greatly enlarging the enclosing space. Practically this method by itself is inconvenient; but it becomes practical when taken in conjunction with a means for abstracting portions of the air. The principle of so rarefying air or other gas is well shown in the simplest form of 'exhausting syringe.'

The region to be 'exhausted' or reduced to a practical 'vacuum' is connected to a tube which contains a small valve, and in which a valved piston works (fig. 72). The principle is in fact identical with the principle of the suction-pump.

At every in-stroke of the piston B the air in the part between the two pistons is forced out through the piston-valve. At every out-stroke the space between the pistons is left nearly empty of air, and the pressure is correspondingly diminished. Hence the pressure of the air in the vessel A, being the greater, forces open the valve, and part of this air expands into the region between the valves. At the succeeding in-stroke the valve in the tube at once closes, and the air in the intervalvular region escapes through the piston-valve. Thus the air in the vessel has been reduced in quantity and its density correspondingly diminished. By a great number of repetitions of this process more and more of the air is gradually abstracted from the vessel, until the pressure is reduced to perhaps the 500th part of its original value.

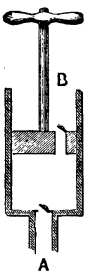


Fig. 72.

The low pressures exerted by moderately-rarefied gases are measured in heights of the corresponding mercury column. Thus, since 760 millimetres of mercury correspond to 1 atmosphere, the 500th part of an atmosphere would be equivalent to 1·52 millimetres of mercury.

The ordinary air-pump, one form of which is shown in fig. 73, is a development of the exhausting syringe; but the various devices for improving its action cannot be entered into here. Leakage of every kind must be prevented, and this can only be effected by perfect fitting of valves and stopcocks. In the most improved air-pump of this type the pressure of air may be reduced to one-tenth of a millimetre of mercury, or 140 millionths of an atmosphere.

To produce still lower pressures, or (as they are usually called) 'higher vacua,' different methods are adopted, the principle of which depends upon the properties of the Torricellian Vacuum. Geissler's and Sprengel's pumps are the best known types of apparatus made on this principle; and by the most improved forms pressures less than a hundred-millionth part of an atmosphere have been obtained.

The action of the Condensing Syringe is just the reverse of the exhausting syringe. The valves are arranged to open towards the other side. At every out-stroke of the piston, air streams in through the piston-valve to fill the otherwise empty space, while the other valve closes should there be any tendency for the air in the vessel to escape. At the succeeding in-stroke of the piston the piston-valve closes, and the air between the valves is forced through the tube-valve into

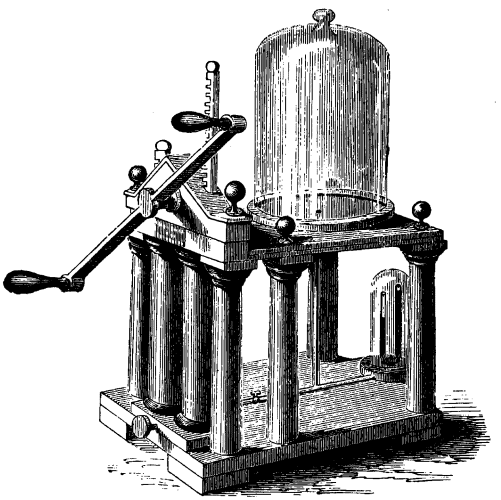


Fig. 73.

the vessel. In the air-gun, air at a high pressure is in this way condensed within the hollow stock, and each time the trigger is pulled a puff of this highly-condensed air escapes, and by its expansive force propels the bullet along the barrel. In the child's toy known as the pop-gun the same principle is exemplified. The piston compresses the air until the increased pressure overcomes the friction which holds the plug in position, and the plug is propelled sharply forward. The condensing syringe plays an important rôle in

these days in connection with the pneumatic tyres of bicyles and other vehicles. Its action is identical with that of the force-pump.

§ 92. **The Manometer.**—The great simplicity of Boyle's Law suggests the use of a gas for measuring pressures. An instrument constructed for this purpose is called a manometer, which in its simplest form is virtually nothing else than the closed end of a Boylean tube (fig. 74). The open end is connected with the region in which is the substance whose pressure is to be measured. This is separated from the closed end by a quantity of mercury which fills the bend. The pressure in the connected region is measured in terms of the volume of air in the closed end, and the difference of the levels of the mercury surfaces.

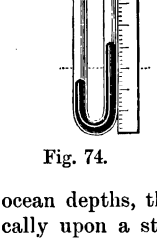


Fig. 74.

Lord Kelvin's apparatus for deep-sea sounding is an air manometer. Sea-water takes the place of mercury. As the instrument descends to the ocean depths, the water is forced up the closed tube, acts chemically upon a strip of prepared paper, and so leaves a permanent mark of the distance to which it was forced. When the apparatus is drawn up, the air of course expands, the sea-water retreats to its original position; but from the mark left the pressure can be at once estimated, and from the pressure the depth is easily calculated. In the prepared tubes the paper strips are graduated to give the depth in fathoms at once.

Having carefully investigated the compression of nitrogen by means of his huge Boylean apparatus, Amagat constructed a nitrogen manometer, and by means of it studied with greater ease the compressibilities of other gases.

§ 93. **Variation of Atmospheric Pressure with Height.**—Since the pressure in the air is due to the weight of the superincumbent column of air, it is evident that the pressure must diminish as we ascend. This may be demonstrated experimentally by measuring the height of the barometer column first at the base and then at the top of a hill. In fact, a good barometer will 'fall' appreciably if taken up a flight of stairs.

The pressures at two points at different heights differ by the weight of the intermediate vertical column of air of unit

section. Hence, as in § 71, if h be the vertical distance between the two points, ρh the mass of the intermediate column of air of unit section, so that ρ is the *mean* density, then the *difference of pressure* is

$$p = g\rho h.$$

In the case of water we could assume ρ to be practically constant; but in the case of air or other gas the density varies directly as the pressure, and the quantity ρ is not even approximately constant. If P is the pressure at the lower of the two points, we may write

$$\mathrm{P} = g\rho\mathrm{H}$$

where H is the height of an ocean of fluid of uniform density ρ, which would give the required pressure P. This quantity H is called the *height of homogeneous atmosphere*, and is easily calculated when the pressure and density are known. For example, if the barometer stands at 30 inches, and if the density of the air is ·00129, that of mercury being 13·6, we at once find the value of H by the simple proportion,

$$\mathrm{H} = 30 \times \frac{13{\cdot}6}{{\cdot}00129} = 316{,}000 \text{ inches} = 5 \text{ miles nearly.}$$

Since, by Boyle's Law, P is directly as ρ, the quantity H is always the same, whatever point it is reckoned for, if the temperature be constant. That is to say, the height of homogeneous atmosphere for the top of a mountain is 5 miles *above that mountain top*, just as it is 5 miles above the sea-level from a point at the sea-level. In short, H is an absolute constant for any equilibrium condition of the atmosphere assumed to be at a uniform temperature.

Dividing, then, the equation in p by the equation in P, we get

$$\frac{p}{\mathrm{P}} = \frac{h}{\mathrm{H}},$$

or the diminution of pressure, due to a small increment of

height, bears to the pressure the same ratio which this increment of height bears to the height of homogeneous atmosphere. Let us take successive increments of height equal to 1 inch; then for every inch of ascent, the pressure diminishes by the same fraction of itself, namely 1/H. This is the principle of compound interest, and may be expressed otherwise thus:

The density and pressure diminish in geometrical progression as the height increases in arithmetical progression.

The formula just given leads by rigorous mathematical processes to a convenient expression for the difference of height between two stations in terms of the pressures there. The expression is

$$h_2 - h_1 = \mathrm{H} \times 2{\cdot}3026 \log.\frac{\mathrm{P}_1}{\mathrm{P}_2}$$

where h_1 h_2 are the heights of the two stations at which the observed barometric pressures are P_1 P_2. The logarithms are the common logarithms to base 10, and H must be calculated for the temperature which is the arithmetic mean of the temperatures at the two stations. Even this is, however, at best an approximation; and it is only from an accumulation of a great many simultaneous pairs of barometric readings at the top and at the base of a mountain that a fair estimate of the height can be made.

§ 94. **Compression of Liquids.**—Liquids are so slightly compressible that it was long before their compressibility was proved, far less measured. Bacon tried to compress water by squeezing and hammering a closed leaden shell full of the liquid. He simply proved that lead was porous, for water exuded like perspiration through the metal. The Florentine academicians (in 1692) had exactly the same experience with a silver shell, even when the shell was covered by a thick layer of gold.

The first to prove the compressibility of water was

Canton (1762), who also made the discovery that water differs from other liquids in being less compressible at higher temperatures. Perkin (1820) showed that the compressibility of water depends upon the pressure, diminishing as the pressure rises. Oersted (1822), by the invention of the **Piezometer,** greatly improved the method of experiment, so that it is now a very easy matter to observe and measure what the Florentine academicians failed to find any evidence of.

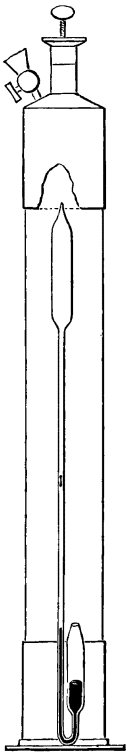


Fig. 75.

In its improved modern form, the piezometer consists of a glass bulb and stem like a thermometer, but open at the end. It is filled with liquid and then inverted so that its open end dips in a dish of mercury, or is turned up like a siphon barometer tube. The whole is then placed in a strong cylindrical vessel full of water (fig. 75), the pressure within which may be increased either by screwing in the top or by having it part of a hydraulic press. As the pressure increases on the exposed mercury-surface the liquid in the piezometer is compressed, and the mercury column begins to rise in the stem, pushing up in front of it a small index, which remains at the highest point reached.

The glass of the piezometer is, of course, also compressed by the hydrostatic pressure of the water outside and the liquid inside. This produces a *diminution* of the volume occupied by the liquid that is being tested; and because of this the mercury column will not rise quite so high as it would do were the piezometer made of an incompressible material. Thus the piezometer gives not the compressibility of the liquid but the difference of the compressibilities of the liquid and the glass. Before the true compressibility of the liquid can be deduced, that of the glass must be measured by some other process. In short, the experimental determination of the compressibility of liquids depends upon a preliminary determination of the compressibility of the substance of the piezometer. But, unfortunately, the determination of the compressibilities of solids is an extremely difficult quest.

§ 95. **Compressibilities of Liquids.**—If v_0 be the original volume of a mass of liquid and v the volume produced by an added pressure p, the average compressibility is measured by the ratio $(v_0 - v)/pv_0$.

Amagat has recently carried out an elaborate series of experiments on various liquids up to pressures of 3000 atmospheres. The compressibilities diminish with increase of pressure, and, except in the case of water, increase with temperature. Thus the average compressibility of water within the first hundred atmospheres is ·0000511 at 0° C. and ·0000499 at 50° C., increasing again at higher temperatures.

At ordinary temperatures and moderate pressures we may take $\frac{1}{22000}$ as a fair estimate of the compressibility of water, estimated per atmosphere of pressure. To find the compressibility in C.G.S. units we must divide this number by $1{\cdot}014 \times 10^6$, which is the equivalent, in dynes per square centimetre, of one atmosphere. This gives $4{\cdot}45 \times 10^{-11}$. The reciprocal of the compressibility, or $2{\cdot}25 \times 10^{10}$, measures what is known as the resistance to compression, estimated in dynes per square centimetre. The compressibility of sea-water is less than that of pure water in the ratio of ·925 to unity.

The following are the compressibilities per atmosphere of various liquids:

Liquid.	Compressibility per Atmosphere.	Liquid.	Compressibility per Atmosphere.
Water at 0° C.	·0000503	Ether at 21·4° C.	·000148
" " 15·9°	·0000450	Alcohol at 17·5°	·000096
" " 61°	·0000389	Sulphurous acid at 14°	·000303
" " 77·4°	·0000398	Mercury at 0°	·0000029
" " 99·2°	·0000409		

These numbers are not to be regarded as absolutely accurate, for there is considerable discrepancy in the results obtained by different experimenters. In all cases there is the doubt as to the true value of the compressibility of the substance of the piezometer.

EXERCISES.

1. A cylindrical beaker, originally full of air, is inverted and immersed in water. How far must it be pushed down so that the water may rise 1 inch in it, the length of the beaker being 6 inches?

2. A little air occupies the space above the mercury in a faulty barometer, which registers only 28 inches when the true barometric height is 30 inches. What is the pressure of the contained air in pounds-weight per square inch?

3. If, in the conditions stated in Question 2, the air occupies a portion of tube 6 inches long, calculate the true height of the barometer when the faulty barometer shows only 26 inches.

4. In a Kelvin sounding apparatus, the volume of the air is compressed to $\frac{1}{10}$ of its original value. Calculate the depth reached in fathoms, the mean density of the sea-water being 1·03.

5. Calculate the approximate height of a mountain at whose summit the atmospheric pressure, as measured on a barometer, is 26 inches, the pressure at the sea-level being 29·5 inches.

6. Two liquids have volumes which are as 2 to 5. They are compressed by the same amount under pressures which are as 2 to 1. Compare their compressibilities.

SUMMARY.

§ 85. Elasticity. Strain and Stress. Elasticity of form and of Bulk. Fluids have Elasticity of Bulk only.

§ 86. Compressibility. Compression. Pressure.

§ 87. Compression of Gas.

§ 88. Boyle's Law. Two Forms of Experiment.

§ 89. Boyle's Law not quite accurate.

§ 90. Compressibility of the Boylean Gas.

§ 91. Rarefaction and Condensation of Gases. Exhausting Syringe. Air-pump. Geissler's and Sprengel's Pumps. The Condensing Syringe.

§ 92. The Thermometer.

§ 93. Variation of Atmospheric Pressure with Height. Height of Homogeneous Atmosphere. Measurement of Heights of Mountains.

§ 94. Compression of Liquids. Piezometer.

§ 95. Compressibilities of Liquids.

CHAPTER XI.

ELASTICITY OF SOLIDS.

§ 96. **Hooke's Law.**—The strains which solids undergo, involving as they do changes of form as well as changes of volume, are much more complicated than the strains discussed in the last chapter. Practically, the most important of these strains are conveniently considered under the headings of Extension, Flexure (or Bending), and Twist. We begin with a few experimental illustrations, so as to bring out the significance of the fundamental law connecting stress and strain. This law was discovered by Hooke (1676), and was enunciated thus: '*Ut Tensio sic Vis*—that is, the power of any spring is in the same proportion with the tension thereof. That is, if one power stretch or bend it one space, two will bend it two, and three will bend it three, and so forward.' In modern phraseology, Hooke's Law is expressed in the words:

Strain is proportional to Stress.

It is important to remark at once that this law, in general, holds only when the strain is small. With a very few exceptions, solids cannot undergo large strains. Any attempt to increase the strain beyond a certain limit results in the rupture or fracture of the material.

§ 97. **Extension, Extensibility.**—The stretching of a piece of india-rubber or 'elastic' is the most familiar instance of extension; but it should be borne in mind that this substance is elastic not because it can be easily stretched, but because it recovers itself when the stretching force is removed. Most solids, although not nearly so extensible as india-rubber, are quite as elastic.

Extension or elongation is defined as the change in unit length of the substance along any line in it. If there is increase in the unit length the extension is positive; if decrease, negative. Extension may take place along one direction only, or along two or more. If a substance undergoes equal elongations in three directions at right angles to one another, it undergoes equal elongations in all, and the result is a simple change of volume. If the changes in this special case are small, the compression will be three times the elongation or extension taken negatively. Again, solids may undergo extension because of change of temperature; and the metals iron, nickel, and cobalt change length when magnetised. But in this chapter we limit ourselves to elastic extension—that extension, namely, which is the result of external forces applied to the substance.

Corresponding to extension as the strain, it might be supposed that we had longitudinal tension or pull as the stress. But this is not strictly the case. To produce equal extensions in all directions we must apply a system of stresses likewise equal in all directions—viz., a hydrostatic tension or pressure according as the extensions are positive or negative. But an extension in one direction only is not produced by a tension acting in that direction. When a longitudinal tension, for example, acts upon a rod of metal, the rod not only lengthens but contracts at right angles to its length. This contraction of the section of the rod is, however, extremely small, so that practically it is left out of account, and the strain of the rod is taken as being measured by the extension in the direction in which the tension acts. The ratio of the extension to the tension which produces it is called the **Extensibility.** Its reciprocal is the important quantity known as **Young's Modulus.**

§ 98. **Young's Modulus** is defined as the ratio of the longitudinal stress to the elongation produced. It is determined by the simple experiment of stretching a wire or rod by appended weights and measuring the resultant increase in length. As with pressure in hydrostatics so with stress

generally—it is estimated per unit area. Thus, let a wire of length L and cross-section A be stretched by the weight W, so that its length becomes $L + l$. Then the extension, which in this case is taken as a measure of the strain, is the ratio l/L; and the longitudinal stress is W/A. Hence, Young's Modulus, being the ratio of the stress to the strain, is

$$E = \frac{L}{l}\frac{W}{A}.$$

The numerical value of E in any case depends upon the unit of force used and the unit employed in measuring the cross-section. For example, if the unit of force is the weight of one pound, and if the section is measured in square inches, the value of E for wrought-iron is about 25 million. This means that a rod of wrought-iron 1 square inch in section will, under a load of 10,000 lb. (4·46 tons), have its length increased by the $\frac{1}{2500}$ part. Thus, if the rod is 6 feet long, its length will be increased by fully $\frac{1}{35}$ of an inch.

Average values of Young's Modulus for various substances are given in the following table. The unit employed is pounds-weight per square inch. The numbers may be reduced to grammes-weight per square centimetre by multiplying by 70·3, and to C.G.S. units—that is, dynes per square centimetre—by multiplying by 69,000.

Substance.	Young's Modulus.	Tenacity.
Bessemer steel	30,000,000	115,000
Iron wire	27,000,000	85,000
Copper wire	17,000,000	60,000
Brass wire	14,000,000	49,000
Ash (along the fibre)	1,600,000	9,800
Oak (" ")	1,300,000	9,000
Glass	8,500,000	2,000

Young's Modulus might be defined as the stress which would double the length of the rod provided Hooke's Law held up to that limit. But in the great majority of cases the material snaps long before this limit is reached. The utmost longitudinal stress which the material can support without rupture is called the *Tenacity*. The tenacities for the substances just named are also given in the

table. In any particular case the ratio of the tenacity to Young's Modulus may be taken as an *indication* of the maximum elongation which the material can suffer under the influence of the breaking stress. It would measure exactly this elongation if Hooke's Law held throughout; but experiment shows that while the stress still falls considerably short of the tenacity, the material begins to yield more than is required by Hooke's Law. Just before the rupture occurs, the material, which is never perfectly uniform, yields most where it is weakest and becomes (comparatively speaking) very much drawn out. Across this diminished section the stress per unit area is correspondingly increased, and rupture rapidly ensues. The *working strength* of the substance — that is, the stress which can with safety be applied to it—is about $\frac{1}{4}$ or $\frac{1}{5}$ of the tenacity. It should be mentioned that across the fibre woods have much smaller tenacities than along the fibre with correspondingly lower values for Young's Modulus.

§ 99. **Flexure or Bending.**—The bow, the tuning-fork, and many kinds of springs fall to be discussed under this heading. Simple and instructive experiments may be made with a bar or plank fixed horizontally at one end and loaded at the other, or with a plank resting by its ends on trestles and loaded at the middle. These are among the experiments made by Hooke himself in establishing his Law.

The character of the strain is beautifully shown by bending a very flexible material like india-rubber. Let the dotted lines in fig. 76 represent a small portion of the block or bar before it is bent, and the full lines what these dotted lines become after the flexure. It will be seen that certain dotted lines originally parallel become inclined as if radiating from a centre O, which is the centre of curvature of the bent bar; while the lines perpendicular to these become circles with their centres at O. The *elastic central line* AB, which passes through the centre of inertia of each section, does not change in length when it becomes deformed into the arc AB′. Every line farther from O than AB′ increases in length, and the material along that line is subjected to a pull or tension. On the other hand, every line nearer to O than AB′ decreases in length, and the material along that line is subjected to a

longitudinal pressure. Any element X of length t, and distant $+x$ from the central line, becomes of length $t\left(1+\frac{x}{r}\right)$ where r is the radius of curvature (OB′) of the central line. The elongation is therefore x/r, and the tension is EAx/r, where E is Young's Modulus and A is the area of section of the element X. Thus, the stress at any point of a section of the bar is normal to the section and is proportional to the distance x from a line drawn through the centre of inertia perpendicular to the plane of flexure, being positive for positive values of x and negative for negative values. These stresses have zero resultant force, but give rise to a resultant couple whose moment is the sum of the moments of the stresses with regard to the line perpendicular to the plane of curvature. The moment of the tension EAx/r about the line through the centre of inertia is EAx^2/r. Since the sum of quantities of the form Ax^2 is equal to the moment of inertia of a disc of the form of the section of the bar and of unit density, we find for the total stress-couple acting across any section the value

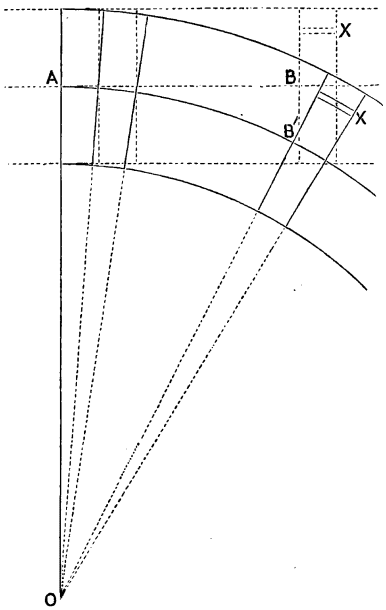


Fig. 76.

$$\frac{\text{Young's Modulus} \times \text{moment of inertia of area}}{\text{radius of curvature.}}$$

The product of the two quantities in the numerator is called the flexural rigidity. However irregular may be the form of section, there are always two lines in the plane of the section at right angles to one another such that the moment of inertia about the one is a maximum and that about the other a minimum. It is only in cases of bending about these lines that the above theory holds. There are consequently two principal flexural rigidities, which are proportional to the corresponding moments of inertia. Thus we see at once why it is easier to bend a plank, say, of elliptical or oblong section, when the longer axis is perpendicular to the plane of flexure than when the shorter axis is.

It is frequently of great importance to have bars or rods of great flexural rigidity. It is evident, from what has been said above, that this is to be effected by making the moment of inertia of the section as great as possible consistent with the other conditions. Thus a T-shaped girder has much greater flexural rigidity than a square-section girder of the same length and mass. Similarly a hollow cylinder will resist bending stresses much more effectively than a solid cylinder of the same mass per unit length; and, because of its comparative lightness, will also be more efficient than a solid cylinder of the same size. In the Forth Bridge the struts (or girders which are in compression) are made of large hollow cylinders, thereby combining lightness and power of resistance to all buckling or bending stresses.

§ 100. **Twist or Torsion.**—The simple twist is a simpler strain than either of the preceding, for it involves only change of form without change of volume. A rod is usually twisted by fixing one end and applying a suitable couple to the other, or by applying opposite couples to the two ends. But it is only in the case of a rod of circular section that end couples can sustain a simple twist. This is the only case considered here.

The central axis of the rod remains unchanged, but every other line originally parallel to this axis takes the form of a screw or helix. A strip of paper held by the ends and twisted shows the kind of change produced in every vertical central section of the cylinder. The twist is measured by the difference in the angular rotations of two sections unit distance apart. Practically we determine it by dividing the angular displacement of the lowest section by the length of the rod. Looked into more closely, the strain is seen to consist of the sliding of each layer over the next in order by an amount depending on their distance apart. A small rectangular parallelepiped included between two sections near to one another will as a whole suffer a displacement proportional to its distance from the fixed end of the cylinder, as from A to A′ (fig. 77). But it will, because of the relative displacement of the sections, also suffer a change of form. The rectangular element will become an equal-volumed but oblique parallelepiped on the same base. Such a strain is called a simple shear, and consists of the sliding of layer over layer (fig. 68).

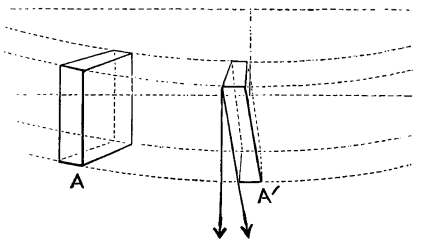
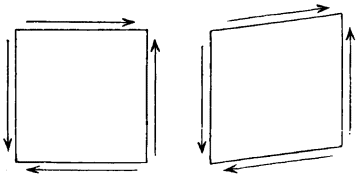


Fig. 77.

To sustain the shear in the elastic solid requires a combination of stresses known as a shearing stress. The stresses acting on a small element may be represented as tangential stresses acting along the sides (see fig. 78). There must be two pairs of them, so that they may balance as regards rotation. Now the stresses acting on any element in the twisted rod will be proportional to the deformation of the element, which is measured by the amount of the shear—that is, the change of angle between the sides. But this change is proportional to the distance of the element from the axis. Let us compare the stresses acting over the sections of two cylinders of the same material but of different radii, subjected to the same


Fig. 78.

twist (fig. 79). Each is similarly divided into elements, so that as regards geometrical dimensions the one may be regarded as a magnification of the other. The corresponding surface elements S and S′ will be in the ratio of the squares of the distances from the respective axes —i.e. as r^2 to r'^2. But the tangential stresses are, as we have just seen, in the ratio of r to r'. Hence the total stresses acting on the surface elements are as r^3 to r'^3; and the moments of these stresses about the respective axes are as r^4 to r'^4.

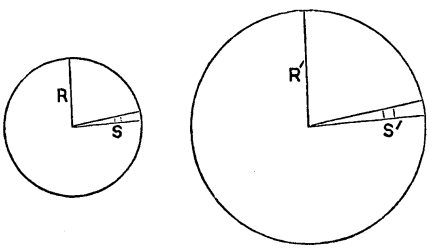


Fig. 79.

This holds for all corresponding elements, for which the ratio r to r' is the same as the ratio of the radii of the cylinders—namely, as R to R′. Thus we conclude that the resultant moment of all the tangential stresses is proportional to the fourth power of the radius, the twist being the same. But this resultant moment is equal to the end couple which produces and sustains the twist. Hence, for a given material, the couple required to produce and sustain twist in a wire or rod is proportional to the twist (Hooke's Law) and to the fourth power of the radius. For example, if we double the diameter of a wire we increase its torsional rigidity sixteen times (2^4); or, to produce a given twist in the thicker wire will require sixteen times the couple that is required to produce the same twist in the thinner wire. If the wires or rods are of different material, the torsional rigidities differ because the *rigidities* or *specific* resistances to change of form differ.

§ 101. **Rigidity,** also known as the torsional elasticity because of the mode of measuring it, is the ratio of the shearing stress to the corresponding shear. The less a substance yields to a given shearing stress the more nearly does it resemble the ideal rigid solid which cannot be distorted, and the greater is its rigidity. The shear is measured by the small change of angle in the unit cube, and the shearing stress is measured by the tangential stress per unit surface. It is not difficult to show that, if n be the rigidity of the

material forming a wire or rod, R the radius, and δ the twist, the couple required to support the twist is

$$C = \frac{\pi}{2} n R^4 \delta.$$

But C, R, and δ are all measurable quantities, so that the rigidity n can be determined.

The usual mode of experimenting is by means of oscillations. A heavy horizontal ring or disc is hung at the end of the wire; and after a sufficient twist is given, the wire is left quite free. The appended ring oscillates about the vertical axis of the wire in a period whose square is directly as the length of the wire, directly as the moment of inertia of the ring, and inversely as the torsional rigidity of the wire. The correct expression for the periodic time of oscillation is

$$T = \sqrt{\frac{8\pi I l}{n R^4}}$$

where I is the moment of inertia of the ring, l the length of the wire, R its radius, and n the rigidity. Hence n can be expressed in terms of measurable quantities. The chief source of error lies in the measurement of R; for any small error is increased fourfold when the quantity is raised to the fourth power.

The following table gives the rigidities in pounds-weight per square inch of certain specimens of the more important substances:

Substance.	Rigidity.
Iron	11,600,000
Copper	5,100,000
Brass	4,480,000
German silver	6,140,000
Aluminium	4,760,000
Glass	3,340,000

These numbers will serve to give a general idea of the relative rigidities of these substances; but different specimens of the same metal are found to give appreciably different results. In general, rigidity diminishes with rise of temperature.

§ 102. **Spiral Springs** are of two kinds. There is the flat spiral spring, like the mainspring of a watch, whose action

depends upon the elasticity of flexure; and there is the helical or screw form of spring, whose action depends partly upon flexure, partly upon torsion. That twisting necessarily takes place when a spiral spring of any form is drawn out is demonstrated by the following very simple experiment: Coil a narrow tape or strip of paper round the forefinger of one hand, and then draw off the upper coils with the other hand. The part that leaves the finger will be found to be twisted, although before extension it was merely bent.

In springs of small step, torsion is of much greater importance than flexure, and is what determines the elasticity of the spring. The stretching of an ordinary spiral spring when loaded is chiefly an effect of this torsion, there being comparatively little stretching of the *material* of the spring. Nevertheless, to the apparent stretching of a spiral spring exactly the same law applies as to the stretching of a wire. The ratio of the load to the extension is constant in accordance with Hooke's Law; but we cannot call this ratio Young's Modulus. Practically, however, it is all we need pay attention to, the torsional character of the strain in the material of the spring being mainly of theoretic interest.

Let n be the numerical measure of the mass which, when appended, will extend the spring by unit length. Then for any extension x the corresponding load will be ngx, measured in absolute units of force. If a mass M be suddenly appended and the spring left to itself, M will continue to oscillate up and down until the various frictional forces bring it to rest. For any momentary extension x the *downward* force acting on M will be

$$\mathrm{M}g - ngx = mg\left(\frac{\mathrm{M}}{n} - x\right).$$

This force vanishes when $x = \mathrm{M}/n$ and becomes negative—that is, *upward*—for greater values of x. But the length M/n is obviously the statical extension under the load M. Hence, putting $\frac{\mathrm{M}}{n} - x = \xi$ the displacement of M from its natural position of rest, we get

$$\textit{Force acting on } \mathrm{M} = ng\xi,$$

or

$$\textit{acceleration acting on } \mathrm{M} = \frac{n}{\mathrm{M}}g\xi$$

acting so as always to diminish ξ. Now this is exactly the same

kind of relation which holds for pendulum motion, and shows that the mass M will oscillate in the period

$$T = 2\pi\sqrt{\frac{M}{ng}}$$

For example, if a spring-balance is extended 2 inches by a load of 10 lb., a 20-lb. mass will stretch it 4 inches, and the periods of oscillation of these masses will be respectively

$$2\pi\sqrt{\frac{2}{12\times 32\cdot 2}} \text{ and } 2\pi\sqrt{\frac{4}{12\times 32\cdot 2}}.$$

The work done in stretching the spring to any extension x is evidently the displacement x multiplied by the average force during the extension. Since the force and extension increase proportionally, the average force is half the final force by which the extension is produced. But the stress corresponding to the extension x is ngx. Hence the work done in stretching, and therefore the potential energy stored up in the spring, is

$$\tfrac{1}{2}ngx^2,$$

n being, as already defined, the measure of the mass whose weight produces unit extension.

Thus the work done on the spring described above when it is stretched by 20 lb. is, since 60 lb. load is what will produce unit extension (1 foot), $\frac{1}{2}\times 60\times 32\cdot 2\times 4^2$ foot-poundals $= 480$ foot-pounds.

§ 103. **Compressibility.**—The direct determination of the compressibility of solids is a matter of extreme difficulty. For isotropic solids a necessary relation holds among the three quantities—Young's Modulus, rigidity, and compressibility. From measured values of the first and second the third may be calculated. It has not been found, however, that the compressibilities estimated from measured values of Young's Modulus and rigidity are in very good agreement with compressibilities measured directly. The only truly direct method of measuring compressibility is that used by Buchanan, who measured the change of length of a glass rod subjected to a considerable hydrostatic pressure. If the material be isotropic, the compression is three times the diminution of unit length; and from this the compressibility at once follows. The value obtained from the particular kind of glass used was 2·9 millionths per atmosphere.

The compressibility of solids has an important application in the construction of instruments for measuring high hydrostatic pressures; for it is found that within very wide ranges of pressure the compressibility of such substances as glass and steel is constant—in other words, Hooke's Law applies. Parrot, in 1833, was the first to use a solid pressure gauge. His apparatus was simply an ordinary thermometer with a bulb thick enough to stand great pressures. The essential part of an aneroid barometer is a thin-walled elastic metal box, exhausted of air, and made delicate enough to measure slight variations of atmospheric pressure.

§ 104. **Limits of Elasticity.**—Excepting in the case of hydrostatic compression, in which the bulk modulus only is involved, Hooke's Law holds rigorously within comparatively small limits of strain. After undergoing a comparatively small strain, the body acquires a 'set,' so that, when the stress is removed, the body does not quite recover its original form. Time also enters into the question. If the body is kept strained a long time, and then left to itself, its recovery towards its original form will not be so complete as when it is kept similarly strained for a short interval of time. And, in like manner, there will be a gradual time change in the form assumed after the stress has been removed—the body, as it were, creeping back towards the original form.

Many important qualities of matter depend upon its imperfect elasticity. For example, there is the property of **ductility,** in virtue of which metals are drawn out into the form of wires. To be ductile, a substance must be at the same time tenacious and plastic. The length is gradually increased at the expense of cross-section, which is reduced step by step by passing the wire successively through a number of holes in a hard steel plate, each hole being a little smaller than the preceding one. **Malleability** is another important property of matter which depends upon a substance being over-strained and yet preserving its continuity. See above, § 6.

Plasticity is a quality which, though usually thought of in

connection with substances like clay, wax, tallow, &c., is possessed in remarkable degree by certain metals. By application of sufficient pressure, Tresca forced lead through an orifice, and the laws regulating its flow were the same as those regulating the flow of a fluid. The one distinction between a plastic solid and a viscous fluid is, that the former does not begin to flow till the stress is greater than a certain critical value, whereas the latter flows under the influence of the smallest force if sufficiently long continued. Ice, especially in large mass, is also plastic. In the form of glaciers, it flows down the valleys exactly as a highly viscous fluid would do.

The question of the strength of tubes, pipes, and boilers, when subjected to fluid pressures inside or outside, is one of far-reaching importance. The effect of an internal pressure so large that the external pressure of the atmosphere may be neglected is to produce the greatest distortion on the inner layer of the tube. Here, accordingly, rupture will begin should the pressure be high enough. For tubes of the same material the distortion depends upon the ratio of the area of section of the whole tube to the area of section of the wall. Tubes of the same material, for which this ratio is the same, will have the same strength. For thin-walled tubes (e.g. boilers) the ratio of the areas is approximately the ratio of the radius of the tube to twice the thickness of the wall; so that for equal-sized tubes the strength is inversely as this thickness. Again, when the wall is thick compared to the diameter of the bore, the area of the wall differs but slightly from the area of section of the whole tube; hence very little strength is added by increasing indefinitely the thickness of the wall. A tube whose diameter of bore is one-tenth the whole diameter is practically as serviceable for standing high internal pressure as one whose diameter of bore is only one-hundredth.

By considering the forces which hold in equilibrium a small element on the surface of a thin-walled cylinder we may easily show that the total tension acting round the cylinder is equal to the

product of the difference of the pressure inside and outside into the radius of the cylinder; or $T = Pr$. [See also below, § 112.] As an example, let us calculate the pressure which an iron boiler 1 inch thick and 4 feet in diameter can safely support—the working strength being 10,000 lb.-weight per square inch. The tension acting round the cylinder must not be greater than the product of the working strength and the thickness; hence, for the limiting difference of pressures, we have

$$P \times 24 = 10{,}000 \times 1,$$

or $P = 417$ lb.-weight per square inch, or nearly 29 atmospheres. The accurate formula gives 409 lb.-weight per square inch. For spherical boilers or shells the equation becomes $2T = Pr$, since there are two directions of curvature.

§ 105. **Static and Kinetic Coefficients of Elasticity; Fatigue.**—By measuring the couple required to keep a wire permanently twisted, we obtain the means of calculating the *static* rigidity of the substance of the wire. If, however, we get the necessary data by taking torsional oscillations, we must regard the rigidity so estimated as being a *kinetic* coefficient. These two rigidities are not quite the same—the kinetic coefficient being a little larger than the static. A similar distinction holds between static compressibility measured according to the definition and kinetic compressibility measured by the velocity with which a state of compression passes through the substance (see §§ 143, 145, 154). In like manner, a longitudinal wave sent along a wire, as when a railway-signal is worked, moves with a speed which depends on the *kinetic* Young's Modulus. A few values of Young's Modulus, static and kinetic, in grammes-weight per square centimetre are given in the following table, reduced from Wertheim:

Metal.	Young's Modulus. Static.	Kinetic.
Copper, drawn	1245×10^6	1254×10^6
Steel, drawn	1881 "	1944 "
Platinum, medium	1704 "	1716 "
Silver, drawn	736 "	758 "
Lead, cast	177 "	199 "

When elastic oscillations are set up, as, for example, in a spring, tuning-fork, or twisted wire, the range of oscillation is found to diminish as time goes on. This decay of oscillation takes place more rapidly with some metals than with others, and cannot, therefore, be wholly due to the resistance of the air. It indicates the existence of molecular friction or **viscosity** in the substance. A curious fact, discovered by Lord Kelvin, is that the viscosity may be greatly increased by keeping a wire oscillating for days together. For example, with one wire in the fresh condition, a given arc of oscillation became reduced to half in 100 vibrations; but with an exactly similar wire, 'fatigued' by enforced vibrations for days, the same arc of oscillation became reduced to half in forty-four or forty-five vibrations.

Very similar in character is the deterioration in elastic quality of metals when they are subjected to periodic variations of stress far within the limits of the working strength. Axles and shafts, the mere rotation of which is necessarily accompanied by periodic stresses, gradually lose their pristine power of resistance, and ultimately break. Railway bridges constructed to carry, say, 100 trains per day would very soon collapse if they were to be crossed by the same number per hour. This kind of fatigue is not appreciably recovered from even after a prolonged rest. Thus the elastic quality of any portion of matter depends not only on the original temper imparted to it, but also upon the treatment it has undergone—upon, in short, its history.

EXERCISES.

1. Compare the extensibilities of two rods whose original lengths are as 3 to 1, and which suffer equal increments of length under tensions which are as 4 to 3.

2. Show that the work done per unit volume in stretching an elastic rod is measured by half the product of the final strain and stress. Hence calculate in foot-pounds the work done in extending a steel bar, 10 feet long and 1 square inch section, by a load equal to 50,000 pounds-weight. If the rod is 3 square inches in section, other things being the same, what is the work done?

3. An india-rubber strip, 3 feet long, is drawn out 1 inch for every pound hung to its end. Calculate the work done in lifting by its means a mass of 20 pounds to a height of 2 feet.

4. A ring of 4 inches diameter is made of the india-rubber strip of last question, and is stretched round a cylinder of 6 inches diameter. What is the tension in the strip and the pressure on the cylinder? (See § 104.)

5. A hollow pipe, curved like a bow, is filled with air. When more air is pumped in the pipe tends to straighten. Explain this.

6. Assuming Hooke's Law to hold throughout for the compression of water, and Boyle's Law for the compression of air, calculate the pressures at which air and water will have the same density. (Density of air under 1 atmosphere of pressure = ·00128.)

7. Compare the strengths of two tubes of the same external diameter when the thickness of the walls are respectively $\frac{1}{5}$ and $\frac{1}{4}$ of the radius.

8. An india-rubber strip is doubled in length under a load of 20 lb., and breaks under a load of 30 lb. At what rate may a pound be whirled at the end of a piece 1 yard long in the unstretched state?

9. Calculate in what proportions work is divided between two springs, according as they are arranged end to end, or side by side.

SUMMARY.

§ 96. Hooke's Law. Strain proportional to Stress.

§ 97. Extension. Longitudinal Tension.

§ 98. Young's Modulus. Tenacity. Working Stress.

§ 99. Flexure or Bending. Depends on Young's Modulus.

§ 100. Twist or Torsion.

§ 101. Rigidity.

§ 102. Spiral Springs. Flat and Helical: Flexure and Torsion.

§ 103. Compressibility.

§ 104. Limits of Elasticity. Ductility. Malleability. Plasticity. Strength of Pipes and Tubes.

§ 105. Static and Kinetic Coefficients of Elasticity. Fatigue. Deterioration.

CHAPTER XII.

MOLECULAR PHYSICS.

§ 106. Under this heading there fall to be considered certain phenomena which bear very directly upon the molecular constitution of matter. Porosity has already been referred to as a property of matter which suggests a coarse-grained structure. However uniform in appearance any substance may be, there is little doubt that in its smallest parts it is intensely heterogeneous. The smallest part with which we deal in physics is the molecule. What its ultimate constitution is we do not inquire. There is abundant evidence that the molecule itself has a complex structure, by virtue of which it possesses many remarkable physical properties. Different substances differ in ways which can only be referred to differences between their molecules. For example, transparency, rigidity, compressibility, magnetic quality, &c., depend not only upon molecular groupings, but upon the very character of the molecule. Then, again, there is the constancy in character of the molecule of any given substance. The molecule of iron is the same, whether it is derived directly from the earth or has come to us out of the depths of space in a meteoric stone. The belief in the absolute sameness of structure in a molecule of a particular kind of matter lies implicitly at the base of all our reasoning as to the materials that compose the atmospheres of the sun and the stars.

§ 107. **The Kinetic Theory of Gases.**—The whole of modern molecular physics is based upon the hypothesis known as the Kinetic Theory of Matter—that is, the theory which tries to explain all phenomena in terms of the *motions* of

molecules. Considerable progress has been made in this direction in regard to fluids, especially gases. The broad distinction between the molecules of solids and the molecules of fluids is that the former are more constrained than the latter. In fluids the molecules are capable of translational motion amongst themselves, like individuals in a loose crowd. In solids, however, the molecules are restrained from shifting about amongst themselves, being comparable to the soldiers of a regiment on parade. Gases, again, differ from liquids in the much greater freedom of the individual molecules, which dart about with great speeds in all directions. This conception of a gas as composed of rapidly moving and, on the whole, independent particles at once explains why a gas almost immediately fills the space in which it is, and why it exerts pressure on the boundaries. Every particle which impinges on the boundary is reflected back again, and the change in its momentum must be due to an impulse whose reaction on the boundary will be felt as an impulsive pressure. In one second a great number of particles will be so reflected from every unit of surface; and the resultant effect will be the pressure on this unit of surface. By doubling the number of particles in any closed region we double the number of impacts that take place in unit time, and get twice the original pressure. Hence pressure varies directly as the density, which is one mode of stating Boyle's Law.

§ 108. **The Speed of a Gas Molecule** is readily calculated according to this theory. Take first the case of air at ordinary pressure and temperature, confined in a cubical box 1 foot to the side. The mass of the cubic foot of air is ·08072 lb. One-third of this may be supposed impinging on any pair of opposite walls. The speed of each particle (supposing for the moment that the particles collide only with the walls and not with one another) measures the number of impacts the particle makes as it darts to and fro between the opposing walls. Hence we may write for this speed

$$v = 2 \times \textit{no. of impacts on one side in one second.}$$

To reverse the momentum of one particle requires

$$\text{an impulse} = 2 \times \textit{mass of particle} \times \textit{speed}.$$

Hence, to reverse the momentum of all the particles that make up one-third of the mass of the whole requires

$$\text{an impulse} = 2 \times \frac{\cdot 08072}{3} \times \textit{speed} = 2 \times \cdot 027 \times v.$$

Finally, to reverse the momentum of the particles that impinge in one second requires

$$\text{an impulse} = 2 \times \cdot 027 \times v \times \textit{no. of impacts due to one particle}$$
$$= \cdot 027 \times v^2.$$

But the impulse is the pressure per square foot

$$= 14 \cdot 7 \times 144 \times 32 \cdot 2 \text{ (foot-pound-second units).}$$

Whence $v = \sqrt{\dfrac{14 \cdot 7 \times 144 \times 32 \cdot 2}{\cdot 027}} = 1589$ ft. per sec.

This is the speed of an air particle at ordinary temperature and pressure.

As a matter of fact, the particles of a gas do undoubtedly collide with one another. If it were not so, the particles, say of ammonia gas, would immediately pass to the farthest limits of the room in which a bottle of liquid ammonia was opened. But we know from our sense of smell that the ammonia takes an appreciable time to pass across the room. This is because the particles encounter the particles of air, and are deflected from their otherwise straight course again and again. Yet, notwithstanding the mutual impacts among the particles, it can be shown that because of the vast number of particles involved, and their assumed perfect elasticity, the *average* result is very nearly the same as it would be were there no mutual collisions. The quantity v^2 is not then the square of the speed of any one molecule, but it is the average of the squares of the speeds of a great number.

If ρ is the density of a gas, and p its pressure, the very simple relation obtained above takes the form

$$p = \tfrac{1}{3}\rho v^2.$$

From this we see that, under similar physical conditions,

the velocity of the molecule is inversely as the square root of the density of the gas. Thus for hydrogen, v^2 is 14·4 times the value for air; hence $v = 6038$ feet per second. When v^2 remains constant, the pressure varies as the density—which is simply Boyle's Law. Here we have a hint that v^2 is a function of the temperature, being constant when it is constant.

§ 109. **Diffusion** is the gradual interpenetration of two substances which are in contact with one another. The phenomenon is most easily studied in liquids. For example, place in the bottom of a glass jar a coloured solution (sulphate of copper, bichromate of potash, &c.), and then introduce very gently above it a quantity of pure water so as to fill the jar. At first there will be a clear surface of division between the coloured and uncoloured liquids. But gradually, as time goes on, it may be for weeks or months according to the nature of the solution, the colour will rise in the jar, and between the lowest layer of the solution and the highest point where colour is beginning to show itself, there will be a gradation of intensity of colour. As the coloured solution rises, the colourless water must sink, so that the phenomenon is strictly a case of interdiffusion.

If a solution of common salt or of sugar be used instead of the coloured solution, exactly the same kind of diffusion will occur, only now it will not be possible to trace the progress of the phenomenon by eye observation. Some other method, such as estimation of density, must be adopted to measure the amount of solution which has penetrated upwards to a given height. In all cases, however, there will be, at first, a gradation in strength of solution from the strongest below to the weakest above. Ultimately, if sufficient time elapse, the solution will become uniform throughout. This uniformly mixed condition is produced very rapidly by stirring the two liquids together. The effect of the mechanical agitation is to increase the area of the surfaces through which diffusion

takes place. The surface of separation, originally a horizontal section of the jar, becomes a surface of many convolutions and of great extent. A rapid stirring for a few minutes can thus do what years of undisturbed diffusion could scarcely accomplish.

The rate at which diffusion takes place across a given surface depends directly upon the rate of variation of the strength of the substance in the direction of diffusion, which is always from regions where the substance exists in greater quantity to regions where it exists in less quantity. As the diffusion continues, its rate of necessity falls off, and ultimately becomes extremely small as the mixture approaches uniformity.

Many pairs of liquids, such as oil and water, water and mercury, &c., do not diffuse into each other. The lighter floats on the heavier, and the surface of separation remains steady and unbroken. In other cases diffusion goes on for some time, but stops as soon as a certain proportion of each liquid has mixed with the other. Two colourless liquids which act chemically on each other so as to produce a coloured compound make a very beautiful diffusion arrangement. The middle layer across which the liquids diffuse becomes more and more deeply tinted and grows in breadth as the action proceeds.

In the case of gases diffusion always takes place. It is impossible to keep two gases from mixing when they are placed in one vessel, however different their densities may be. Many familiar experiences demonstrate the rapidity with which gaseous diffusion takes place. An escape of coal gas speedily makes itself known even in a large room. All odours, indeed, are due to the effect upon the organs of smell of gaseous exhalations diffusing through the air, or of minute solid particles borne on air diffusing into itself. For, although it is only in the case of the interdiffusion of different substances that we are able to make experimental measurements, there is no doubt that in every fluid interdiffusion between

contiguous portions is constantly taking place. This, in fact, is simply another way of stating the kinetic theory.

The phenomena of diffusion are so simply expressed in terms of the molecular theory that they form one of its most powerful supports. It is evident that the rate of diffusion must depend upon the velocities of the molecules and the frequency of their mutual collisions. Now, in such collisions there is a constant interchange of momenta and of kinetic energies, and the tendency is to an average equality in these throughout the whole mass. Suppose, for example, that a current of air is bounded by air at rest or moving with a different velocity. Across the boundary molecules will pass carrying with them the average momentum of the molecules in the region they have left. Thus the more swiftly moving air will lose momentum and the more slowly moving air will gain momentum, the end being equalisation of average molecular momentum in the two bodies of air. This *diffusion of momentum* (so to speak) explains the existence of what was called above viscosity or fluid friction. By an exactly similar process diffusion of kinetic energy will take place. This, which depends on the quantity ρv^2, is a function of the temperature—exactly what function will be discussed later under Heat. For the present it is sufficient to note that equalisation of molecular kinetic energies means equalisation of temperature. Hence what might be called the *diffusion of kinetic energy* is closely connected with the manner in which heat is conducted through a gas.

The theory, mathematically worked out, gives expressions for the viscosity and thermal conductivity in terms of the effective size of the molecules and the average number of encounters per second. Consequently, from the experimental determinations of the viscosity and conductivity of a gas these molecular constants can be deduced. The theory is, of course, only approximate, so that in the following table, reduced from Maxwell, only the speeds of the molecules can be regarded as accurate, the collisions per second being only approximate, while the other estimated quantities are largely conjectural.

TABLE OF MOLECULAR DATA.

At one Atmosphere of Pressure.	Hydrogen.	Oxygen.	Carbonic Acid.
Velocity (of mean square) in ft. per sec. at 0° C.....	6,100	1,525	1,300
Collisions per second, *in millions*.....	17,750	7,646	9,720
Diameter of molecule in hundred-millionths of an inch (10^{-8}).....	2·3	3	3·7
Number of molecules in a cube of side one-millionth of an inch.....	310	310	310

Thus, if the molecules were arranged symmetrically in rows, there would be 6,800,000 to the linear inch.

The fact that all gases at the same temperature and pressure have the same number of molecules per unit volume is another way of stating the important law established by Gay-Lussac that the densities of gases are proportional to their molecular weights. It is a simple deduction from the kinetic theory. For equality of pressure means that $\frac{1}{3}\rho v^2$ is the same for all. Then equality of temperature means that the average kinetic energy of a molecule is the same in all. If, then, there are n molecules, the quantity $\frac{1}{2}\frac{\rho}{n}v^2$ must also be the same for all. Hence n is the same for all.

§ 110. **Diffusion through Septa; Transpiration; Osmose.** —The constancy of ρv^2 for all gases at one pressure shows that if a gas be allowed to diffuse from one region into another at first empty, the rate of diffusion will be inversely as the square root of the density. Thus hydrogen will diffuse four times more quickly than oxygen. When diffusion takes place through a thin porous wall—a process called by Graham *Effusion*—the process takes place very much as if there were no wall at all. When, however, the wall is not very thin the process of *Transpiration* (as it was termed by Graham) becomes more complicated, and the rates of transpiration for different gases are no longer inversely as the square roots of the densities.

The following simple experiment proves that air and coal-gas differ in their rates of transpiration. A porous

clay ball is fitted with a tubular glass stem, the open end of which dips in water, coloured for convenience (fig. 80).

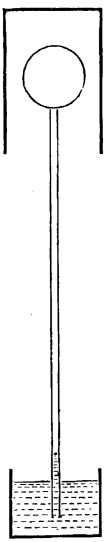


Fig. 80.

At first air fills the ball and tube. When a glass jar full of coal-gas is placed cap-wise over it, the coal-gas begins to pass through the pores into the ball, air at the same time diffusing out of the ball. But because of its greater rate of transpiration, coal-gas passes in in greater quantity than air passes out. This is shown by the bubbles of air and gas which are forced out below through the liquid. After some time let the glass jar be removed. Then, because the ball contains a considerable quantity of coal-gas, while immediately outside it is surrounded chiefly by air, the transpiration will be greater outwards. Hence the total quantity of mixed gases in the ball and tube diminishes, and liquid is pushed up the stem by the greater pressure outside. When the liquid has reached a certain height in the tube it begins to fall again; and it will gradually fall back to its normal level as air passes into the ball to take the place of the escaped and escaping coal-gas.

If a mixture of gases is allowed to diffuse through a porous wall, the part that has diffused will probably differ in density from the rest, because of the different diffusivities of the individual gases in the mixture. The process forms a valuable test of the purity of a gas.

When two miscible liquids diffuse into each other through a porous membrane, certain very curious phenomena present themselves, which are grouped under the general title of **Osmose.** The main feature is that the one liquid diffuses through the membrane more quickly than the other, so that there is a tendency for liquid to accumulate on one side of the membrane. For example, let a flask, whose bottom has been removed and replaced by a piece of bladder, be half-filled with

alcohol and then half-immersed in a vessel of water—the water will diffuse into the flask more quickly than the alcohol will diffuse out. If the flask be continued above by means of a long narrow tube (see fig. 81), the water will gradually ascend in the tube, and may finally run over at the top. If brine be used instead of alcohol, an exactly similar result is obtained. In each case the process will then go on until the pressure on the one side of the septum exceeds that on the other by an amount which depends on the character of the septum and on the kinds of liquids used. The final difference of pressure which can be produced in this way is called the *Osmotic Pressure* of the substance held in solution. A very remarkable kinetic theory of solutions, originated by Van't Hoff, connects the osmotic pressure with the number of the molecules of the substance held in solution in unit volume. The laws that govern osmotic pressure are in many respects similar to the laws that govern gaseous pressure, and the theory expresses this similarity in a more or less emphatic manner.

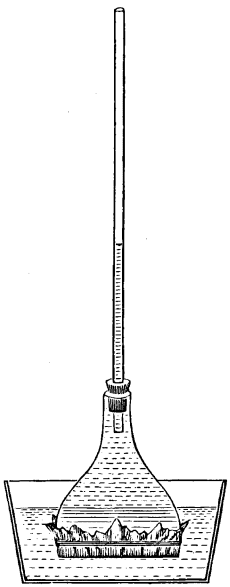


Fig. 81.

Crum Brown's view of the action has the merit of great simplicity, and is based on two recognised principles of solution and diffusion. As pointed out in last paragraph, the rate of diffusion depends upon the rate of variation of the strength of the diffusing substance in the direction of diffusion. Thus water diffuses across a septum in the direction of the drier or less wet side. When the septum is in a homogeneous state as regards wetness, diffusion will cease. The

difference in wetness between the two sides of the bladder in the experiment described above is due to the fact that all solutions tend to a state of less concentration, and the stronger the solution the stronger this tendency, so that the solvent has less wetting powers the greater the concentration of the solution. Thus the comparatively pure water diffuses through the septum towards the drier side where the solution is, and this diffusion goes on until, under the increased pressure, the wetting powers of the solvent become equal to those of the water and the septum is brought into a homogeneous condition. As a general rule diffusion takes place in both directions, each of the substances passing at its own rate through the septum in the direction of diminishing concentration; for, clearly, where the salt is most concentrated, there the water is least concentrated. Nernst has simplified the experiment by using a septum which is impervious to the substance held in solution. On the one side is pure ether, and on the other a solution of benzol in ether, and between them a bladder thoroughly soaked in water. The septum is, in fact, water; the bladder simply serving the purpose of stiffening the water septum. Now, benzol is insoluble in water, and cannot, therefore, diffuse through it. But the ether diffuses readily, and continues to diffuse until, under the increased pressure, the smaller concentration of the ether in the solution of benzol is made up for, and the water septum is brought into a state of homogeneity as regards the ether contained in it. If both substances diffuse, the ultimate state must be equality of pressure and of strength of solution on both sides; but, because of the greater rate of diffusion of the one substance, there is in the earlier stages a difference of strength of solution on the two sides and a corresponding difference of pressure. Could we obtain a porous substance impervious to one kind of gas, we should be able to make a transpiration experiment very similar to Nernst's osmotic experiment described above. The explanation of the action just given is in satisfactory agreement with

the established laws of osmosis—namely, that the osmotic pressure is proportional, in the case of weak solutions, to the concentration of the solution, and that the osmotic pressure increases with temperature. The details of the action in different cases are, however, chemical rather than physical.

The great differences in the rates of diffusion of substances led Graham to group them under two great classes, *Colloids* and *Crystalloids*. The colloids, such as albumen, gum, glue, &c., diffuse very slowly compared to crystalloids, which include sugar and salt solutions. When a mixture of colloids and crystalloids in solution is placed on one side of the bladder or parchment which forms the septum, pure water being on the other side, it is practically the crystalloids alone which pass through. This process of separating the two classes of substances is called *Dialysis*. It is used in the detection of poisons, which are mostly crystalloids, and are thus easily separated from the large admixture of colloids in the viscera. It is also the method by which the colloid oxide of iron, known in pharmacy as *Dialysed Iron*, is prepared. In many natural processes osmosis plays an important rôle; as, for example, in the swelling of beans and peas soaked in water, and in the intercellular motion of sap and lymph.

§ 111. **Cohesion,** the property in virtue of which the particles of matter hold themselves together, brings into evidence certain aspects of molecular action altogether different from those just discussed. This property is particularly characteristic of solids, which can (in general) be broken up or disintegrated only by means of (comparatively) powerful forces. As noted in last chapter, a rod will sustain longitudinal tension up to a limit called the Tenacity. At this limit the force of cohesion is overcome and rupture takes place. The molecular forces are overcome because the molecules are drawn asunder so as to be beyond the spheres of mutual action.

Each molecule may be supposed to be a centre of force, which is sensible all round at distances far smaller than any

we can directly measure. Two molecules, to be under each other's action, must not be farther distant than twice the radius of this sphere of molecular influence of each. Hence to make two distinct bodies cohere (or adhere) we must bring their surface molecules into very close contact. Thus a moderate pressure will make two newly-scraped surfaces of lead adhere strongly together. The welding of iron or platinum at high temperatures illustrates the same principle. Pressure applied to graphite powder makes it cohere in a form suitable for blacklead pencils; and the same transformation is revealed in every sandstone rock, which not infrequently bears the footprints and ripple-marks impressed on the original loose sandy surface.

To obtain a true measure of the cohesive forces in liquids or of the adhesive forces between solids and liquids is a quest of extreme difficulty. When a square inch of clean glass is in contact with the clean surface of mercury it requires a force of 180 grains-weight to separate them. The same plate will be pulled away from a similar surface of water by a force of only 60 grains-weight. But in the latter case the plate carries off some of the water with it. Hence the experiment measures the cohesion of water rather than the adhesion of glass and water. But then again it is demonstrable that the kind of cohesion which is overcome when a drop of liquid detaches itself is quite different from that which holds together the two halves of a spherical drop. Before the drop falls away, the neck of liquid connecting it with the rest of the liquid becomes very narrow, and the real effective force of cohesion involved is in the surface skin.

The molecular theory of the constitution of matter almost of necessity implies that peculiar phenomena should make their appearance at the boundary between two different kinds of matter. For in the neighbourhood of this boundary the molecules must be under conditions very different from those which hold for molecules in the heart of the substance. These latter are in equilibrium under the symmetrical system

of molecular forces acting all round. But it is quite otherwise with molecules whose distance from the boundary is less than the range of molecular action. We proceed to consider the experimental evidence for peculiar dynamic surface conditions.

§ 112. **Surface-tension.**—As already stated, there are certain liquids which do not diffuse into each other. The boundary between them remains as distinct as the surface which separates a liquid from the vapour or air above it or from the solid walls which confine it. Now, there are many phenomena which indicate that these bounding surfaces are in a dynamic condition best described as a state of tension or stretch. The most striking of these are shown in the behaviour of soap-bubbles and soap-films.

For example, in blowing a soap-bubble we experience a considerable resistance, work being done in extending the bubble. Then when the pressure of air necessary to keep the bubble at a given size is diminished till the inside of the bubble is exposed to the same atmospheric pressure as the outside, the bubble will begin to contract, and will go on contracting until it is reduced to a film at the narrowest part of the bowl or funnel on which it is supported. As it contracts it drives out the air within it, the work done being at the expense of its own energy. This energy evidently increases with the surface, and is given to it originally when the bubble is being blown. In short, the soap-bubble behaves like a stretched elastic film.

Again, we may show that a soap-film formed within a loop of wire possesses the same power of shrinking by the following simple experiment due to Van der Mensbrugge. Include in the film a small loop of thread whose ends have been knotted together. It will lie in the film in an irregular form. But, as soon as the part of the film that is within the thread is destroyed by contact with a sharp point, the thread at once takes the form of a circle, being drawn out equally in all directions by the pull upon it of the film which lies between the thread and the wire.

If one side of a loop be made of thread, the action of the film will be to draw the thread inwards as far as possible, so that the area of the film is made as small as possible.

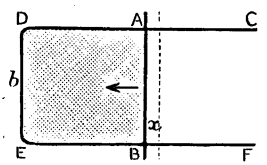


Fig. 82.

Suppose the framework to consist of a wire CDEF, bent as in fig. 82, and a cross-piece AB able to slide along the two parallel sides. Let T be the tension of the enclosed soap-film acting across every unit length. Then the total tension acting across AB is Tb, where b is the breadth of the film. The work done in moving AB through a distance x is

$$Tb.x = T.bx$$
$$= T \times \text{increase of area.}$$

The energy of the surface is therefore increased by this amount. Now, the value of the surface-tension depends only upon the character of the substances separated by the surface. In the case of the soap-film there are two surfaces, and the whole superficial energy is therefore 2TS, where S is the extent of either surface and T is the surface-tension per unit length. If AB be sufficiently light, and the contacts be smooth, the tension will pull AB up against gravity.

In the case of a free soap-bubble, which necessarily assumes a perfectly spherical form, the energy is $2T \times 4\pi r^2$, where r is the mean radius of the bubble. Suppose the bubble to be made a little larger because of a slight increase of the pressure, p, inside. Then, if r' is the new value of r, we have

work done by the total pressure = gain of surface energy;

or, $$4\pi r^2 p\,(r' - r) = 2T\,(4\pi r'^2 - 4\pi r');$$

hence $$pr^2 = 2T\,(r' + r) = 4Tr$$

ultimately for very small increase in r, and thus finally

$$pr = 4T$$

—a relation giving the increase of pressure within a soap-bubble in terms of the surface-tension and the radius.

For water and air the value of T is about $3\frac{1}{2}$ grains-weight per linear inch. Hence, for a soap-bubble 6 inches in radius the value of p is

$$p = \frac{4 \times 3\frac{1}{2}}{6} \text{ grains-weight per sq. in.}$$

$$= \frac{14}{6 \times 7000} \text{ pounds-weight per sq. in.}$$

$$= \frac{1}{3000} \text{ pounds-weight per sq. in.}$$

and the energy stored up in the surfaces is

$$2\mathrm{TS} = 4\pi \times \frac{1}{4} \times 7 \times 12 \text{ foot-grains}$$

$$= \frac{84\pi}{7000} = \cdot 038 \text{ foot-pounds.}$$

In the case of a cylindrical soap-bubble with each of its ends terminated by a funnel, the relation of pressure and tension may be similarly found to be

$$pr = 2\mathrm{T}.$$

The general dynamical principle referred to in Chapter VI. § 53, that the potential energy is a minimum for stable equilibrium, has very simple applications here. For, clearly, the potential energy is a minimum when the whole area of separation is a minimum. The sphere is the surface of smallest area which contains a given volume; hence the perfect spherical form of free soap-bubbles, raindrops, small glass beads, small shot, and so on. With a given perimeter the circle contains the greatest area. Hence the perfect circular form of the thread in Van der Mensbrugge's experiment described above; for, since the blank circular area is the greatest possible, the area of the soap-film itself will be the least possible.

When the film is supported on a flat loop of wire it will (neglecting gravity) be a plane surface. But should the wire be bent, so that the loop no longer lies in one plane, the soap-film will become curved. Now, curvature in one direction means an increased pressure towards the concave side. But since the soap-film has the same pressure on both sides, there must be an equal curvature in the other direction, so that equilibrium may be sustained. In other words, whenever a soap-film, freely exposed to the air on both sides, has a curvature, it must be a curvature like that of a saddle, concave for one section and convex for the section at right angles thereto. In mathematical language, the sum of the curvatures at

any point in any pair of mutually rectangular normal planes must be zero.

When the film is supported in a framework so as to enclose a space, the pressure within this space will differ from the pressure outside by an amount which is proportional to the sum of the curvatures at any point of the film in any pair of mutually perpendicular planes. The surface must fulfil the minimum condition consistent with the boundary conditions; or, in mathematical language, the sum of the curvatures at any point in mutually perpendicular planes must be constant.

§ 113. **Three Fluids in Contact.**—Surface-tension exists at the common boundary of any two fluids which do not mix. If there are three fluids in contact with one another there will be three distinct surfaces, each possessing its own tension. In almost all practical instances, one of these fluids is air. We shall suppose this to be the case throughout; and the other two fluids will be distinguished as fluid 1 and fluid 2. Then T_1, T_2, and T_{12} will be taken as representing the surface-tensions between air and fluid 1, air and fluid 2, and fluids 1 and 2 respectively. If fluid 2 rest on fluid 1 in the form of a finite drop (see fig. 83), then the surface-tensions will act upon a common point O in the directions of the tangents to the corresponding boundaries which meet there. The point O will be in equilibrium only if these tensions can be represented as the sides of a triangle. If, however, one of the tensions is greater than the sum of the other two, it will not be possible for them to form a triangle. In such a case, neither liquid will rest as a drop on the surface of the other. In the following table, reduced from Quincke, the values of T_1, T_2, T_{12} are given in grammes-weight per

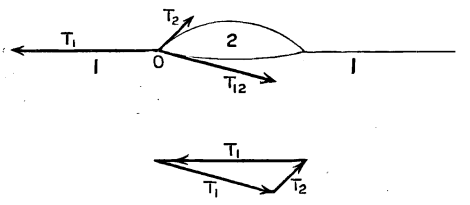


Fig. 83.

linear metre for several pairs of liquids. To reduce to grains-weight per linear inch, divide by 2·55.

	T_1	T_2	T_{12}
Mercury—Water	55·03	8·25	45·11
" —Alcohol	55·03	2·60	40·71
" —Olive-oil	55·03	3·76	34·19
Water—Olive-oil	8·25	3·76	2·09
" —Turpentine	8·25	3·03	1·18
Olive-oil—Alcohol	3·76	2·60	·23

In none of these cases can a drop of the second substance rest on the *pure* surface of the first; for in all T_1 is greater than $T_2 + T_{12}$. The difference is particularly marked in the case of water and turpentine. A little turpentine dropped on the surface of water is drawn out in all directions till it forms a uniform sheet resting on the water. The same is true for all oily substances and pure water. This explains the great difficulty of preserving a really pure water-surface, the slightest impurity that may fall upon it being drawn out by surface-tension till the whole surface is contaminated.

A small piece of camphor when placed on water moves about in a most irregular and fitful fashion. The camphor dissolves more readily at some places than at others. This produces unequal changes of surface-tension in different directions, and the camphor is drawn in the direction of resultant pull.

§ 114. **Fluids in Contact with Solids.**—When two fluids are in contact with a solid there are, as in the case of three fluids, three surface-tensions to consider. Suppose the solid to be glass and the one fluid to be air. A drop of water or alcohol placed on the glass at once begins to spread out in all directions; whereas a drop of mercury does not spread out, but remains as a drop resting on the glass. A bubble of air introduced into a closed vessel filled with water or alcohol forms an 'air-drop' pressing up against the top. [This is the principle of the spirit-level.] On the other hand, a bubble of air similarly introduced into a closed

vessel full of mercury flattens itself out against the top. Thus air behaves towards glass in the presence of mercury as water behaves towards glass in the presence of air; while air in the presence of water behaves like mercury in the presence of air. The general law is thus expressed by Maxwell: 'If the tension of the surface separating the solid from the first fluid exceeds the sum of the tensions of the other two surfaces, the first fluid will gather itself up into a drop, and the second will spread over the surface.' If the plate be well greased the water will gather in drops like mercury.

When a clean glass plate is dipped into water the water rises a little up the sides so that the plate is wet to a level higher than the true level of the liquid. This is the same phenomenon as the spreading of the water over the horizontal plate; only gravity comes in to act against the tension of the air-glass surface. Many other solids and liquids show the same effect. On the other hand, if the plate is dipped into mercury the mercury is depressed a little along the sides, so that the highest point of contact of the mercury and the plate is at a level lower than the true level of the liquid. If zinc be substituted for the glass, the mercury will rise like water; for mercury 'wets' zinc.

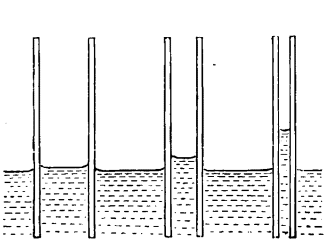


Fig. 84.

Let now a second similar plate be brought near to the first. The water between them will begin to rise, and the mercury will begin to be depressed; and the effect will increase as the plates are brought nearer. In the former case the included liquid will be concave outwards, as shown in fig. 84. In the latter case the included liquid will be convex outwards, somewhat as shown by fig. 84 inverted.

The concavity or convexity is obviously a direct consequence of the manner in which the liquid rises or falls to meet either plate—that is, a direct consequence of the relation between the three surface-tensions. But because of this concavity or convexity the tension acting over the surface makes the pressure on the convex side less than the pressure on the concave side. Thus, in the case of water, the pressure in the liquid immediately within the concave surface is less than the atmospheric pressure in the air without. Consequently, the water just below the inner surface belongs to a higher hydrostatic level than the water-surface outside, and must therefore rise. The rise will be inversely as the relief of pressure, which increases directly as the curvature. And the curvature will increase as the distance between the plates diminishes. Hence the rise is greater as the distance between the plates is less, being indeed inversely as this distance. With mercury, on the other hand, the fluid immediately below the convex surface is at a pressure higher than the atmospheric pressure, and belongs accordingly to a lower hydrostatic level than the mercury-surface outside. It is, therefore, depressed.

When two glass plates, touching along one pair of vertical edges and kept slightly apart by the insertion of, say, a thin blade between the other pair, are set standing in a trough of coloured water, the water rises between the plates to different heights at different distances from the touching edges (fig. 85*a*). The contour of the enclosed water film is a curve whose height at any point measured from a definite horizontal line is inversely as its distance from a definite vertical line. Such a curve is a rectangular hyperbola, and demonstrates to the eye the relation given above.

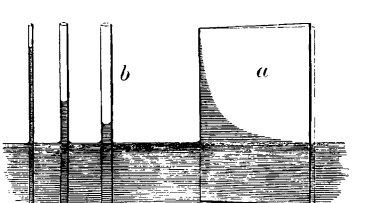


Fig. 85.

Instead of plates, fine-bored tubes are usually employed;

and from this form of experiment is derived the name **Capillarity**, a term now applied to the whole range of phenomena that depend on surface-tensions. In the tube the curvature of the liquid surface is spherical, not cylindrical; and water will rise twice as high as between plates whose distance apart is equal to the diameter of the tube (fig. 85*b*).

This form of experiment gives a convenient mode of measuring surface-tensions. For, if h be the height to which the liquid is raised or lowered, the pressure ($\rho g h$) due to this height of column must be balanced by the pressure due to the surface-tension acting over the concave or convex liquid-surface. But this pressure is measured by $2T/r$, where T is the surface-tension and r the radius of curvature of the surface. Equating these two expressions for the same pressure, we get T in terms of easily measurable quantities. In the case of water the radius may be assumed to be equal to half the diameter of the tube, for the water-surface meets the glass-surface tangentially, or (in other words) the angle of contact is zero. In the case of mercury the liquid-surface does not meet the glass-surface tangentially; and before the value of T can be found the angle of contact must be measured. The angle of contact of clean mercury and clean glass is about 130°. Impurities on the surface tend to increase the angle of contact.

One of the most striking illustrations of capillarity is given by the boy's 'sucker,' a disc of moist leather which adheres strongly to any flat surface against which it is pressed. The film of water between the two surfaces becomes concave outwards all round its margin, and this diminishes the pressure within, so that the surfaces remain pressed together by the atmospheric pressure.

Highly porous substances, such as lump sugar, blotting-paper, sponges, &c., draw in water by virtue of capillarity; and many similar phenomena occur in plant and animal tissues.

In the action of the *Atmometer* we have a particularly instructive case. This instrument consists of a hollow ball of unglazed clay with a glass tube as stem. The whole is filled with water and inverted so that the open end of the stem dips in mercury.

The water evaporates gradually from the exposed surface in the pores, and the mercury rises to take its place in the stem, sometimes to a height of nearly 2 feet. This column of mercury must be supported by the surface-tension acting on the highly curved surfaces of water in the pores. Now, 2 feet of mercury correspond to fully 27 feet or 324 inches of water. Taking, then, half-an-inch as the rise of water in a tube $\frac{1}{10}$ of an inch in diameter, we find for the diameter of the pores the value

$$\tfrac{1}{2} \times \tfrac{1}{10} \times \tfrac{1}{324} = \tfrac{1}{6480} \text{ of an inch.}$$

§ 115. **Crystalline Structure.**—Many solid bodies tend to assume certain regular geometrical forms known as crystals. The molecules, under the influence of their mutual forces, marshal themselves in accordance with a regular law of growth. The problem is one of molecular statics: given a number of exactly similar molecules, each having the same sphere of action and occupying a definite region of space, find the groupings of stable equilibrium. That crystalline forms necessarily result is at once seen in the ordinary piling up of shot in pyramids. Beginning, for example, with a square base, we may build up successive layers of marbles, each marble resting in contact with a definite number touching it on all sides. The pile will at once suggest the geometrical figure known as the octahedron—a form belonging to the cubical type of crystallisation. Or we may build upon a layer of marbles arranged in an equilateral triangle; this will give the tetrahedron. Spheroidal shot arranged in the same way, with their axes all pointing in the same direction, will give a form of crystal which can be got from the corresponding cubical form by application of a definite homogeneous strain. Such examples will suffice to show that crystalline form is a result of regularity of distribution of molecules all exact copies of one another.

Crystals of salts are usually formed by gradual deposition out of a saturated solution. As the water of the solution is partly driven off by boiling or simply evaporates, the quantity of solvent left cannot retain in solution the whole of the substance dissolved, part of which crystallises out in the solid form. By careful treatment it is possible in certain cases to form *super-saturated* solutions, in which the solvent retains in solution more of the solute than it could be made to retain by simply mixing them together. For example, acetate of soda is more soluble in hot water than in cold. If we form a saturated solution at a high temperature, and then allow the liquid to cool in a closed flask, it will remain a liquid down to lower temperatures. The solution is, however, in an unstable condition. When a crystal of the salt is dropped into it

crystallisation at once begins, and the liquid is transformed into a white crystalline solid. There is, at the same time, an appreciable evolution of heat, and the solid becomes quite warm to the touch. The molecules falling into positions of stable equilibrium—that is, of minimum potential energy—lose energy in some molecular form, and this appears as heat.

Analogous cases of instability are met with in solids. A drop of molten glass cooled rapidly in water will resist a hard blow; but if a small part of the tapering end be broken off, the apparently solid glass breaks explosively into a powdery heap of small glass particles. As soon as the constraint of the stable surface layer is removed at any point, the internal particles assert their instability and the glass is shattered.

SUMMARY.

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§ 107. The Kinetic Theory of Gases. Explanation of Pressure.

§ 108. The Speed of a Gas Molecule. Boyle's Law.

§ 109. Diffusion of Liquids and Gases. Diffusion of Momentum and of Energy. Viscosity and Thermal Conductivity. Table of Molecular Data.

§ 110. Diffusion through Septa. Effusion and Transpiration. Osmose. Osmotic Pressure. Colloids and Crystalloids. Dialysis.

§ 111. Cohesion. Mercury and Glass.

§ 112. Surface-tension. Work done in blowing a Soap-bubble. Energy on the stretched Surface. Minimum Potential Energy. Sum of the Curvatures Constant.

§ 113. Three Fluids in Contact. Oil on Water.

§ 114. Fluids in contact with Solids. Water and Mercury on Glass. Capillarity. Mode of measuring Surface-tension. The Boy's 'Sucker.' The Atmometer.

§ 115. Crystalline Structure.

HEAT.

CHAPTER XIII.

HEAT AND TEMPERATURE.

§ 116. By our sense of touch we are able to discriminate between bodies according as they are hot, warm, lukewarm, cool, or cold. It is also a familiar experience that when a hot body and a cold body are placed together, the hot body becomes less hot and the cold body becomes less cold. If left together for a long enough time, the two bodies will reach final steady states, so that by successive tests we shall not be able to notice in either of them any further change either of increasing or of decreasing warmth. The two bodies are then said to be at the *same temperature*. Before the final steady states were reached the bodies were at *different temperatures*, the one which was becoming colder being at a higher temperature than the other. Thus the temperature of a body in relation to surrounding matter may be defined as that condition which determines whether it is to get colder or warmer. The terms cold and warm refer primarily to the sensation produced when the hand or other part of the skin touches the substance; and from this sensation we draw an inference as to the relative temperatures of the substance and the hand. But did we trust entirely to the indications of this sense of touch we should frequently be misled. For example, the blade of a wooden-handled knife under ordinary

conditions *feels* colder than the handle; and yet, in accordance with the definition just given, their temperatures are the same, for each has reached a steady state. Further, if the knife be left for some time in a hot oven until the whole is brought to the same temperature, the blade will feel hotter than the handle. Thus it is evident that our sense of touch is quite untrustworthy when the temperatures of different substances are to be compared, although it is a safe enough guide in determining whether a given substance is at a higher or lower temperature than the skin. To obtain a satisfactory method of comparing temperatures we must appeal to some definite physical change which is a constant accompaniment to change of temperature. The most convenient for our purpose is change of volume.

§ 117. **Volume Changes accompanying Change of Temperature.**—A metal ball which, at ordinary temperatures, just passes through a metal ring will be unable to pass through the ring when it is heated red-hot in a fire or furnace. But as it cools it shrinks in size, and will again pass through the ring when the temperature has become low enough.

Again, fill a flask having a long narrow neck with alcohol, and then pour hot water over it. The first effect, due to the *expansion* of the flask, is a retraction of the alcohol in the neck; but this is immediately followed by a rising of the alcohol in the neck not only up to the former level but past it, so that the liquid overflows. When allowed to cool, the alcohol will contract; and if it be all restored to the flask at the original temperature will just fill the flask again.

Finally, to show how the volume of a gas is affected by change of temperature, let a glass tube with a bulb blown on one end be inverted with the open end dipping in a liquid (see fig. 86*a*). When the hand is placed over the bulb, the air within at once begins to expand, and part of it is forced out at the lower end and escapes as bubbles through the liquid. When the hand is removed, the air

within the bulb begins to contract as it cools to the original temperature, and the liquid in consequence rises in the tube (fig. 86*b*). When the hand is brought back again, the liquid is forced down by the expanding gas, simply to rise once more as soon as the hand is removed. Let a lighted match be brought near so as to heat gently any part of the bulb. Down goes the liquid column, indicating an almost immediate increase of volume of the air in the bulb.

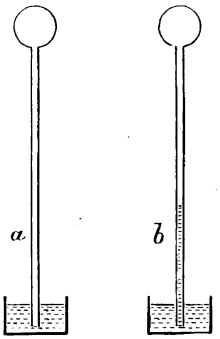


Fig. 86.

These and similar experiments show that solids, liquids, and gases change in volume as they change in temperature. The most sensitive in this respect is the gas, and the least sensitive the solid. Any substance whose volume changes are used to indicate temperature changes is called a thermometric substance.

§ 118. **Thermometers.**—The most useful thermometric substance is Mercury. Under similar conditions alcohol is more sensitive as a thermometric substance; but mercury, although less expansible, has several peculiar merits which make it eminently serviceable in this respect. It is heated very easily and expands very regularly when being heated. It is opaque, so that the boundary is distinctly seen. It remains a liquid through a range of temperature extending from a point well below the temperature of melting ice to a point far above that of boiling water.

Any form of apparatus in which the changes of volume of the thermometric substance can be readily observed is called a thermometer. The unqualified word thermometer will always be understood to refer to a mercurial thermometer.

In its simplest form the thermometer consists of a glass tube terminating in a bulb, the bulb and part of the tube being filled with mercury, while the rest of the tube is

empty. The greater bulk of the mercury is contained in the bulb, and any change of temperature in it is at once indicated by the motion of the end of the mercury column in the tube. The larger the bulb relatively to the capacity of unit length of the tube, the more sensitive is the thermometer to small temperature changes. When the bulb is placed in close contact with some other substance, the mercury and bulb will, if originally at a different temperature, soon come to the same temperature as the substance. This will be indicated by the end of the mercury column becoming steady in the tube. Should the mercury rise to this final steady state, we conclude that the substance was originally at a higher temperature than the thermometer. But if, on the other hand, the final state is reached after a fall of mercury in the tube, the conclusion is that the thermometer was originally at the higher temperature. Should no change occur when the thermometer is placed in close contact with the substance, then the temperatures of the two are the same. If a graduated scale be attached to the tube, or if a series of equidistant marks be made on the tube itself, changes of temperature may not only be indicated, but to some extent *measured*, by the number of degrees or intervals of graduation moved through by the end of the mercury column.

Every time this instrument is plunged into a mixture of ice and water the mercury will be found to stand almost exactly at the same level. The temperature so indicated is called the *Freezing-point*—that is, of water. Again, if the thermometer is set above boiling water so as to be wholly enveloped in the steam that is being given off, the mercury will rise to a level which will be always the same at the same atmospheric pressure. The temperature so indicated when the barometric pressure is 30 inches is called the *Boiling-point*. All thermometers of modern construction are graduated with reference to these two definite temperatures. In the Centigrade thermometer the freezing-point is called *Zero* (0°) and the boiling-point, at 760 millimetres

pressure, 100°, and the interval between the two is subdivided into 100 equal parts, each of which is called a degree centigrade. In the Fahrenheit thermometer, which is commonly used in this country, the freezing-point is called 32°, and the boiling-point, at 30 inches pressure, 212°, so that 180 Fahrenheit degrees correspond almost exactly to 100 centigrade degrees. To reduce a temperature given in the one scale to its value in the other is a mere question of arithmetic. For example, to express 98° Fahrenheit, the normal temperature of the human body, in the centigrade scale, we first subtract 32, and then, reducing the remainder in the ratio of 100 to 180, or 5 to 9, obtain 36·7°. To reduce centigrade to Fahrenheit, multiply the centigrade number by $\frac{9}{5}$ and add 32. In the Réaumur thermometer the freezing-point is 0° and the boiling-point 80°. This thermometric scale is still used on the Continent; but is being gradually superseded by the centigrade.

The graduation may be extended below the freezing-point and above the boiling-point, each successive degree indicating the same volume increment of the thermometric substance. On the centigrade scale, temperatures lower than the freezing-point of water are represented as *negative* quantities: $-5°$, $-10°$, and so on. This change of sign is not a little troublesome at times, especially in tabulating and discussing meteorological returns. In the Fahrenheit scale the change of sign does not occur till the temperature 32° below the freezing-point is reached; so that in this respect the Fahrenheit scale has a distinct advantage over the centigrade. Zero Fahrenheit corresponds to $-17·8°$ centigrade. It is usual to employ the initial letters F. and C. to represent the thermometric scale used in measuring a temperature.

For the meantime we shall take as our provisional thermometric scale the centigrade scale of the mercurial thermometer. Let V_0, V_1 be the volumes of a quantity of mercury at the temperatures respectively of melting ice and boiling water under an atmospheric pressure of 760 mm.; then any volume

v of the same quantity of mercury indicates a temperature which is measured by the number

$$t = 100 \times \frac{v - V_0}{V_1 - V_0}.$$

§ 119. **Heat a Quantity.**—When the mercury column of a thermometer is rising the substance is said to be *gaining heat;* and when the column is falling the mercury is said to be *losing heat.* In other words, a body at a particular temperature possesses or contains more heat than the same body at a lower temperature. Thus, in ordinary phraseology we recognise heat to be something which may be transferred from body to body, and which by its presence determines the temperature of the body. Heat is, in short, a physical quantity which can be measured in terms of some chosen unit. We shall find later on that there is a dynamical unit of heat involving the fundamental units of length, time, and mass. But in the first place it is usual and convenient, besides simplifying the discussion of the varied effects of heat, to take provisionally an arbitrary unit of heat expressed in terms of change of temperature.

Definition: The unit of heat is the amount of heat required to raise unit mass of water one degree in temperature.

The quantity of heat so defined as the unit will vary with the unit of mass and the scale of temperature employed, and also with the particular temperature at which the water is. There are (see § 121) strong reasons for taking this temperature at 10° C. (50° F.), which, moreover, is not far from the ordinary average temperature of temperate climes. But, whatever be the precise form of definition agreed upon, we obtain, in every case, a definite quantity with which other quantities of heat are readily compared. When the kilogramme is taken as the unit of mass and the centigrade as the temperature scale the corresponding unit of heat is called the *Calorie.* Thus 100 calories will raise the temperature of 100

kilogrammes of water from 10° C. to 11° C. But we must not conclude that this amount of heat will raise the temperature of 1 kilogramme of water from 0° C. to 100° C. That would be equivalent to the assumption that the same amount of heat is needed to raise the temperature by 1°, at whatever temperature the substance may chance to be. Experiment shows that this is not so. We might by definition make each degree of rise of temperature correspond to the in-take of exactly the same amount of heat; but the temperature scale so obtained would differ from the scale determined by the dilatation of mercury. Nevertheless, the difference is not very great, and for ordinary purposes we may assume that the unit of heat, defined as above, is approximately the hundredth part of the heat required to raise unit mass of water from 0° C. to 100° C.

§ 120. **Thermal Capacity and Specific Heat.**—The capacity of any body for heat is the amount of heat taken in by the body while its temperature rises through one degree. Different portions of the same substance have capacities which are proportional to their masses. On the other hand, equal masses of different substances have in general different capacities. The ratio of the capacity of a given body to the capacity of an equal mass of ice-cold water is called the **Specific Heat** of the substance composing the body. Or, since the unit of heat is the amount required to heat unit mass of water through one degree of temperature, it follows that the specific heat of a substance is measured by the amount of heat required to raise by one degree the temperature of unit mass of it.

Of all substances, water is the most difficult to heat. Its specific heat is unity, so that the specific heats of other substances are measured by numbers less than one.

The determination of specific heats is effected in several ways, the simplest in theory being the method of mixtures. It depends upon the almost self-evident principle that the amount of heat *given out* by the body as it cools through a

given range of temperature is exactly equal to the amount of heat taken in as its temperature rises through the same range. For example, suppose we have a pound of water at 10° and a pound of mercury at 100°. Let the two be mixed together and well stirred till they come to the same temperature. This common temperature will be found to be nearly 13°, more accurately 12·9°. That is to say, the pound of water has been heated through 2·9°, while the pound of mercury has been simultaneously cooled through 87·1°. But, if we neglect the loss due to the heating of surrounding bodies, the heat that has been lost to the mercury has been gained by the water. Hence we get the relation

$$\text{specific heat of mercury} = \frac{\text{rise of temperature of water}}{\text{fall of temperature of mercury}} = \frac{2{\cdot}9}{87{\cdot}1} = \frac{1}{30} = {\cdot}033.$$

If the same experiment be tried with equal *volumes* of water and mercury, the final common temperature of the mixture will be fully 38°. For in this case the mass of the mercury is 13·6 times the mass of the water, so that its relative capacity for heat is 13·6 times its specific heat.

The general expression for the ratio (s/s') of the specific heats of two substances whose masses are m and m' and original temperatures t and t' is

$$\frac{s}{s'} = \frac{m'(\mathrm{T} - t')}{m(t - \mathrm{T})}$$

where T is the common temperature after mixing. This is at once deduced from the equation

$$ms(t - \mathrm{T}) = m's'(\mathrm{T} - t')$$

which asserts that the heat lost by m is equal to the heat gained by m'. For effectual mixing, one of the substances must be a liquid, and is generally water. Thus the ratio on the right hand side gives at once the value of s, for s' is in this case equal to unity. The quantity ms, the product of

the mass of a body into its specific heat, is called the Water Equivalent of the body. It measures the mass of water which is thermally equivalent to the mass m of the body —the same amount of heat will raise the temperature of either through the same number of degrees.

In order to obtain a satisfactory result by the method of mixtures, many precautions must be taken and corrections applied. As little heat as possible should be allowed to escape to neighbouring bodies, and what little does escape should be carefully measured. Of these neighbouring bodies the most important are the vessel which contains the water, the thermometer which measures the temperature changes, and the rod used to stir the water as its temperature is changing. Their water equivalents must be estimated and taken into account in the final reduction.

A second method for determining specific heats is by means of the Ice Calorimeter, which depends on the important fact (fully discussed in next chapter) that a definite amount of heat is required to convert a given mass of ice at 0° centigrade into water at the same temperature. There are three hollow vessels one within another, the innermost of all being the vessel which receives the body whose specific heat is to be determined. The region between the outer and middle vessel and the region between the middle and inner vessel are both packed with pieces of ice, and each drains by a separate stopcock (fig. 87). The lid to the outer vessel, which is placed in position immediately after the body is introduced into the inner vessel, is also filled with ice. Because of the equality of temperature between the two regions which are packed with ice, no heat can pass from the one to the other. Whatever heat the outer region receives must

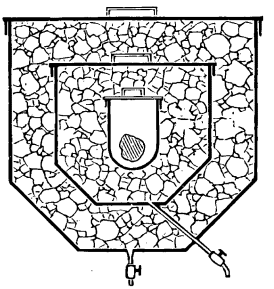


Fig. 87.

come from the outside and is used up in melting part of the ice in this region, but does not affect the temperature. Whatever heat the inner region receives must come from the inside and is used up in melting part of the ice in this region. Suppose for example that a piece of iron thoroughly heated to 100° centigrade in steam is dropped into the innermost vessel. It will begin to cool and the heat it loses will be *wholly* employed, in the long run, in melting the ice in the middle region; and this melting will go on until the iron is reduced to 0° C. If m is the mass of the iron and s its average specific heat between the temperatures of 0° and 100°, the amount of heat lost by the iron as it cools will be 100 ms. This will melt, say, a mass m' of ice, as measured by the quantity of water which drains off from the middle region. But since 80 units of heat are required to melt unit mass of ice (§ 128) we have the equation

$$100ms = 80m'$$

from which s can be at once calculated.

The one doubtful point in this form of experiment is the uncertainty as to whether all the melted ice has drained off as water. To obviate this Bunsen devised a form of ice calorimeter in which the amount of ice that was melted was measured by the change of volume in the mixture of water and ice contained in the middle region. The accuracy of both of these forms of calorimeter depends upon the previous careful determination of two important effects of heat.

A third method for comparing the specific heats of liquids is the method of cooling. The liquid is contained in a closed vessel and is allowed to cool from a high temperature. The rate at which heat is emitted from the surface of the vessel depends only on the condition of the surface. Hence, whatever be the contained liquid, whether water or oil, the amount of heat lost per second at each definite temperature (that of the air being constant) is exactly the same. But the rate of cooling, as measured by a thermometer dipping in the liquid,

will probably differ in the two cases; for (1) their specific heats are different, and (2) their masses are different. By observing, at close enough intervals of time, the successive temperatures, we obtain data from which the rate of change of temperature under given conditions can be estimated. This rate of change, multiplied by the mass of the cooling substance, and by its specific heat, must be constant for the different liquids. Thus the specific heat is inversely as the mass of the liquid, and inversely as the rate of change of temperature. For example, if r, R are the rates of change of temperature for masses m and M of, say, oil and water, otherwise in identically the same conditions of volume, temperature, and bounding surface, then the specific heat of oil is given by the ratio

$$s = \frac{RM}{rm}$$

It is important to bear in mind that the higher the specific heat, other things being equal, the slower the rate of cooling, a truth which finds many illustrations in ordinary daily experiences.

The following table gives the specific heats of a few substances, arranged in order of magnitude:

Water	1	Iron	·114
Alcohol	·566	Copper	·095
Ice	·505	Silver	·057
Chloroform	·233	Mercury	·033
Sulphur	·203	Lead	·031

A very important law, discovered by Dulong and Petit (1819), connects the specific heats and atomic weights of the chemical elements which exist as solids at ordinary temperature. The law is that the specific heat is inversely as the atomic weight, the product of the two being approximately 6·4 for all.

The discussion of the specific heats of gases will be taken up later (§ 125).

§ 121. **The Production and Nature of Heat.**—Heat is generally produced by combustion—a complicated process involving chemical changes — that is, transformations of material configurations; but heat may be and often is produced by much simpler processes, a careful study of which will lead us to a clear understanding of its nature.

For example, heat is produced by friction—a truth well known to many savage tribes who obtain their fire by rubbing two pieces of wood together. A brass button rubbed vigorously on cloth may be made so hot as to be disagreeable to the touch. Metals briskly polished by means of sand-paper or emery undergo an appreciable rise of temperature. Sir Humphrey Davy melted two pieces of ice by rubbing them together, thereby disproving the theory, at that time universally accepted, that heat was a substance in union with ordinary matter. For, according to this theory, the heat that melted the ice must have come *out* of the ice, but it is only by putting heat *in* that we can melt ice. Again, Count Rumford showed that any desired quantity of heat could be obtained in the boring of a cannon by simply making the borer sufficiently blunt and working long enough. A meteoric stone, entering the earth's atmosphere at a high speed, is raised to a high temperature as the resistance to its passage through the air rapidly destroys the motion. By hammering a piece of lead we raise its temperature; and, generally, when the motion of a body is suddenly arrested heat is produced. Finally, in all forms of machinery it is necessary to keep well lubricated the various surfaces which rub on each other. This diminishes the friction, and the heat developed by the rubbing is diminished proportionally. In all these cases heat is generated at the expense of motion destroyed; and a careful experimental investigation into the whole circumstances proves that the heat so generated is proportional to the dynamical *energy* which has disappeared. The work done against the various resistances to motion has its equivalent in the heat generated. This is the great principle of the

equivalence of Heat and Work—an equivalence which was first thoroughly established by the experimental labours of Joule.

The experiment, from which Joule obtained the most accurate results, is extremely simple in principle. It consists in setting a quantity of water into commotion by means of a paddle-wheel driven by a falling weight. The amount of energy supplied is known from the height through which the weight has fallen, and this, because of fluid friction, finally disappears altogether as dynamical energy; but at the same time the temperature of the water rises, and the rise can be measured on delicate thermometers. But, simple though the experiment be in principle, the practical carrying out of it is a matter of the greatest difficulty. Endless precautions must be taken and numerous corrections applied. The specific heats of all parts of the apparatus must be known, and the loss of heat through radiation, conduction, and convection must be carefully measured. In none of Joule's experiments did the rise of temperature exceed a few degrees Fahrenheit.

The important truths demonstrated by Joule may be enunciated very nearly in his own words, but with Rowland's reduction to air thermometer temperature (§ 124) applied to the quantitative result:

1. **The quantity of heat produced by friction of bodies is always proportional to the energy spent.**

2. **The quantity of heat capable of increasing by 1° Fahr. the temperature of one pound of water at 60° F., weighed *in vacuo* at the sea-level in the latitude of Greenwich, requires for its evolution 777·92 foot-pounds of work.**

This number is called the **dynamical equivalent of heat**, or Joule's Equivalent. Its value is 1400·3 foot-pounds, if we define the unit of heat in terms of the pound and centigrade degree. Expressed in centimetre-gramme-second units, the amount of heat required to raise 1 gramme of water from 15° to 16° C. is equivalent to $4{\cdot}188 \times 10^7$ ergs. Joule's

experiments have been repeated with improved apparatus by Rowland of Baltimore, who measured the equivalent at temperatures ranging from 5° C. to 36° C. A few of his results, reduced to C.G.S. units, are given below, the value of g at Baltimore being assumed to be 980 centimetres-per-second per second:

Temperature.	Joule's Equivalent.	Temperature.	Joule's Equivalent.
5° C.	4212×10^4	20° C.	4179×10^4
10° "	4199 "	25° "	4173 "
15° "	4189 "	30° "	4171 "
		35° "	4173 "

From these numbers we conclude that, to raise a given mass of water from 5° to 6° C., more heat is required than is needed to raise the same mass from 30° to 31°. In other words, the specific heat of water varies appreciably with temperature, passing through a minimum value at 29° C. It is important to notice that 42 million ergs is the value of Joule's Equivalent when the water is at a temperature of nearly 10° C.; and for ordinary purposes this number may be taken as the measure of the unit of heat, which, practically defined as the amount of heat required to raise 1 gramme of water from 10° to 11° C., is really defined as the equivalent of a definite amount of energy. The dynamic unit of heat is the amount of heat which is equivalent to the unit of work—the erg in the metric system, the foot-poundal in the foot-pound-second system of units. In this way we obtain a definition of the unit of heat quite independent of the character of any particular substance.

When heat is produced in a way not obviously dynamical, a careful inquiry into the whole process in the light of modern theories of the constitution of matter reveals that here also its origin is essentially dynamical. For example, when water is mixed with sulphuric acid a perceptible rise of temperature occurs. The explanation is that the molecules of the liquids fall together to form new groups characterised by a lower potential energy. The energy set free appears as heat. Simi-

larly in all cases of ordinary combustion, the new chemical combination formed is a molecular configuration of greater stability—that is, of lower potential energy, and the energy lost in this readjustment appears as heat, just as the kinetic energy of the falling stone is transformed into heat when the stone is brought to rest on the surface of the ground.

EXERCISES.

1. Express 0° Fahrenheit in the centigrade scale, and −100° C. in the Fahrenheit scale.

2. What temperature has the same numerical symbol in both the Fahrenheit and centigrade scales?

3. A piece of copper weighing 45·1 gm. was immersed in 52·5 gm. of water at 10° C. The temperature of the copper before immersion was 99·6°, and the common temperature of copper and water after immersion was 16·8°. Find the specific heat of the copper.

4. A 10-lb. mass of iron of specific heat ·112 is immersed in 12 lb. of water, whose temperature consequently rises from 10° C. to 25° C. What was the temperature of the iron immediately before immersion?

5. What volumes of water and mercury should be mixed together so that the final temperature may be the arithmetic mean of the original temperatures of the two substances? The density of the mercury is 13·6, and its specific heat ·033.

6. How much higher is the temperature of the water at the bottom than that of the water at the top of a waterfall 100 feet high, assuming that the kinetic energy due to the fall is wholly converted into heat?

7. A mass m, attracted towards the earth from a great distance, falls upon its surface with an amount of energy mgR, where R is the earth's radius (see § 67). If this energy is wholly transformed into heat, and half of this heat is absorbed by the mass itself, to what temperature will the mass be raised, assuming an average specific heat of ·2?

8. An iron vessel, weighing 12 ounces, contains 2 lb. of water and half-a-pound of mercury. How much heat must be applied to raise the temperature of the whole from 10° C. to 20° C.?

9. Calculate the water equivalent of a copper vessel and its contents, the copper weighing ½ lb., and the contents being 10 lb. of alcohol and 10 ounces of iron. If the whole be at 10° C. and

another pound of alcohol at 50° C. be added, what will be the resulting temperature?

10. If 1400 foot-pounds of work can raise the temperature of 1 lb. of water 1° C., how many degrees will 10 lb. of mercury be raised by the expenditure of half that amount of work?

11. 10 grammes of a substance are placed in a Bunsen calorimeter, and the volume of the ice and water is found to decrease by $\frac{1}{10}$ cubic centimetre. Water expands $\frac{1}{11}$ of its volume on freezing, and the latent heat of ice is 80. Find the specific heat of the substance.

SUMMARY.

§ 116. Hot and Cold. Temperature.

§ 117. Volume Changes accompanying Change of Temperature. Expansion of Solids, Liquids, and Gases. Thermometric Substance.

§ 118. Thermometers. Mercury peculiarly serviceable as a Thermometric Substance. Freezing and Boiling Points of Water. Centigrade, Fahrenheit, and Réaumur Scales.

§ 119. Heat a Quantity. The Unit of Heat. Calorie.

§ 120. Thermal Capacity and Specific Heats. Determination of Specific Heats: by mixture, by melting of ice, by cooling. Water Equivalent. Dulong and Petit's Law.

§ 121. The Production and Nature of Heat. Combustion. Friction. Hammering. Equivalence of Heat and Work. Joule's Experiments and Results. Rowland's Experiments. Change of Specific Heat of Water with Temperature.

CHAPTER XIV.

EFFECTS OF HEAT.

§ 122. The changes which usually accompany the application of heat to matter are conveniently classified as follows:

(*a*) Volume changes in the solid state;

(*b*) Liquefaction, or change of solid into liquid;

(*c*) Volume changes in the liquid state;

(*d*) Volatilisation, or change of liquid into vapour;

(*e*) Volume or Pressure changes in the vaporous and in the gaseous states.

As a rule, these changes are also the successive changes that occur when a solid substance is heated persistently until it becomes a gas.

In the converse case of heat being abstracted from matter, the order of these changes will be reversed, liquefaction (of vapours) will take the place of volatilisation, and solidification the place of liquefaction (of solids). Many solids, such as ice, paraffin, lead, iron, &c., can be first liquefied and subsequently volatilised by continued heating. Others, again, such as glass, can be liquefied, but have not been obtained in the gaseous state. Then there are certain substances (e.g. iodine) which seem to pass from the solid to the gaseous condition without passing through the intermediate state of liquid; and there are liquids, such as alcohol, which have not been obtained as solids. Finally, carbon is a substance which has never been obtained in the liquid or gaseous state, although it may be dissolved in certain molten metals.

The volume changes which accompany rise of temperature have already been referred to in connection with the construction of the ordinary thermometer. It is necessary to

consider these changes more in detail; and because of their comparative simplicity and far-reaching importance we shall first discuss the changes of volume and pressure which accompany change of temperature in gases.

§ 123. **Charles'** (also called **Gay-Lussac's) Law** asserts that the volume of a given mass of a gas kept at constant pressure increases uniformly with rise of temperature. Thus if V is the volume at 0° C., and v the volume at t° C., then

$$v = V(1 + \alpha t)$$

where α, the *coefficient of dilatation*, has very approximately the same value for all gases. A similar law holds for the change of pressure at constant volume. If Boyle's Law held accurately at all temperatures, the coefficient of increase of pressure would indeed be exactly the same as the coefficient of dilatation, and Boyle's and Charles' Laws would be combined in the simple form

$$pv = A(1 + \alpha t),$$

where A is the value of pv at 0° C.

The careful experiments of Regnault and others have shown that the pressure and dilatation coefficients differ slightly—the difference being more marked in the more easily liquefiable gases. The values for a few of the gases are shown in the accompanying table:

	Dilatation Coefficient at Constant Pressure.	Pressure Coefficient at Constant Volume.
Hydrogen	·003661	·003668
Air	·003671	·003665
Carbonic Oxide	·003669	·003667
Sulphurous Acid	·003902	·003845

For all ordinary purposes, however, the Boyle-Charles Law may be assumed to hold true, and the value of α to be about ·00366 or $\frac{1}{273}$. The law may then be written in the simpler form

$$pv = R\left(\frac{1}{\alpha} + t\right) = RT,$$

where T is the temperature measured from a point 273°

below the centigrade zero, and 273R is the value of pv at 0° C. At the temperature of − 273° C. the product pv for the ideal gas would vanish—either the gas would cease to have any volume (which is absurd) or it would cease to press upon the walls of the containing vessel.

Now, according to the kinetic theory of gases (§ 107), the pressure of a gas can be expressed as being proportional to the product of the density into the square of the speed of the particle. In other words, we can regard the product pv as being proportional to the mean kinetic energy of translation of the molecules; and thus the temperature, measured from − 273° C., is also proportional to the same kinetic energy. This conclusion is in complete harmony with the view that heat is energy; for rise of temperature, which means increase of molecular kinetic energy, is the equivalent of heat supplied.

§ 124. **The Air Thermometer; Absolute Temperature.**—This extremely simple relation between the pressure volume and temperature of a gas at once suggests the advisability of making air our standard thermometric substance. We may keep the pressure constant and measure the temperature by means of the volume, or keep the volume constant and measure the temperature in terms of the pressure. Having noted the indications of the air-thermometer at the chosen standard temperatures of melting ice and boiling water, we call these temperatures 273 and 373 respectively, interpolate 100 equal degrees between them, and continue the graduation down to the lowest and up to the highest attainable temperatures. We thus approximate to what is known as the Absolute Scale of Temperature, the zero of reckoning being the absolute zero of temperature.

It is called the Absolute Scale because it agrees very closely with the scale of temperature suggested by Carnot's Theory and independent of the properties of any special substance. This scale will be described later (§ 137).

For ordinary purposes the scale of the air-thermometer

does not differ markedly from the scale of the mercurial thermometer. The merits of air as a thermometric substance are that it is one of a group of substances which follow almost exactly the same law of dilatation, and that through the whole range of attainable temperatures it continues to obey the same law. But practically the air-thermometer is inconvenient for use. Its very sensitiveness makes it difficult to manipulate, slight variations of atmospheric pressure, for example, producing appreciable alteration in its indications.

When, once for all, the dilatation of mercury has been carefully determined with the absolute scale as the scale of temperature, measurements of temperature on the mercurial scale can be readily reduced to their values in the absolute scale. Henceforward all temperatures will be expressed in terms of the scale given by the gaseous law

$$pv = \mathrm{RT} = \mathrm{R}(273 + t),$$

where T is the absolute temperature and t the measure of the same on the centigrade air-thermometer scale.

§ 125. **The Specific Heats of a Gas.**—The application of heat to a gas must change two at least of the quantities, temperature, volume, and pressure—perhaps all three. The specific heat is the amount of heat required to raise the temperature of unit mass of the substance by 1° C. This rise of temperature may be accompanied (1) by change of pressure only, (2) by change of volume only, (3) by change of both. In the first case we have to do with the specific heat at *constant volume;* in the second, with the specific heat at *constant pressure.* The third case is of comparatively little interest.

Consider, now, the work that is done when the heat is applied to the gas under each of these conditions. In both the temperature is raised, and this, according to the kinetic theory, implies increase of the kinetic energy of the particles. In effecting this increase a definite quantity of energy in the form of heat passes into the gas. In the case

of constant volume the pressure increases, because of the more rapid motion of the particles, which impinge oftener on the walls. But in the case of constant pressure the gas is allowed to expand, so that, although the kinetic energy of the molecules has increased with the temperature, the volume through which the particles move has been enlarged, and the number of impacts per second upon the walls has not changed. Suppose the gas to be enclosed in a piston chamber, within which there works a piston on which a definite pressure is sustained. As the gas expands, the piston is pushed outwards through a certain distance, which, multiplied by the area of the piston, will measure the increase of volume. But the work done by the expanding gas is evidently equal to the product of the total pressure on the surface of the piston into the displacement of the piston outwards. Hence

Work done = pressure × area of surface
× outward displacement of the surface
= pressure × increment of volume.

If we assume that this work is done by the heat supplied, then it is evident that the heat required to raise by 1° the temperature of a gas at constant pressure must be greater than the heat required to effect the same temperature change at constant volume; for in the former case extra work must be done in expanding the gas. From the values of the two specific heats of air we may readily calculate Joule's Equivalent; or from the known value of Joule's Equivalent together with one specific heat we may calculate the other. The calculations lead to concordant results, so that the assumption made above is justified.

Thus 1 lb. of air at 1 atmosphere of pressure and 0° centigrade has a volume of 12·55 cubic feet. Hence Boyle's and Charles' Laws take the combined form

$$pv = 12{\cdot}55 \times (14{\cdot}5 \times 144).$$

The specific heats at constant pressure and volume are respectively ·2389 and ·1705. Hence their difference, ·0684, may be assumed to be the heat consumed in expanding the gas. But the change of

volume for 1° rise of temperature is $v \times \cdot 003665$. Consequently the mechanical work done is

$$pv \times \cdot 003665$$
$$= 12 \cdot 55 \times 14 \cdot 5 \times 144 \times \cdot 003665 \text{ foot-lbs.}$$

But this must be the dynamical equivalent of ·0684 unit of heat. Hence, if J is Joule's equivalent, its value is given by the equation :

$$J \times \cdot 0684 = 12 \cdot 55 \times 14 \cdot 5 \times 144 \times \cdot 003665,$$

or $$J = 1395 \text{ foot-lbs. per degree centigrade.}$$

Thermodynamic considerations show that, for a gas obeying the Boyle-Charles Law, the specific heats should be in the ratio of 1·41 to 1. From the specific heat at constant pressure that at constant volume could be calculated. It is only recently, indeed, that Joly, by means of the heat given out during the condensation of a measured quantity of steam upon a vessel containing air, has made accurate determinations of its specific heat at constant volume.

§ 126. **Dilatation of Liquids.**—Liquids generally expand or dilate when heated, and the coefficient of dilatation is defined as the change of unit volume due to unit rise of temperature. It may be measured in either of two ways.

In the first method the quantities of any liquid which at different temperatures just fill a narrow-necked bottle are determined by weighing. If the bottle were of a material that did not change bulk when heated, these measured quantities of the liquid would all have the same volume. But no such material is known to exist. Hence, before the true dilatation of the liquid can be measured, the cubical dilatation of the material of the bottle (usually glass) must be determined independently. As will be shown in the succeeding section, the cubical dilatation of an isotropic solid is three times the linear dilatation, a quantity which is determined with comparative ease. Having determined it for the material of the bottle, we can from the measured mass of liquid filling the bottle at temperature $t°$ C. readily calculate the mass that would fill a volume equal to that of the bottle

at 0° C. Calling this corrected mass M_t, and the mass at 0° C. M_o, we find the diminution of density from the ratio

$$\frac{M_o - M_t}{M_o},$$

a small quantity which also measures the increase of volume due to rise of temperature t. Hence, if K is the average value of the coefficient of dilatation throughout this range of temperature, the ratio just written is equal to Kt, and finally

$$M_t = M_o(1 - Kt).$$

The second method, devised by Dulong and Petit, and used by Regnault in his classical experiments, is independent of the expansion of the solid which contains the liquid. The principle is that of the experiment described in § 71, in which the heights of the columns of different liquids in the branches of a U tube above the common surface where the liquids meet are inversely as the densities of the liquids. In the present application the liquids are the same, but differ in being at different temperatures. The essential feature of the arrangement is shown in fig. 88. The tube is a double U, and an air space separates the two columns of liquid. One half is enclosed in a vessel full of ice-cold water, and the other half in a vessel whose liquid contents may be kept at any desired temperature. The differences of level ab, $a'b'$ are then carefully measured, and as these are inversely as the densities, the average coefficient of dilatation throughout the given range of temperature can be at once calculated.

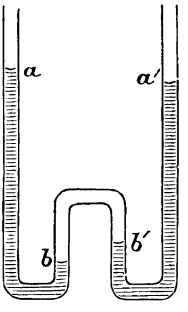


Fig. 88.

The most important liquids from a thermal point of view are mercury and water, the former because of its value as a thermometric substance, the latter because of its wide distribution and the part it plays as a physical agent.

The results of Regnault and other investigators show that

mercury does not expand quite uniformly with rise of temperature. The following table gives the values of the mean coefficients between 0° C. and various other temperatures according to Regnault:

Temperature $t°$.	Mean Coefficient from 0° C. to $t°$ C.
50°	·0001803
100°	·0001815
200°	·0001840
300°	·0001866
350°	·0001878

Thus the rate of expansion of mercury per degree rise of temperature increases with the temperature. The rate of increase is small and fairly constant.

As compared with mercury, all other liquids at ordinary temperatures have, in general, greater coefficients of dilatation, which increase even more decidedly with rise of temperature. Thus, at temperatures between 0° and 40° C. the coefficient of dilatation of alcohol averages ·0011.

In the relation between temperature and volume, water presents some remarkable features. For example, when ice-cold water is heated it *contracts*. This contraction continues until the temperature has risen to 4° C., after which rise of temperature is accompanied by expansion in the usual way. The temperature at which water has its smallest volume—that is, its greatest density—is called the **maximum density-point of water.** The following table contains the volumes at different temperatures of a mass of water whose volume at 4° C. is unity:

Temperature.	Volume of Water.
0°	1·000127
4°	1·000000
10°	1·000265
20°	1·001751
50°	1·01201
100°	1·04315

The exact temperature at which water under ordinary pressures attains its maximum density is 4·04° C.

Imagine a quantity of fresh water at a temperature of about 10° C. being cooled down gradually by contact of its upper surface with cold air. At first the upper layer as it cools becomes denser than the underlying water, and consequently sinks, while the warmer layers rise up from below to take its place and become cooled in their turn. Thus the process of cooling is accompanied by a constant down-streaming of the surface waters and an up-streaming of the bottom waters, until the whole is reduced to the temperature of the maximum density-point. Thereafter cooling is accompanied by expansion, so that the surface layer at a temperature say of 3° C. cannot sink, but remains floating on the slightly warmer layer beneath it. And thus the cooling surface layer remains always the surface layer and continues to cool until it begins to freeze. The heat in the water beneath is slowly conducted upwards through the colder surface layer into the cold air above, and thus the temperature becomes gradually reduced through an ever thickening layer of water. The phenomenon is well shown in Hope's Experiment. A cylindrical vessel, filled with water, is provided with two thermometers, one near the top and the other near the bottom. The centre of the column between the two thermometers is cooled by means of a freezing mixture of broken ice and salt packed all round it on a shelf constructed for the purpose. The lower thermometer at once begins to cool, and cools steadily till the temperature falls to 4° C., the upper thermometer meanwhile remaining unaffected. Thereafter the lower thermometer remains steady, but the upper begins to cool briskly, and continues to cool down to the freezing-point.

§ 127. **Dilatation of Solids.**—The change in volume of solids when heated or cooled has been already noticed. One mode of measuring this change of volume is suggested by the experiment first described as affording a measure of the dilatation of liquids. As pointed out, the true dilatation of the liquid contained in the glass flask could not be ascertained unless the dilatation of the glass were known. But it is evident that if the dilatation of any one liquid is determined by some other process, the experiment with the flask will enable us to find the dilatation of the flask itself. Now, Regnault's modification of Dulong and Petit's method has led to very accurate determinations of the coefficient of dilatation of mercury. Hence, from observation of the

apparent dilatation of mercury in a particular glass vessel, the dilatation of the glass can be calculated. In very accurate determinations of temperatures by means of the air-thermometer, the dilatation of the glass in which the air is for the most part confined requires to be taken into account.

Generally, in regard to solids, it is the *linear* dilatation or change of length, and not the *cubical* dilatation or change of volume that is the important quantity. For example, a rod when heated increases most perceptibly in *length*, the accompanying increase in section being comparatively insignificant. If a rod of length L at 0° C. increases to length l, when the temperature is raised to $t°$, the quantity

$$\frac{l - \mathrm{L}}{\mathrm{L}} \div t = k$$

is called the average linear dilatation between the given limits of temperature, or

$$l = \mathrm{L}\,(1 + kt).$$

A unit cube will have each side increased to $1 + kt$, and hence the volume of the cube will be,

$$(1 + kt)^3 = 1 + 3kt + 3k^2t^2 + t^3.$$

But k is usually a very small quantity, so that terms containing its square and cube may be neglected, and hence unit volume becomes $1 + 3kt$. In the quantity $3k$ we recognise the coefficient of cubical dilatation; and thus in isotropic solids the cubical dilatation is three times the linear dilatation. Occasionally it is important to take account of the increase of surface of a plate when heated. It is easy to show that the *areal* dilatation is twice the linear dilatation.

Linear dilatation can be measured with an accuracy depending on the accuracy with which lengths can be measured; and, by the use of screw micrometers, lengths can be measured with very great precision.

In fig. 89 is shown a simple form of apparatus for demonstrating the linear dilatation of a solid rod when heated. The rod, which is fixed at the one end, A, by means of a bind-

ing screw, passes easily through a hole at B, and bears with its free end on C the short arm of the lever DC. A spring keeps the short arm pressing always on the end of the rod. When the rod is heated by a row of gas-jets or spirit flames (as shown in the figure) it begins to lengthen, and

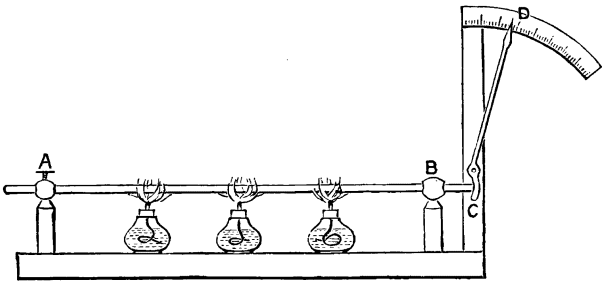


Fig. 89.

this lengthening is indicated and measured by the motion of the end D of the long arm of the lever in front of a suitable scale. The apparatus may be modified in various ways, so as to make it suitable for the accurate observation of the temperature of the rod and the corresponding length.

The following table contains the linear dilatations of some of the more important metals in the neighbourhood of 30° C., as determined by Voigt:

Aluminium	·00002306	Nickel	·00001315
Cadmium	·00002470	Silver	·00001925
Copper	·00001709	Steel	·00001147
Gold	·00001414	Tin	·00002220
Iron	·00001158	Zinc	·00002510

To which may be added:

Platinum		·00000891
Platinum-Iridium		·00000876
Glass	from	·000008
	to	·000009

It will be seen that platinum has a coefficient of dilatation differing very slightly from that of glass, whereas the other

metals have distinctly larger dilatations. This approximate equality of dilatations enables us to fuse platinum wire into glass and obtain a hermetically-sealed joint, an operation which would be utterly impossible to effect with, say, copper or brass. The copper wire could, no doubt, be inserted into the soft, semi-molten glass; but on cooling the metal would contract more than the glass, so that the junction would not be secure.

When stretched india-rubber is heated it contracts, and recovers its original length on cooling. The simplest way to observe this anomalous phenomenon is to drive steam through an india-rubber tube which is stretched by an appended weight. In close relation to this phenomenon is the appreciable rise in temperature which occurs when rubber is suddenly extended. It is worthy of note that, though the linear dilatation of stretched india-rubber is negative, the cubical dilatation is positive. This means that the dilatations are different in different directions, a phenomenon also characteristic of many crystalline substances.

The fact that different metals have different dilatations is taken advantage of in the construction of a steady-going clock or watch. In the case of the clock the regulator is the pendulum, which should be of a definite and constant length. If the pendulum be constructed of one metal, its length will change with the daily and seasonal changes of temperature, and the period of its oscillation will correspondingly alter. Thus the clock will lose time in summer and gain time in winter. To *compensate* for this temperature effect, the Compensation Pendulum has been devised. In one form, known as the Gridiron Pendulum, the compensation is effected by correcting the expansion downwards of one metal by the expansion upwards of another. The principle of construction of the pendulum is shown in fig. 90. The outer and middle rods, shaded darker, are of one kind of metal —the intermediate rods, not shaded, are of another kind of metal. It is obvious that the effect of the expansion of the intermediate rods is to raise the bob, thereby

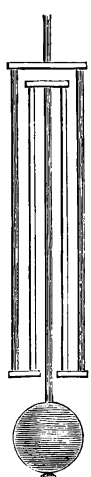


Fig. 90.

counteracting the lowering of the bob because of the expansion of the other rods. To make the compensation perfect, the *total* expansion upwards of the intermediate rods should be exactly equal to the total expansion downwards of the rest of the pendulum. Consequently the intermediate rods must have their coefficient of dilatation greater than that of the other rods in the inverse ratio of their lengths. Brass and iron are very suitable metals for use in this way; and gridiron pendulums are usually made of these metals.

The rate of a watch is regulated by the balance-wheel, which oscillates to and fro in a period depending upon its moment of inertia and upon the strength of the spring which sustains the motion. Temperature changes will affect chiefly the moment of inertia if the wheel is made all of one piece. The compensation in this case is effected by making the rim of the wheel discontinuous, each portion being composed of two strips of different metals, the more expansible being outside (see fig. 91). A rise of temperature will be accompanied by an increase in the diameter; but at the same time each arc will become more curved, and the weights at the end will be brought in nearer the centre. By careful adjustment, this inward displacement of the end weights can be made to compensate for the expansion of the radius, so that the moment of inertia remains unchanged. Decrease of temperature will be accompanied by an *outward* displacement of the end weights so as to compensate for the contraction of the radius.

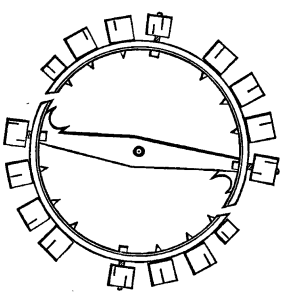


Fig. 91.

Bréguet's metal thermometer consists of a spiral formed of three metal ribbons fixed together. The outer ribbons are of silver and platinum, and are separated by a ribbon of gold, whose coefficient of dilatation is intermediate to those of the other two. The spiral is fixed above, and to its lower free end a horizontal pointer is attached. A very slight change of temperature causes the spiral either to uncoil or to coil up more, and this movement is at once shown by the pointer moving on a graduated scale placed below.

§ 128. **Liquefaction, Fusion, or Melting of Solids: Solidification or Freezing of Liquids.**—Most solids when heated to a sufficiently high temperature become liquid; and most

liquids when cooled to a low enough temperature become solid. The temperature at which this change of state occurs is called the **Melting** (or **Fusing**) **Point,** or **Freezing Point,** according to the manner in which the transition is being regarded. As a rule we speak of the melting-points of substances which are solid at ordinary temperatures, and of the freezing-points of substances which are liquid at ordinary temperatures; but both refer to one and the same temperature for any one substance. The following table contains the melting or freezing points of various bodies:

Mercury	-39°C.	Silver	1000°
Ice	0°	Copper	1100°
Paraffin (solid)	46°	Gold	1200°
Wax	62°	Cast-steel	1375°
Sulphur	115°	Wrought-iron	1600°
Tin	230°	Platinum	1775°
Lead	326°	Iridium	1950°

The laws of melting, or fusion, are these:

1. As heat is applied to the melting substance the temperature remains constant (if the whole be well stirred) till the last particle is melted.

2. For every unit mass of substance melted a definite quantity of heat, called the **Latent Heat,** is absorbed.

By way of illustration, consider the important case of ice and water. The water which comes from melting ice is at the same temperature as the ice. A mixture of the two is at a temperature of 0° C. Any heat supplied to the mixture results in the melting of some of the ice; any heat taken from it results in the freezing of some of the water. Since heat is a form of energy, we conclude that water in virtue of its molecular arrangements contains more energy than does an equal quantity of ice. To alter the arrangement of the particles and change the ice into water, a certain amount of work must be done. It is the heat which supplies this work. Thus Latent Heat—so named by Black before the true nature of heat was discovered—is not heat in any true sense of the word. It signifies a transformation of energy, that which was supplied originally as heat having been transformed into some form of molecular energy.

The amount of heat which is absorbed during liquefaction

may be measured in the following manner: Suppose we have in separate vessels one pound of ice at 0° C. and one pound of water at the same temperature, and let there be added to each of them one pound of water at 100°. The water and water mixture will result in two pounds of water at 50° C.; but the ice and water mixture will result in two pounds of water at 10°. In the latter case the original hot water has lost 90 units of heat. Ten of these has heated the *melted ice* (water) from 0° to 10°; consequently the remaining 80 units must have been used up in melting the ice. Thus, one pound of ice at 0° C. requires for its liquefaction as much heat as will raise one pound of water 80° C. In other words, the latent heat of water is 80 units of heat.

The following are the latent heats for a few substances when melted:

Water	80·0	Sulphur	9·4
Zinc	28·1	Phosphorus	5·0
Silver	21·1	Lead	5·3

Thus water is remarkable for its great latent heat as well as for its great specific heat. As a meteorological agent its effect is to prevent rapid changes of temperature. For example, to mention one result among many, insular climates are much more equable than continental climates, because of the presence of the sea, and the consequent large amount of cloud and vapour.

Most bodies expand when they melt, so that the liquid is lighter than the solid at the same temperature. But in certain cases contraction accompanies melting, and the solid floats on the liquid of the same temperature. In this respect water again calls for special remark. The density of ice is about ·91, so that the volume of water increases by 9 per cent. as it freezes. When water at 4° C. or lower fills a closed vessel and is cooled down steadily, it does not freeze but exerts increasing pressure on the containing walls, which are at the same time contracting. Under these conditions the walls frequently yield, and the water expands and freezes.

Since water expands in freezing, work is being done against the pressure of the air. If this pressure be increased, more work will have to be done during the solidification, and it will be more difficult to effect the freezing. In this remark we have a hint of the reasoning by which James Thomson showed that the freezing-point of water must be lowered by increase of pressure. Lord Kelvin subsequently included the case under a general law, which is to the following effect:

Substances which expand on melting have their melting-points raised by pressure; whereas substances which contract on melting have their melting-points lowered by pressure.

Thus wax, which belongs to the first class, is made more solid when compressed. On the other hand, ice at 0° C. melts under increased pressure. To lower the melting-point of ice by 1° C. requires 133 atmospheres of pressure; and Mousson has shown by experiment that under 13,000 atmospheres water is still fluid at a temperature of – 18° C. This phenomenon gives the explanation of what is known as the *regelation* of ice—namely, the freezing together into one lump of two pieces of ice placed in contact. It also explains the making of a snowball, shows why skating is possible, and has an important bearing on the theory of the motion of glaciers.

§ 129. **Evaporation and Boiling of Liquids; Condensation or Liquefaction of Gases.**—When water is left exposed in an open dish it gradually diminishes in quantity. This diminution can be made evident very soon by weighing the dish and its contents from time to time on a good balance; but a few hours generally suffice for the diminution to be visible to the eye, especially if the experiment be made in a warm room. This process is called *Evaporation*, the surface liquid passing off into the air in the form of vapour. It is by this process that wet objects become dry and that the air becomes filled with the moisture which ultimately reappears in the liquid or solid form as mist, rain, dew, snow, hail, or hoar-frost.

The rate of evaporation is greatly accelerated by rise of temperature, so that at about 40° C. the water visibly 'steams' —that is, gives off vapour which, condensing again by contact

with the cooler air, forms a visible cloud of minute drops of water. As the temperature rises this *steaming* becomes more and more apparent. If the water is being heated by a flame placed below it, a steady state of temperature is ultimately reached, and the steam or vapour is given off freely and with considerable commotion. The water is then boiling; and while this boiling continues the temperature of the liquid does not rise beyond a certain point. The highest temperature so attainable is called the **Boiling-point.** Other liquids behave in a similar manner, each having its definite boiling-point.

Corresponding to the laws of melting already enunciated are the following laws of boiling:

1. As heat is applied to the boiling substance the temperature remains constant under constant pressure until the last particle of liquid has become vapour.

2. For every unit mass of substance so transformed into vapour there is absorbed a definite quantity of heat called the **Latent Heat of Vaporisation.**

3. The boiling-point is raised by increase of pressure and lowered by decrease of pressure.

In the following table the boiling-points at the standard atmospheric pressure (760 mm. of mercury) of a number of substances are given:

Carbonic Acid	−78° C.	Alcohol	78° C.
Ammonia	−38 "	Water	100 "
Chlorine	−34 "	Oil of Turpentine	159 "
Sulphurous Acid	−10 "	Mercury	357 "
Ether	+35 "	Sulphur	447 "
Chloroform	61 "	Zinc	1040 "

The latent heats of vaporisation of a few of these are as follows:

Water (Steam)	536 units of heat.
Alcohol	202 " "
Ether	91 " "
Oil of Turpentine	69 " "

The high latent heat of steam calls for special remark. To transform 1 pound of water at 100° C. into steam at 100° C. requires 5·36 times as much heat as is required to raise the pound of water from 0° C. to 100° C. Or, in other words, 1 pound of steam at 100° passed into 5·36 pounds of water at 0° will raise that water to 100° and become itself reduced to water at the same temperature — the final result being 6·36 pounds of water at 100° C. (no heat being supposed to have escaped).

§ 130. **Cooling by Evaporation.**—Evaporation takes place at all temperatures, and when it is taking place a certain quantity of liquid is passing into the gaseous state. This transformation of state is accompanied by the disappearance of a definite quantity of heat for every grain of liquid vaporised; and this heat must be supplied from some source. If no external source of heat is being used, the heat that is to become latent must be supplied out of the substance itself and the bodies in contact with it. Hence the unassisted **evaporation of a liquid is accompanied by a cooling effect.**

This effect is particularly observable in the case of ether, which has a low boiling-point and evaporates much more rapidly than water at ordinary temperatures. If a little be poured on a piece of thin cloth wrapped round the bulb of an ordinary thermometer, its rapid evaporation will produce an appreciable fall of the mercury column. Again, if the hand after being dipped in warmish water is allowed to dry off naturally by evaporation in the air, the cooling effect will be distinctly perceptible. Water carafes made of porous earthenware material act on the same principle—the evaporation of the water filling the pores keeping the water contents cool. In India water is similarly kept cool in canvas bags. Then, again, the rapid evaporation of water in a dry air is sufficient under certain conditions to freeze the water—a method used with effect in the northern parts of India.

§ 131. **Vapour Pressures.**—The rate at which a liquid

evaporates in air depends upon the amount of its vapour already in the air. Thus in very *damp* air the process of 'drying' is very slow, and entirely ceases when the air is what is called saturated. In this state of saturation the air contains the greatest possible amount of water vapour under the existing conditions. Now the one condition which determines the amount, and therefore the pressure, of the vapour needed to saturate a given volume of air is the temperature. The principle may be enunciated thus:

A vapour in contact with its liquid has a maximum pressure which depends solely on the temperature and curvature of the liquid surface.

Let there be a closed vessel containing a quantity of any liquid. The space not occupied by the liquid will be found to contain, in addition to gases or vapours which may be present, a certain quantity of the vapour of the liquid. With rise of temperature more vapour will be given off; with fall of temperature part of the vapour will be condensed to the liquid form. Should the space over the liquid surface be diminished in volume (the temperature being unchanged) part of the vapour will also be condensed to the liquid form. On the other hand, increase in the volume of the space occupied by the vapour will be accompanied by further evaporation of the liquid. In short, other things being the same, the *quantity* of vapour existing in the space above the liquid is proportional to the volume of this space. For only thus can the pressure of the vapour remain the same. If there is nothing but the vapour of the liquid present, then compression of the region containing the liquid and its vapour does not produce increase of pressure. The effect of the compression is simply to liquefy part of the vapour.

The manner in which the pressure of saturated water vapour depends on the temperature is shown in the following

table, in which the pressures are in terms of the height of the mercury barometer measured in millimetres:

Temperature. °C	Vapour Pressure. *mm.*	Temperature. °C.	Vapour Pressure. *mm.*
−30	0·4	40	54·9
−20	0·9	50	92·0
−10	2·1	60	148·8
0	4·5	70	233·1
10	9·2	80	354·6
20	17·4	90	525·5
30	31·6	100	760·0

At 100° the pressure of the water vapour equals the normal atmospheric pressure. At this temperature consequently water boils. If water, heated say to 60° C., is placed under the receiver of an air-pump and the air exhausted until the pressure falls to between one-fifth and one-sixth of an atmosphere (148·8 mm.), the water will begin to boil. And similarly in any other case. The diminution of barometric pressure with height produces quite an appreciable fall in the boiling-point of water. Thus, on the top of Mont Blanc water boils at about 85°C. An apparatus constructed on this principle, and known as the Hypsometric Thermometer, is much used by travellers for estimating altitudes. For every 972 feet of ascent the boiling-point of water is lowered 1° C.

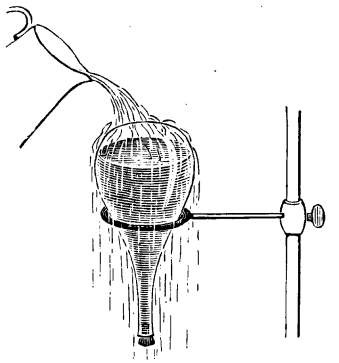


Fig. 92.

The following simple experiment is particularly instructive. A glass flask is partly filled with water, which is kept boiling briskly for some minutes until nearly all the air has been expelled. A tightly-fitting cork is then inserted into the mouth of the flask, and the flame is removed from under it. The flask is then inverted and set, stopper down, in the ring of a retort-stand, as shown in fig. 92.

The space above the water is filled almost entirely with water vapour, whose pressure is determined by the temperature of the liquid surface. If, now, cold water be poured over the flask, the vapour in contact with the cooled glass will be condensed into the liquid form, and the pressure of the vapour over the liquid will be diminished. To restore the pressure of saturation the liquid gives off more vapour—and, in short, begins to boil briskly. The boiling can be stopped at once by pouring hot water instead of cold over the flask.

When by diminution of pressure a liquid is made to boil, the evaporation involves a cooling. It is possible, by rapid exhaustion of a receiver containing a flat dish of water and a substance to absorb the water vapour, to make the water *boil* until it *freezes*. This beautiful experiment is due to Sir John Leslie.

The temperatures at which saturated water vapour acquires certain specified pressures greater than the normal pressure of the atmosphere are given in the following table, the pressures being measured in *atmospheres*:

Temperature.	Atmospheres.	Temperature.	Atmospheres.
100·0° C.	1	170·8° C.	8
120·6	2	180·3	10
133·9	3	213·0	20
144·0	4	236·2	30
152·2	5	252·5	40
159·2	6	265·9	50

Thus water can be kept liquid at any high temperature by simply heating it in a closed vessel capable of standing the pressure.

An extremely important climatic factor is the water vapour present in the atmosphere. This vapour is supplied originally by the water surfaces (liquid or solid) in contact with the air, seas, lakes, rivers, rain-drops, snow, ice, &c. When the pressure of the vapour in the air at a given temperature approaches its saturation value, the air is said to be humid or moist. A slight fall in temperature may then be accompanied by a condensation of part of the vapour to the liquid form. On the other hand, when the vapour pressure is far below its saturation value, the air is dry; and it

requires a considerable fall in temperature before any deposition takes place in the form of rain or dew. But whatever be the amount of water vapour present, its pressure will correspond to a certain definite temperature, known as the **Dew-point.** Should the air be cooled to this temperature, it will become saturated, and a further cooling will result in deposition. Or, if a substance, at a temperature a little below the dew-point, is introduced into the air, it will become coated with a liquid film. This, indeed, is what occurs whenever dew is formed.

The *humidity* of the atmosphere is the ratio of the pressure of vapour actually present to the saturation pressure corresponding to the temperature. If the dew-point is known, the humidity can be calculated. Suppose, for example, that the temperature of the air is 20° C., and that the dew-point is found to be 10° C.—then from the table on p. 228 we calculate the humidity by the ratio of 9·2 to 17·4; that is, 0·53.

The dew-point may be measured by means of Daniell's Hygrometer, in which a bulb is cooled by the evaporation of ether within it, until its polished surface is observed to become dim by the deposition of moisture. The temperature is measured by a thermometer dipping in the ether. The experiment is an extremely difficult one to carry out with any degree of accuracy; and practically meteorologists measure the humidity of the atmosphere by means of the Wet and Dry Bulb. Two similar thermometers are set side by side; but the bulb of the one is surrounded by a piece of cloth, which is kept wet by having its lower end dipping in a small vessel of water. The Wet-Bulb thermometer generally registers a lower temperature because of the cooling due to evaporation from the wet cloth. But if the air be saturated there is no evaporation, and the Wet-Bulb thermometer registers the same temperature as the Dry-Bulb thermometer. The drier the air the greater the difference between the readings of the two thermometers. From these readings the humidity is calculated with the aid of hygrometric tables, in the preparation of which much elaborate research has been made. The only *accurate* way, however, of measuring the humidity of the air is to measure directly by weighing the quantity of vapour in a given volume, and compare that with the amount necessary to saturate the air at the given temperature.

When the dew-point is below the freezing-point of water, any dew that may form because of the cooling of surfaces in contact with the air will be in the form of hoar-frost. In this case the vapour condenses to the solid state without passing through the intermediate liquid condition.

§ 132. **Distinction between a Vapour and a Gas: Critical Temperature.**—A liquid is distinguished from other kinds of fluid by having a free surface; and in such a case there is always vapour present over and in contact with the surface. Imagine any liquid with its vapour to be enclosed in a strong glass tube, the surface of demarcation between the liquid and the vapour being clearly visible. Let this be heated. Then at a certain temperature, peculiar to each substance, the line of demarcation between the two fluid states of the substance will vanish. At and above this temperature it is impossible to have the liquid state existing in presence of the vapour or gaseous state. The full significance of this statement will be more easily grasped if we take a particular case, say that of carbonic acid gas, the substance studied by Andrews in his classical experiments on this subject. The discussion is facilitated by use of the *isotherm*, or graphical representation of the relation between pressure and volume at constant temperature.

The isotherms of the ideal gas, obeying Boyle's Law, are hyperbolas, and are shown in fig. 93*a*—the product of pressure and volume being constant at constant temperature.

The volume of a given mass of gas is measured horizontally, and the pressure vertically; and to every volume there corresponds a definite pressure at a chosen temperature. The point whose ordinates are the volume and pressure so related lies on the isotherm corresponding to the chosen temperature; and every such point specifies fully a particular state of the substance. As we pass along an isotherm, we take the substance through a succession of states all at the same temperature. If, keeping the volume constant, we heat the gas till its temperature rises 1°, we increase the pressure, and the new state will be represented by a point lying above the isotherm first considered. And now, keeping the new temperature constant, we can take the substance through a new succession of isothermal states. Each state will be represented by a point upon the corresponding isothermal line. In

this way we may suppose isotherms drawn for successive temperatures 0°, 1°, 2°, 3°, 4°, &c. The higher the temperature the higher will the corresponding isotherm be. It is obvious that two different isotherms can never meet or cut.

The isotherms for the ideal gas (fig. 93*a*) are very similar to what are really obtained for air, hydrogen, oxygen, nitrogen, marsh gas at ordinary temperatures, and for 'vapours' at high temperatures. But what are the forms of the isotherms of a vapour at low or moderate temperatures? We have seen that

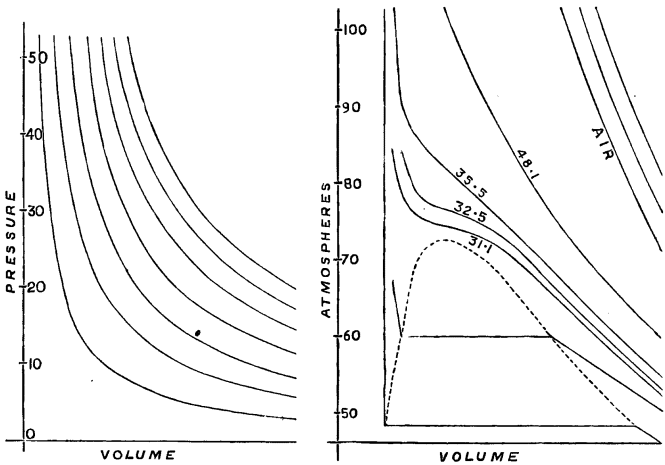


Fig. 93.

when a vapour is saturated and is kept at constant temperature, compression is not accompanied by increase of pressure. Under these conditions there is steady diminution of volume, but no change of pressure at all. The isotherm will be a straight line parallel to the axis along which volume is measured. Thus, as shown in fig. 93*b*, the isotherm for 21·5° is, at the higher volumes, not unlike the isotherm for air. But at a certain point—the point, namely, where the vapour becomes saturated—the isotherm quite changes its character. From

this point until by compression the last portion of vapour has been condensed, the isotherm runs parallel to the volume axis. If we raise the temperature, say to 25°, the corresponding isotherm is similar to the one just described, but the straight line part is not so extensive. As the isotherm is taken higher and higher the straight line part becomes shorter and shorter, indicating narrowing limits within which liquid and vapour can exist together. Finally, when the temperature reaches 31°, the straight line part disappears. The isotherm instead shows a curious dip, which as the temperature is taken still higher becomes less pronounced, until at 50° the isothermal becomes throughout a curve lacking any striking peculiarity.

The dotted curve drawn through the extremities of the straight line portions of the successive isotherms shows at a glance the limits within which the substance can be, simultaneously and in the same vessel, in two different physical states. Every isotherm below that of 30·9° passes through this region. Hence, if we begin with a quantity of vapour at a temperature lower than this, we can by mere compression liquefy it. But if we take the substance at a temperature higher than 30·9°, it is not possible by mere alteration of pressure to get it to change its state. It remains a gas at all pressures and volumes. In order to liquefy carbonic acid gas at, say, 40°, we must cool as well as compress it. According to Amagat's recent experiments (1893), the Critical Temperature of carbonic acid is 31·35° C., the corresponding pressure 72·9 atmospheres, and the density ·464 — results which differ slightly from Andrews' early determinations.

The temperature, which corresponds to the isotherm that just ceases to have a straight line portion, is called the Critical Temperature. Below this temperature the substance is either a *liquid* or a *vapour;* above this temperature it is a true *gas.* In other words, a true gas cannot be liquefied by pressure alone—a vapour can.

The following are values, determined or estimated, of the critical temperature of various other substances :—

Ether	+ 175° C.	Carbon Bisulphide	+ 258° C.
Alcohol	+ 248° "	Oxygen	− 113° "
Water	+ 360° "	Hydrogen	− 174·2° "

Thus it is not possible to liquefy oxygen and hydrogen at ordinary temperatures. They must be cooled to very low temperatures before they can be obtained in the liquid form. The liquefaction of these and the other so-called permanent gases is consequently a difficult operation, which has, however, been successfully carried out by Pictet, Von Wroblewski, Dewar, and others.

EXERCISES.

1. A gas is heated till its pressure and volume are both doubled. What is the final temperature, the original temperature being 10° C. ?

2. The volume of a gas originally at 15° C. is observed to change from 10 cubic inches to 10·2 cubic inches when its temperature is raised to 100° C. Compare the initial and final pressures.

3. Iron is 7·8 times as dense as water at 4° C. Compare the relative densities at 100°. (See tables in §§ 126, 127.)

4. A glass bottle holds at 0° C. 679·22 grms. of mercury, while at 100° it holds only 668·68 grms. Assuming the coefficient of dilatation of mercury, calculate the coefficient of expansion of the glass.

5. Two pendulums are made exactly similar, except that the rod of the one is iron, and of the other brass. They beat seconds at a temperature of 10° C. Compare their times of oscillation at 20° C., the linear expansion of the brass being 1·4 times that of the iron.

6. Graham's Compensation Pendulum consists of a metal rod with a vessel containing mercury attached to the lower end. How does it act ?

7. How much heat is needed to convert 10 lb. of ice at − 10° C. into water at 20° C. ? What is the dynamical equivalent of this amount of heat ?

8. The surface of contact between a skate and the ice on which it rests is $\frac{1}{5}$th of a square inch. The skater weighs 150 lb. Calculate the pressure on the ice, and the consequent amount by which the freezing-point is lowered.

9. If kept very still, water can be cooled below its freezing-point without ice being formed. Let a quantity of water be in this way cooled down to $-10°$ C., and then shaken, so that it begins to freeze. How much of it will become ice?

10. If 10 lb. of boiling water be poured over 3 lb. of ice at 0° C., what will the final condition of the substance be? If only 2 lb. of boiling water be used, what will be the result?

11. An iron vessel 10 cubic inches in volume is filled with mercury at 10° C. It is then heated to 80° C. How much mercury will overflow?

12. How far apart on the centigrade scale will the freezing and boiling points be when the water substance is under a pressure of 50 atmospheres?

SUMMARY.

§ 122. Effects of Heat—Changes of Volume and of State.

§ 123. Charles' Law.

§ 124. The Air Thermometer. Absolute Temperature.

§ 125. The Specific Heats of a Gas. Calculation of Joule's Equivalent.

§ 126. Dilatation of Liquids. Case of Mercury. Maximum Density-point of Water.

§ 127. Dilatation of Solids. Fusion of Platinum into Glass. Compensation Pendulum and Balance-wheel. Bréguet's Thermometer.

§ 128. Liquefaction, Fusion, Melting. Solidification, Freezing. Latent Heat of Liquefaction. Effect of Pressure on Freezing or Melting Points.

§ 129. Evaporation and Boiling. Liquefaction of Gases. Latent Heat of Vaporisation.

§ 130. Cooling by Evaporation.

§ 131. Vapour and Vapour Pressures. Saturated Vapour. Hypsometric Thermometer. Dew-point. Hygrometer. Wet and Dry Bulb.

§ 132. Distinction between Vapour and Gas. Isotherms. Critical Temperature. Condensation of the Typical Gases.

CHAPTER XV.

TRANSFERENCE AND TRANSFORMATION OF HEAT.

§ 133. **Transference of Heat.**—Heat is transferred from one portion of matter to another in three distinct ways, namely, by Radiation, Convection, and Conduction. In many practical cases the process of transference is a co-operation of two of these, and sometimes of all three; but as a rule, in any particular case, some one of the three processes named is the most important.

The result of radiation is familiar to us in sunshine and in the glow of an open fire. Strictly speaking, it is not so much a transference of *heat* as a transference of *energy which becomes heat* on encountering material obstacles. The source of this energy is no doubt a heated body, the heat in which is, however, first transformed into the energy of motion of the medium surrounding the body. Radiant Heat is in fact a particular case of what is now generally called Radiant Energy, and its laws of propagation are identical with those of Light. The subject will be considered as a whole in Part II., Chapters XXI. and XXII.

In convection the heat is carried from one place to another by motion of the heated matter. It is the most important of all the processes for conveying heat through a liquid or a gas.

In conduction, finally, the heat passes from particle to particle of a substance by a process which seems to depend upon the agitation of the molecules. The final result of convection is aided by conduction, and it is by conduction that bodies in contact gradually come to the same temperature. Wherever a difference of temperature exists in matter conduction of necessity takes place.

§ 134. **Conduction of Heat.**—Certain familiar experiences show us that substances differ greatly as regards their conducting powers. We can hold a short piece of glass by one end while the other is being fused in a hot flame. If we treat similarly a short piece of copper, the temperature of the end not in the flame will soon become too high for us to hold it. With an equal-sized piece of iron a distinct rise of temperature will also be apparent, but it will not take place so rapidly as in the case of the copper, and may not make the iron too hot to be held in the fingers. With a piece of lead, again, the end in the flame will melt and drop off before the other end has appreciably risen in temperature. Of the four substances named, copper is the best conductor of heat; iron comes next; and then, in order, come lead and glass.

The difference in the conducting powers of copper and iron may be shown by the following simple experiment: Take two rods, one of each metal, and support them horizontally end to end, as shown in fig. 94. Attach by means of wax a series of

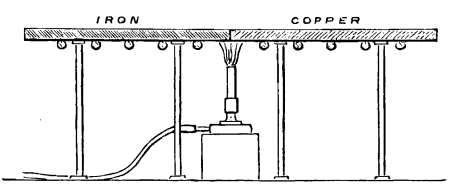


Fig. 94.

balls at equal intervals along them, and apply at the junction of the rods a flame so as to play equally on both. Gradually heat will be conducted along the bars, and when the temperature at any ball rises high enough to melt or loosen the wax the ball will fall. It will be found that the first ball on the copper falls first, and is followed soon after by the fall of the first ball on the iron. After a little time the second ball on the copper falls, and shortly after the corresponding ball on the iron. Then the third ball on the copper drops off, and possibly the fourth ball also before the third ball on the iron

has been loosened, and so on. If the specific heats of copper and iron were the same, this experiment would give us some idea of the relative conducting powers of the two metals; but copper has a slightly smaller specific heat than iron, and therefore heats up more rapidly. Consequently less heat will be required in the copper to loosen the balls than in the iron; and copper will appear to be, as compared with iron, a better conductor than it really is.

It is obviously possible that this kind of experiment might be quite misleading. The substance with the better conducting power might have so much higher a specific heat that the balls fixed to the worse conductor might drop off first.

The true method for measuring the conducting power of a substance was first given by Fourier, whose *Theorie Analytique de la Chaleur* (1822) is one of the classics in physical science. To give definiteness to the problem, imagine the case of an infinite slab of some material one foot thick, and suppose the surfaces to rest in contact with bodies kept steadily at definite temperatures which differ by unity. Then ultimately there will be a steady gradient or slope of temperature from the warmer to the colder side, and every point of the slab will remain at a steady temperature. Hence no heat on the whole is being taken in or given out by any portion of the slab. Whatever amount of heat flows in on the one side flows out on the other. Across every square foot of the slab a certain amount of heat will flow in one second. This amount of heat is called the **Thermal Conductivity** of the substance, and is the accurate measure of its conducting power.

Definition: The Thermal Conductivity of a substance is the number of units of heat which pass in unit time through unit surface of a slab of unit thickness, the difference of the temperatures of the sides of which is unity.

It is not possible to experiment exactly in the way imagined above, since it is not possible to deal with infinite slabs or with bodies at absolutely steady temperatures. Fourier's methods,

however, enable us to calculate the conductivity from observations of the gradient of temperature along a bar, one end of which is kept at as steady a temperature as possible. The form of apparatus used by Forbes is shown in fig. 95. The one end A of the bar AB is immersed in a vessel of molten lead, and the other end (if the bar be not too short) is left exposed

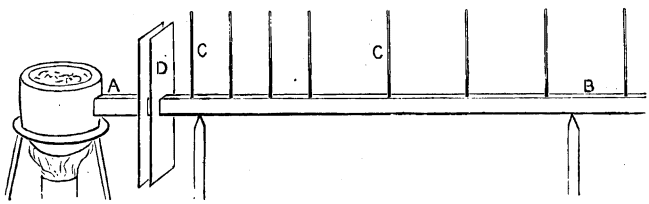


Fig. 95.

in air. Thermometers C dip into holes bored at convenient intervals along the bar, and indicate the mean temperature of the section where they are. A screen D placed near the source of heat shields the greater part of the bar from the direct radiation of the heated vessel and flame. After some six or seven hours the temperature at each point of the bar becomes steady; and because of radiation and convection from the surface of the bar the temperature diminishes gradually as we pass from the heated to the cold end. The rate of change of temperature along the bar—in technical language, the temperature gradient—is greater the nearer the part considered is to the source of heat. The amount of heat which flows in across any section is proportional to the temperature gradient, the cross-section of the bar, and the conductivity. Now, since the temperature of every succeeding part of the bar is steady, this amount of heat must be given off by radiation and convection *wholly* to the air. But by means of another experiment upon a shorter bar of the same material, which is allowed to cool from a high temperature, we can determine the amount of heat that the bar loses to the air at every temperature. Hence, knowing the temperatures of all parts of the long bar, we are able to estimate very accurately the heat that is lost every second by the part of the bar beyond the section under consideration; but this, being the amount of heat entering the section, is equal to

temperature gradient × section of bar × conductivity.

The first two are obtained by direct measurement, and hence the conductivity can be at once calculated.

The determination of the thermal conductivity of liquids is a quest of great difficulty. In the first place, convection currents must be carefully guarded against. This is done by placing the source of heat in contact with the upper surface* and noting the distribution of temperature downwards. But there is a further difficulty arising from the comparatively low conductivity of the liquid. Unless due care be taken heat might be carried down to the lower strata through the walls of the containing vessel, which is probably constructed of material of comparatively high conductivity. The difficulty is obviated by using a wide vessel, so that the containing walls are well removed from the source of heat, and are, besides, screened off from its radiation.

According to the accepted view of the constitution of matter, there is an obvious difference between the process of conduction of heat in solids and in fluids. In fluids, and especially in gases, there is constant interdiffusion of the molecules; hence there is, strictly speaking, a molecular *convection* going on which is one important factor of the process of conduction. According to the kinetic theory of gases, temperature is determined by the average energy of motion of the molecules. Thus conduction depends on the diffusion of energy, just as viscosity depends on the diffusion of momentum.

The conductivities of some of the more important substances are given in the following table. The numbers are expressed in centimetre-gramme-second and centigrade units—the unit of heat being the amount of heat required to raise 1 gramme of water 1° C.:

THERMAL CONDUCTIVITIES.

Silver	1·096	Marble	·0073
Copper	·940	Ice	·0057
Aluminium	·619	Crown Glass	·0016
Gold	·507	Flint Glass	·0014
Brass	·262	Water	·0012
Iron	·151	Mahogany	·0005
German Silver	·085	Alcohol	·00049
Lead	·080	Glycerine	·00063
Manganese Steel	·033	Silk	·00022
Mercury	·020	Air	·000056

* Except in such a case as water between 0° C. and 4° C.—that is, below its maximum density-point.

These values must not be regarded as in any sense absolute. Different specimens of the same nominal material differ greatly in their conductivities; and there is considerable discrepancy among the results obtained by different experimenters for the same substance. The experiments are extremely laborious and difficult; and in many cases the specific heat, on which the final estimated values of the conductivity depends, is not accurately known.

The following table of conductivities, deduced from Mitchell's carefully repeated experiments on Forbes' and Tait's bars, shows how widely different two specimens of copper may be, and how the conductivity increases with temperature:

THERMAL CONDUCTIVITIES.

Temperature.	Iron.	Copper.	Copper (C.).	Lead.	German Silver.	Manganese Steel.
0° C.	·1509	·9396	·6296	·0786	·0847	·0328
100° "	·1771	1·0043	·7031	·0876	·1021	·0463
200° "	·1831	1·0485	·7631	...	·1291	·0624
300° "	·2396	1·0928	·8247	...	·1495	...

The great range in the values of thermal conductivities is very striking, and both good and bad conductors have many important uses. The blade of a knife feels colder than the wooden handle because of the better conducting power of the metal, which causes a more rapid abstraction of heat from the part in contact with the hand, and therefore from the hand. Teapots and kettles should be furnished with handles of bad conducting material, so that there may be no risk of the hand which grasps them being burned. Delicate trees and shrubs are surrounded with straw in winter to protect them from the cold—strictly speaking, to prevent a too rapid cooling; similarly, ice is protected from heat by being packed in straw or sawdust, or other bad conducting substance. Clothes are worn practically for the sake of warmth, and are

made of bad conducting material, so that the heat of the body is kept in. The interior of a metal box may be kept cool in the hottest day by having double walls with air or some convenient material of low conducting power between.

A very striking effect of high conductivity is shown by the experiment, fig. 96. A piece of wire gauze is stretched over a Bunsen burner, which is then lighted. When the gauze is lifted up an inch or two the flame appears to be carried up with it. No combustion takes place between the gauze and the top of the burner. The reason is, that because of the good conductivity of the gauze the heat of the flame is conducted outwards along the metal wires, and the temperature immediately below the gauze is prevented from rising to the combustion point. The principle was made use of by Sir Humphrey Davy in the construction of his safety-lamp for mines. The oil-lamp was simply surrounded by a cylinder of gauze. The presence of any combustible gas in the mine would be made evident by the lamp flaring up inside the gauze. Under conditions in which a naked flame would have produced a violent explosion, Davy's lamp was perfectly safe, and by its behaviour warned the miner of danger.

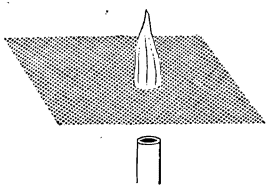


Fig. 96.

§ 135. **Earth Temperatures.**—Another method of experimentally determining conductivities is to subject one of the surfaces of the substance to a periodic variation of temperature and study the manner in which the waves of temperature so produced pass through the material. This method, as applied to metal bars, was first used by Angström; but its most interesting application is to the measurement of the conductivity of the crust of the earth. Here the daily alternation of day-heat and night-cold, and the annual alternation of summer-heat and winter-cold, produce thermal waves which pass down into the crust of the earth. By sinking thermometers to different depths we can observe the variations of temperature at these depths hour by hour and day by day. At a few feet depth the daily variation which is so well marked at the surface can

no longer be distinguished; but the annual variation can be traced to a depth of about 30 feet. The law is, that the depth at which a periodic surface variation of given range is sensible varies inversely as the square root of the period. Now $\sqrt{365}=19$ fully. Hence the annual variation penetrates about 19 times deeper than the daily variation. Another general law is, that the Amplitude, or half the total range of the variation, diminishes in geometrical progression as the depth increases in arithmetical progression. For example, if the annual range of temperature at a depth x is half that at the surface, the range at depth $2x$ will be one-fourth, at depth $3x$ one-eighth, and so on. Finally, the crest of the wave of temperature at any depth will come later and later as the depth is greater.

The most extensive series of earth temperatures that we have is that commenced by Forbes in 1837. Four gigantic thermometers, with their scales above ground, were fixed in the rock of the Calton Hill, Edinburgh, with their bulbs at depths of 3, 6, 12, and 24 French feet respectively. In 1879 a new set were completed to take the place of the old set, which had been broken by a madman in 1876. With the exception of these three years, the observations are continuous from 1837 to the present time. The most important results directly deduced from these observations are:—

(1) The mean temperature increases from about 7·5° C. at the highest thermometer to 8·1° at the lowest.

(2) The mean annual ranges of temperature for the four thermometers in order are 8·2°, 5·6°, 2·7°, and 0·7° C.

(3) The dates of the maximum temperatures—the crests of the heat-waves—are, in order, August 19th, September 8th, October 19th, January 6th.

Thus it takes about half a year for the summer heat to penetrate 24 feet down into the porphyry rock of Calton Hill. The results of (2) and (3) give data from which the conductivity of this rock can be calculated—it comes out ·00415 C.G.S. units. From (1) we learn that the mean tem-

perature of the earth increases as we descend, and that consequently the earth is a cooling solid. Not only so, but with the value of the conductivity just determined we can proceed to calculate the amount of heat flowing outwards through the crust. The value given in (1) for the mean temperature gradient, namely, 0·6° in 21 feet, or nearly 0·03° C. per French foot, is greater than the average for various localities over the earth's surface. This seems to be about 1° C. for every 100 English feet or 30·5 metres; from which we find, taking ·005 as a rough estimate of the average conductivity of the earth's crust, that the yearly supply of heat to a square centimetre of surface from the interior is about 54 units of heat, expressed in gramme-degrees.

§ 136. **Convection of Heat** is well illustrated in the heating and boiling of a liquid such as water. If the water is contained in a glass vessel and heat is applied below, the up-streaming of the heated and expanded water in the centre and the down-flow of the cooler and denser water at the sides can be readily observed. The nature of the effect is indicated in fig. 97. The process of convection depends, in fact, upon the laws of buoyancy. Gravitation and the effect of heat in changing the density of substances act together in producing the various phenomena included under the term.

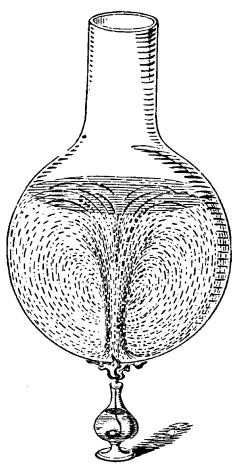


Fig. 97.

Another familiar and important example is the ascent of smoke in a chimney. The fire heats the air, causing it to expand and become lighter, bulk for bulk, than the air in the immediate neighbourhood. Hence it rises, and is replaced by colder air flowing in to continue the combustion. The powerful up-draught of warm air carries with it the

lighter products of combustion in the form of smoke, the grosser particles of which gradually fall down towards the earth again when out of the influence of the up-draught. The lighter particles may remain suspended because of the viscosity of the air until they become nuclei for the deposition of moisture, and are then brought down in rain-drops.

Winds are produced primarily in much the same way. The radiant energy from the sun, falling on the earth's surface, with its motley variety of rock, soil, and water, produces corresponding inequalities of temperature in that surface and in the air in contact with it. The warmer portions of air rise, and the neighbouring cooler air streams in to take its place.

§ 137. **Transformation of Heat into Work.**—To complete the study of heat in its energy relations (§ 121, Chapter XIII.), we must now consider the laws of its transformation into work. The fact of this transformation is familiar to us in the ordinary steam-engine. Heat, derived from the combustion of fuel, acts upon certain substances and mechanical combinations. These are set in motion and mechanical work is done. Now it is readily recognised that when heat is to be in this way utilised as the source of work a difference of temperature is indispensable. One at least of the substances involved in the action of the engine must be at a higher or lower temperature than the others. There must be, in fact, a *boiler* or generator, and a *condenser* or refrigerator.

But such a difference of temperature involves of necessity a transference of heat *as heat*—for example, by conduction. This conducted heat is not transformed into another form of energy, but remains heat. Consequently all the heat that is supplied by the boiler to the working substance is not transformed into mechanical energy. There is no difficulty in effecting a *complete* transformation of other forms of energy into heat. The difficulty is rather in preventing this transformation. But when we try to reverse the process and obtain the dynamical equivalent of so much heat

supplied, it is found to be practically impossible to effect even an approximately complete transformation.

§ 138. **Carnot's Thermo-dynamic Principles.**—We owe to Sadi Carnot (1796–1832) the first clear statement of the foundations of Thermo-dynamics. One of the most important of his conceptions is the *Cycle of Operations.* When a substance, on which work is being done and from which heat is being derived, or to which heat is being supplied and by which work is being done, is brought back to its original condition as regards temperature, pressure, and volume, it is said to undergo a cycle of operations. It is only under these conditions that we can reason as to the equivalence of heat and work. Suppose, for example, that an experimenter were carrying out Joule's Paddle-wheel experiment with water at 0° C. It is possible that, unknown to him, a small particle of ice might be floating about in the water. Theoretically Joule's experiment is a cycle, the first part being mechanical work done and temperature raised, and the second part being the abstraction of heat until the temperature is brought back to its original value. But unless the precaution were taken of bringing the water back to its original *volume*, thereby obtaining again an equal-sized particle of ice, the conclusion as to the equivalence of the work done in the first part, and the heat abstracted in the second part, of the cycle would obviously be wrong.

Another important conception is the principle of *Reversibility.* Carnot imagined an ideal engine known as the Reversible Engine, the essential characteristic of which is that it can work with equal facility either as a machine for producing work from heat or a machine for producing heat from work. When such an engine is used to take heat from the boiler to the condenser, the work done during a complete cycle is the equivalent of the heat that has disappeared. When, on the other hand, this amount of work is done upon the engine, heat is made to pass from the condenser to the boiler, and exactly that amount of

heat comes into existence which disappeared in the direct operation first described.

For simplicity take as working substance a definite quantity of air contained in a chamber, whose volume may be varied indefinitely by the outward or inward motion of a piston. We shall assume that the walls of this chamber can be made either perfectly impervious to heat or perfectly diathermanous—that is, either of zero conductivity or infinite conductivity. When in this latter condition the walls are to be kept in contact with another substance at the same constant temperature. Volume and pressure changes, which take place in this condition in the working substance, take place *isothermally*, there being no change of temperature. On the other hand, when the walls are impervious to heat, the changes which take place in the working substance take place *adiabatically*, there being no loss or gain of heat. When the temperature of air is kept constant we know by Boyle's Law that the pressure varies inversely as the volume. This relation and all similar relations may be represented graphically by means of an *isothermal* line, every point of which (as explained in § 132) denotes a definite state of pressure and volume. In fig. 98 two isothermals, AB, CD, corresponding to temperatures S and T, are shown. T is supposed to be the higher temperature. Volume is measured horizontally from the origin O, and pressure vertically. On the other hand, the lines CA, DB are *adiabatic* lines. They show how volume and pressure vary with one another when no heat is allowed to leave or enter the substance. Just as along each isothermal the temperature is constant, so along each adiabatic the entropy, as it has been named by Clausius, remains constant. To pass from one isotherm to another we must change the temperature. In like manner, to pass from one adiabatic to another we must change the entropy. Adiabatic lines are also called isentropic.

Begin with the working substance in the state A—i.e. with volume O*a*, pressure *a*A, and temperature S. Compress

adiabatically—that is, with walls impervious to heat—till the temperature rises to T and the state C is reached. In this first operation a definite amount of work is done, but no heat is gained or lost. The work done is measured by the area AC*ca*, the product of the change of volume into the average pressure (§ 125). Next, let the substance expand isothermally to any state D, doing work and at the same time taking in heat from the source, which is kept permanently at the temperature T, and which is in connection with the working substance through the diathermanous wall. In this case the work is done by the substance and is measured by the area CD*dc*. In the third operation let the substance expand adiabatically until the temperature falls to S and the state B is reached. In this operation a definite amount of work is done by the substance, and its measure is the area DB*bd*. Finally, compress the substance isothermally until the original state A is reached. Here work is done on the substance to an amount represented by the area BA*ab*, and heat is given out to the refrigerator, which is kept permanently at temperature S. The cycle is now complete, for the substance has been brought back to its original condition of volume, temperature, and pressure. The work done by the working substance or engine is represented by the area ACDB, for this is the algebraic sum of all the expressions formed by multiplying the changes of volume by the average pressure corresponding; and this work done must be equivalent to the heat which has disappeared. If Q units of heat are taken in in the second operation (CD), and q units of heat given out in the fourth (BA), Q must be

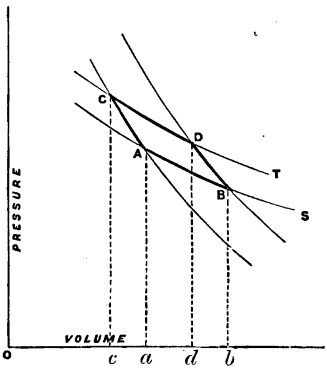


Fig. 98.

greater than q, and the difference $(Q - q)$ will be dynamically equivalent to the work (W) done by the engine. Now if the engine be reversible in Carnot's sense, it will be possible to go round the cycle in the opposite direction, reversing all the physical processes involved, generating $(Q - q)$ units of heat, and taking q from the condenser, by the expenditure of W units of work. Carnot's principle is that this reversibility is the test of a perfect engine. A more efficient engine than the reversible engine cannot exist. To prove this, let N be the reversible engine, and suppose that M is a more efficient engine than N. In other words, M can, with a given supply of heat, do more work than N. The $(Q - q)$ units of heat taken in by M during the cycle will be transformed into $(W + w)$ units of work; and of these, W units will be changed back into $(Q - q)$ units of heat by the reversible engine, N, working backwards. Thus in a complete double cycle w units of work will be done, while the heat originally taken from the source at temperature T has been restored to it. To account for this overplus of work we must suppose an equivalent of heat to be taken from the refrigerator at temperature S. Hence if a more efficient engine than the reversible engine existed it would be possible to do work by means of heat taken from the refrigerator. By taking as refrigerator any limited part of the universe, we should be able to cool that part until all heat was removed from it, and so to obtain from it useful work. Such a result is contradicted by all our experience. Hence we conclude that the reversible engine is the perfect engine. This conclusion, or the experimental truth on which it is based, constitutes the **Second Law of Thermo-dynamics**, the **First Law** being the equivalence of work and heat established by Joule's experiments.

The transformation of heat into work is well shown in the action of freezing mixtures and other artificial means of producing very low temperatures. Thus, when snow and salt are mixed together, the snow melts and the salt dissolves. These changes imply an absorption of heat, and the mixture cools. The rapid

volatilisation of a substance, which has by pressure been condensed from its normal vaporous state into a liquid state, is also a most efficient method of producing intense cold.

§ 139. **Absolute Temperature.**—Lord Kelvin has shown how to deduce from Carnot's Principle a scale of temperature, quite independent of the properties of any particular substance, but agreeing very closely with the temperature scale based on Boyle's and Charles' Laws (§ 124). Returning to the cycle of operations, we see that Q units of heat are taken in at temperature T, and q units of heat given out at temperature S; and by experience we know that Q is greater than q and T higher than S. Kelvin's absolute scale of temperature is obtained by *defining* T and S such that their ratio is the same as that of Q and q. In words, the heat given out is to the heat taken in as the temperature of the condenser is to the temperature of the boiler. In symbols, $q/Q = S/T$.

Now the *efficiency* of an engine is the ratio of the useful work done to the energy supplied. By the First Law of Thermo-dynamics the useful work done is equivalent to the heat which had disappeared, i.e. to $Q - q$; and the energy supplied is the quantity of heat (Q) taken from the boiler. Hence

$$\text{Efficiency} = \frac{Q - q}{Q} = \frac{T - S}{T}.$$

Experiment shows that this scale agrees well with the scale of the air thermometer, the zero being – 273° C., or 273° below the freezing-point of water.

The greatest possible efficiency of a steam-engine working at 53 lb. pressure per sq. in., or $3\frac{1}{2}$ atmospheres, when the temperatures of boiler and condenser are 150° C. and 43° C. respectively, is

$$\text{Efficiency} = \frac{150 - 43}{423} = \frac{107}{423} = \frac{1}{4} \text{ fully.}$$

Practically, because of loss by radiation and conduction, the efficiency is much less than this—perhaps one-tenth.

Thus we see that unless we can have a condenser at the

absolute zero, we can never transform the heat supplied wholly into work. But, since there is no known material impervious to heat, it would be impossible to keep the condenser at this low temperature, even if we had it there to begin with. Dewar, in his experiments on the liquefaction of air, has reached no lower than $-220°$; and this has been done at the expenditure of a great amount of energy. Obviously, then, the effective transformation of heat into work is largely beyond our control. Heat is energy, but it is energy in a comparatively useless form. Its transformability to useful work depends upon inequalities of temperature, and these tend of necessity to diminish through the diffusion of heat. By no known dynamical process can this decay of availability for transformation be prevented. In other words, the physical universe is tending towards a state of absolute inactivity, in which all the energy will have become transformed into uniformly diffused heat. This is Kelvin's doctrine of the **Dissipation** or **Degradation of Energy**. It is conceivable, however, that in its hidden processes in the cell, life may be able to arrest this dissipation. At all events, Maxwell has pointed out that if we were able to deal with the *individual* molecules of a gas, we could without expenditure of energy separate those moving more quickly than the average from those moving more slowly, and so keep up a difference of temperature in contiguous regions. If this could be done, the Second Law of Thermo-dynamics would no longer be fulfilled.

SUMMARY.

§ 133. Transference of Heat.

§ 134. Conduction of Heat. Fourier. Thermal Conductivity. Forbes' Bar. Davy's Safety-lamp.

§ 135. Earth Temperatures.

§ 136. Convection of Heat.

§ 137. Transformation of Heat into Work.

§ 138. Carnot's Thermo-dynamic Principles. Cycle of Operations. Maximum Efficiency of Heat Engine.

§ 139. Absolute Temperature. Degradation of Energy.

WAVE-MOTION.

CHAPTER XVI.

TRANSMISSION OF ENERGY THROUGH MEDIA.

§ 140. **Wave-Motion.**—When the surface of a sheet of water is disturbed, waves are invariably produced. These may vary in magnitude from the huge rollers of the Atlantic to the tiniest observable ripple. The broad characteristic of such wave-motion is, that as the wave passes over the surface at a considerable speed, the liquid itself simply rises and falls with a slight to-and-fro motion in a steady rhythmic manner. The wave, in short, is a particular state of motion handed on from one portion of the water to another. It is energy, not matter, that is transmitted.

There are various kinds of wave-motion besides the familiar type just mentioned; but all consist in the transmission of a disturbance through or over a medium. The conditions necessary for the existence of a wave are (1) the possibility of a disturbance being produced; (2) the tendency of the medium to recover from this disturbance. In other words, the disturbance must bring into play some force not only resisting it but tending to undo it. Thus the effective force in ordinary sea-waves is gravity. Once the wave is formed, the tendency is, in accordance with hydrodynamic principles, for the crest to be pulled down and the trough pushed up.

In the waves of condensation and rarefaction in air which constitute sound, it is the elasticity of the medium which is the effective agent. Other instances will be given later.

As a rule, we deal with trains of waves following one another at regular intervals. Even when a single disturbance has been given, as in the case of a stone falling into a pool of water, the result is not infrequently a succession of waves. Such wave-motion we find to have certain characteristics which are common to all types. These are the *Wave-length*, the *Period*, and the *Amplitude*.

In terms of the familiar characteristics of water waves, we may describe the wave-length as the distance from crest to crest, the period as the time between the passing of two successive crests through any chosen particle of the fluid, and the amplitude as half the height from trough to crest. But these descriptions apply only to one kind of wave, and cannot be taken as definitions generally applicable. To get at their real meaning, we must look more closely into the essential nature of wave-motion, and more particularly into the manner in which each successive particle moves as the wave passes along. In all cases of steady periodic wave-motion, it will be found that every particle moves in exactly the same way, describing an equal and similar path in equal intervals of time. But as we pass from particle to particle in

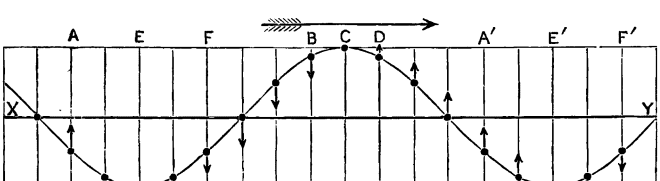


Fig. 99.

the direction in which the wave is moving, the *simultaneous positions* occupied by the particles in their respective paths are different. For example, in fig. 99 there is represented

diagrammatically a series of waves passing in the direction of the arrow through a set of small balls originally distributed uniformly along the straight line XY. For simplicity, suppose each ball to be constrained to move up and down along a straight line perpendicular to XY. This supposition, though limiting the character of the wave, will not seriously affect its value in typifying the general characteristics of the wave-motion. The particle at C, where the crest of the wave is, is in the position of maximum positive displacement. A short time earlier B was in that position, but is now moving downwards towards its original position in XY. A little later in time D, which is now moving upwards, will reach its highest position, and C will be moving down just like B. In fact, each particle attains its greatest displacement a little later than the one preceding it, and a little sooner than the one succeeding it. At any given instant the successive particles forming one wave are in different positions or in different *phases*, to use the accurate word. Not until we have passed along the wave one wave-length do we encounter a second particle in the same phase as the one we begin with. Thus A and A′ are in the same phase, and so are E and E′, F and F′, and so on. A and F, although they are at the same distance below their mean positions, are not in the same phase, because A is moving down and F is moving up. Equality of phase implies equal displacements and equal and similarly-directed speeds. Hence we have the following definitions, which will be found to apply to all dynamical types of wave-motion:

(1) The distance in the direction of wave propagation between two particles in the same phase is the **Wave-length.**

(2) The time taken by each particle to describe completely its path is the **Periodic Time or Period.**

(3) The maximum displacement of each particle from its mean position is the **Amplitude.**

(4) The **speed** of propagation of the wave is the ratio of the wave-length to the period; for, since the period is the

interval between the passages of two successive crests through one point, it is also the time taken by one crest to advance through a wave-length.

A little consideration will show that the above definitions apply equally well to the case of a row of particles, each of which moves to and fro about its mean position along the line in which they lie. This is indicated in fig. 100, where the

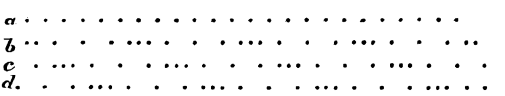


Fig. 100.

first line (a) shows the particles at rest, the others (b, c, d) in different successive configurations. Here there is no *crest* in the popular acceptance of the term; but the densest parts of the configuration may be regarded as strictly analogous to crests, and the least dense parts to troughs. Differences of phase are determined in exactly the same way as before, and the definition of wave-length follows at once.

§ 141. **Simple Harmonic Motion** is the simplest kind of periodic motion which a particle can have. It is defined as the projection on a diameter of uniform circular motion. Thus in fig. 101, if Q moves uniformly in the circle whose centre is O, the point P, which is the orthogonal projection of Q on any diameter, AB, oscillates to and fro about O with a simple harmonic motion.

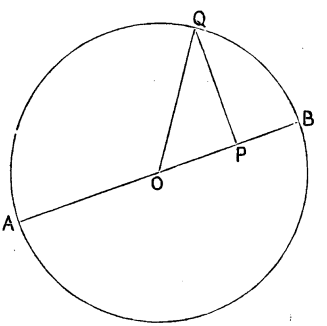


Fig. 101.

If the particles in the last paragraph describe simple harmonic motions, then the wave-form is the simplest that can be obtained. A very important theorem, known as Fourier's Theorem, tells us that the most complex wave-form can

be analysed into a series of simple harmonic wave-forms of different periods, which are as the numbers 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c.

A particular case of simple harmonic motion has already been discussed in § 42, in which the law of simple pendulum motion was derived from that of the conical pendulum. Imagine two spectators to be situated, the one due north and the other due east of a conical pendulum, and to be at the same level as the pendulum. To each the pendulum will appear to describe an approximately simple harmonic motion. But when to the one spectator the bob appears to be at the extremity of its swing, to the other it appears to be at the middle, and *vice versâ*. Thus the simple harmonic motions visible to the two spectators differ a quarter of a period in phase. The same truth may be expressed in this way: Two simple harmonic motions taking place along lines at right angles to one another, and having equal periods and amplitudes, but phases differing by a quarter of a period, compound into uniform circular motion. More generally, two simple harmonic motions of equal periods, but differing in all other respects, compound into elliptic motion about the centre.

§ 142. **Waves and Ripples on Water.**—Two simple cases of water waves are usually distinguished according as the wave-length is great or small compared with the depth of the liquid. In the propagation of *Long Waves* in shallow water, the wave-length being great compared to the depth of the water, each particle of liquid describes an ellipse with major axis horizontal. This ellipse is more elliptical at greater depths, and at the bottom closes into a straight horizontal line. The speed of the wave is equal to the square root of the product of the total depth of the liquid into the acceleration due to gravity—in symbols, $v = \sqrt{gh}$.

In the propagation of the *Oscillatory* or Deep-Sea wave, in which the wave-length is small compared to the depth, each particle of the fluid describes a circle, which rapidly diminishes as the depth increases. Thus at a depth equal to one wave-length, the disturbance of the water is only $\frac{1}{535}$

of that at the surface; and at a depth of two wave-lengths this fraction squared, or $\frac{1}{286000}$. With the largest ocean waves the agitation has an inappreciable effect at even moderate depths. The speed of propagation of the wave is given by the formula:

$$v^2 = \frac{gl}{2\pi}$$

where l is the wave-length, and π the number 3·14159. Thus with a wave-length of 4π (or about 12·6) feet the speed is 8 feet per second, or 5·6 miles per hour. With a wave-length of 100π feet—a length by no means uncommon with Atlantic rollers after a gale—the speed is 40 feet per second, or 27·3 miles per hour. A wave half a mile long has been observed to take 23 seconds to pass. The formula gives 22·7. So far, then, as the value of the velocity is concerned, the simple theory of the oscillatory wave is very satisfactory.

As waves run in upon a shelving beach they gradually change character. The troughs become flatter and the crests become sharper. At length the crest begins to curl over, and finally topples as a breaker upon the beach. If we regard these waves as long waves in shallow water, we see that their velocity should diminish with the depth. Hence there is a tendency for the higher parts of an advancing wave to move faster than the lower parts; the water in any crest tends to outstrip the lower parts of the wave; and the result is the breaker.

With water waves whose wave-lengths are longer than 1 foot, gravity is practically the efficient agent in propagating the wave-motion. There is, however, another agent, which becomes more and more important as the waves get smaller, namely, the pressure due to the surface-tension of the curved surface of the water (see § 112). This pressure acts downwards over the convex crest and upwards over the concave trough. Under the combined effects of gravity and surface-tension, the expression for the square of the speed is

$$v^2 = \frac{gl}{2\pi} + \frac{2\pi T}{\rho}$$

where ρ is the density or mass per unit volume and T the

surface-tension. Putting in the known values for g, π, T, and ρ, in pounds, inches, and seconds, we find for water,

$$v^2 = 61{\cdot}4l + \frac{31}{l}.$$

Thus the speed increases with increasing wave-length because of the action of gravity, but diminishes with increasing wave-length so far as surface-tension is concerned. For longer wave-lengths gravity is more important; for shorter wave-lengths surface-tension is more important. When the wave is sufficiently long, the term depending on surface-tension is negligible. When the wave is sufficiently short the term depending on gravity is negligible. Gravity and surface-tension have equal influence when $61{\cdot}4l = 31/l$—that is, when the wave-length is 0·71 inch. This is the critical wave-length which separates between *waves* and *ripples*. The shorter wave-length gives a ripple which runs mainly by surface-tension; the longer wave-length gives a true wave which runs mainly by gravity. The speed of the ripple-wavelet whose wave-length has the critical value is 9·34 inches per second. This is the minimum speed possible; all other waves and ripples have higher speeds.

For a given speed greater than 9·34, l is determined by means of a quadratic equation which has two real roots. In other words, to every wave there corresponds a ripple having the same speed. The product of the wave-lengths of any pair of corresponding wave and ripple is equal to $31 \div 61{\cdot}4$ or ·505. Thus a wavelet of 2 inches wave-length has the same speed as a ripple of nearly a quarter of an inch wave-length.

A rod moving through smooth water with a speed greater than 9·34 inches per second forms ripples in front and wavelets behind, which accompany it in its advance. The ripples get shorter and the wavelets get longer as the speed of the rod increases. The experiment is a very pretty one, the corresponding wavelets and ripples combining to produce a beautiful pattern on the water surface.

§ 143. **Tides and Tidal Waves.**—Every 12 hrs. 27 min. on the average a wave, known as the *tidal wave*, passes each point of ocean. Roughly speaking, the water reaches its highest level, or high tide, twice a day, and similarly each day witnesses two low tides. If the ocean waters were continuous round the equatorial regions of the earth, the distance from crest to crest of the tidal waves would be about 12,500 miles, and the speed nearly 1500 feet per second. This wave evidently cannot be classified along with the water waves discussed in last paragraph. It was early recognised as being in some way connected with the moon's motion round the earth; and Newton showed that it was a necessary consequence of the law of gravitation.

In the general explanation given by him, Newton supposed the ocean to cover the whole earth, and to assume at each instant a figure of equilibrium under the combined gravitational influence of earth, sun, and moon. The real problem is, however, a kinetic one, and as such was attacked by Laplace. Both the equilibrium and kinetic theories agree in explaining the broad features of the tides. Their results differ in important details, and neither satisfactorily represents the facts. The kinetic problem that calls for solution is to find the *forced* oscillations (see § 152) of the ocean under the periodic tidal action of the sun and moon.

The tidal action depends on the *difference* of the gravitational attractions on different parts of the earth. For example, let r be the distance of the tide-producing body of mass m from the earth's centre, and let a be the earth's radius, small compared to r. Then the difference of the attractions at distances r and $r-a$ (the distance of the nearest point of the earth to the tide-producing body) is proportional to

$$m\left(\frac{1}{(r-a)^2}-\frac{1}{r^2}\right)=m\frac{2ar-a^2}{r^2(r-a)^2}=m\frac{2a}{r^3} \text{ approximately,}$$

where a is neglected in comparison with r. Thus the tidal action of any distant body is directly as its mass, and inversely as the cube of its distance. The masses of the sun and moon are in the ratio of 2,700,000 to 1, and their distances as 390 to 1; hence their tidal actions are in the ratio of 2,700,000 to 390^3, or, roughly, 5 to 11. The moon is, therefore, the more important tide-producing body, and the tidal wave has a period approximating to the value of half a lunar day.

Each luminary produces its own tide, and what we observe at any instant is the resultant of these superposed waves. At the

times of new moon and full moon, the sun and moon are so situated that their tidal waves will come crest to crest and trough to trough. On the other hand, at the times of half-moon, the crest of the lunar tidal wave will coincide with the trough of the solar tidal wave. If the heights of waves produced were proportional to the tidal actions, we should have the total tides at these times proportional respectively to the sum and difference of the tidal actions, namely, as 16 to 6. Here we have at once the explanation of the spring and neap tides. In general the spring and neap tides do not differ by nearly so much as these numbers indicate. Thus at London Bridge the average ranges of spring and neap tides are 20 feet 8 inches and 17 feet 3 inches; and at Leith, 16 feet 4 inches and 12 feet 9 inches. Again, the equilibrium theory would lead us to expect spring-tides exactly at times of new and full moon. As a matter of fact they occur about one and a half days later.

As the moon passes through its first quarter, from conjunction to quadrature, the solar tide, having a shorter period, gradually shoots ahead of the lunar tide. For any position between new and half moon the crest of the resultant tidal wave will, therefore, pass a little sooner than it would if the moon alone were acting. Similarly, for any position between half-moon and full moon—that is, during the second quarter—the crest of the resultant tide will lag behind the position it would occupy were the moon the sole agent. In the former case the tide is said to *prime*, and in the latter to *lag*. The same phenomena occur in the third and fourth quarters. The priming and lagging of the tide are very evident phenomena, causing the intervals between successive high tides to vary considerably, as may be seen in any table of times of high water.

The height of the tide at any place depends on conditions that are still far from being fully understood. Round the Pacific Islands, generally, where the openness of the ocean is little disturbed, the tides rarely exceed 2 or 3 feet. In seas communicating by narrow straits with the open ocean tides are also small, being, for example, inappreciable in the central parts of the Mediterranean, although evident at the head of neighbouring gulfs or long bays. Because of the configuration of the land, the tidal wave may get broken up into two parts, which may meet again and produce very peculiar effects. The four daily tides at Colombo are referred to this cause; and a similar explanation probably holds for the extraordinary case of Papiete, one of the Solomon Islands, where the high tide occurs invariably between noon and 2 P.M. The exceptionally high tides in the Bay of Fundy are due partly to its being

a long, narrow bay, and partly to the fact that two tidal waves meet off its mouth.*

§ 144. **Waves along a Stretched Cord.**—When a stretched cord is drawn aside at one point and then let go, the disturbance will propagate itself along the cord as a wave-motion. If the tension be great enough, the speed of propagation of the wave will depend upon the tension and the mass, but will be practically independent of gravity. Suppose a wave of any form, such as A, to be passing along the stretched cord (fig. 102) with speed v; and let T be the tension of the cord.

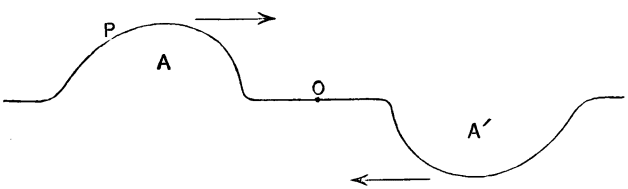


Fig. 102.

Instead of considering the wave as running along the cord, we may think of the cord as passing through the wave-form with an equal speed in the opposite direction. The relative motion of cord and wave will remain unchanged. But in the latter case we see at once that the centrifugal force of any element, P, moving along the curved path of the wave-form must balance the normal pressure due to the tension. Hence, if r is the radius of curvature at the point P, and ρ the mass per unit-length of the cord, we find

$$\frac{\rho v^2}{r} = \frac{\mathrm{T}}{r} \text{ or } v = \sqrt{\frac{\mathrm{T}}{\rho}}.$$

Thus the speed of any wave along a stretched string, cord, or wire is equal to the square-root of the ratio of the tension to the line-density or mass per unit-length.

Now, while the wave-form A is travelling to the right

* The unfortunate custom of applying the term tidal wave to those destructive waves that occasionally visit the shores of countries subject to earthquake disturbances cannot be too strongly condemned. Such waves are in no sense *tidal*.

along a string of indefinite length, let an equal but opposite wave A′ be travelling towards the left to meet it. The speeds are equal, but oppositely directed, and the waves will ultimately meet at the point O originally half-way between them. But this point O will always remain at rest even while the two waves are passing through O. For, whatever upward displacement O might have at any instant because of the A wave, it will have an equal and opposite displacement because of the wave A′. Since, then, the point O never moves, it may be fixed and the cord on one side of it removed. In the half that is left the effect will be the same as if the wave A were reflected at the fixed end O.

Imagine two trains of equal waves, *a* and *b*, to be passing along the infinitely long cord in opposite directions, as indicated in *c* (fig. 103). There will be a succession of equi-

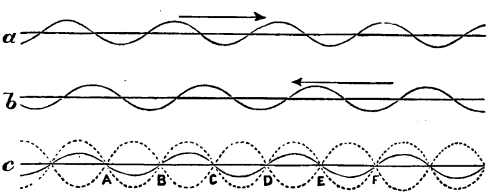


Fig. 103.

distant points always at rest, each of which at any instant lies half-way between a crest of the one set and a trough of the other. These steady points are called the **Nodes,** and the distance between any contiguous pair is evidently half the wave-length of the waves passing along the cord.

Any two of the nodes may be fixed and the motion of the intermediate part of the cord will be unchanged. Thus any vibrating finite string with fixed ends may be regarded as a part of an infinite string along which equal and opposite waves are running. What is really happening in the cord with fixed ends is, that the waves are being continually reflected from these ends.

We may throw the string into vibrations which correspond to the passage of waves of different wave-length so long as the whole length of the string contains an exact number of half wave-lengths. But every such wave runs with the same speed, the tension and density being constant. This speed is:

$$v = \frac{\text{wave-length}}{\text{periodic time}}$$

$$= \text{wave-length} \times \text{number of oscillations per second.}$$

Hence in a given stretched string the wave-length of any wave passing along it is inversely as the **frequency** or number of oscillations per second. Thus, since a finite string can be thrown into vibrations whose wave-lengths, in terms of twice the length of the string, are 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c., the frequencies of these vibrations are as the numbers 1, 2, 3, 4, &c. This result is an important one in the theory of sound, and will be further considered in next chapter.

§ 145. **Elastic Waves** are propagated through media because of the elasticity of these media. At some point there is produced a molecular disturbance, from which, in virtue of elasticity, the substance recovers itself and passes on the disturbance to the contiguous parts. The disturbance may be a change of density, or a change of form, or a combination of these two. During the recovery the particles are set in motion, and continue to oscillate about their mean configuration of rest; under the influence of viscosity the kinetic energy gradually decays.

In the case of liquids and gases the elasticity is an elasticity of bulk, and hence elastic waves must be waves of condensation and rarefaction. The speed of propagation of such waves depends upon the density of the substance and upon the resistance to compression, which is measured by the reciprocal of the compressibility. The greater this resistance to compression is, the more powerful will be the recovery, the more quickly will any molecular group oscillate about its position of equilibrium, and the faster will the

waves run. On the other hand, the greater the density, the greater will be the mass that has to be kept in motion; the slower, consequently, the oscillation, and the less the speed of the wave. In symbols, if E is the resistance to compression or bulk-modulus of elasticity, and ρ the density, measured in any scientific system of absolute units, then the speed is given by the formula:

$$v = \sqrt{\frac{E}{\rho}}.$$

Exactly the same formula holds for all kinds of elastic waves, if E, the particular coefficient of elasticity involved, be given its true value. It applies, of course, to the propagation of compressional waves in solids. But it also applies to the propagation of *distortional* waves in solids—those, namely, which have to do with change of form only. In this case the quantity E is the *rigidity* or resistance to shearing (§ 101). In all known cases the rigidity is less than the resistance to compression. Hence the distortional wave is propagated at a less speed than the compressional wave.

When a sudden pull (as in working a railway signal) is given to the end of a wire, a wave of dilatation runs along it. In this case the quantity E is Young's Modulus, the strain being the somewhat complex strain which accompanies a longitudinal pull and involves changes of both form and bulk.

The following table gives some idea of the speed in feet per second of the compressional, distortional, and longitudinal waves in different substances, the last referring only to wires or rods:

Substance.	Compressional Wave.	Distortional Wave.	Longitudinal Wave.
Air	1,090	...	...
Hydrogen	4,140	...	...
Water	4,700	...	...
Iron	15,000	11,000	17,000
Copper	14,000	7,000	12,000
Oak	...	...	11,000
Glass	11,000	9,000	14,000
Hard Rock	12,000	7,000	...

It should be noted that the elastic moduli used in calculating these speeds are kinetic moduli under adiabatic conditions (see § 138)—that is, when there is no gain or loss of heat accompanying the variations of strain. In the case of gases the adiabatic bulk modulus is very different from the isothermal bulk modulus (see § 155).

The numbers given for 'Hard Rock' are average numbers, calculated from T. Gray's experimental values of the elastic constants of granite, marble, tuff, slate, &c. In the propagation through the earth's substance of elastic waves from earthquake centres we have, however, a means of measuring the speeds directly. Delicate seismometers set up in this and other European countries have recorded minute earth vibrations which have been traced to an earthquake origin in Iceland, Japan, and other disturbed regions. Speeds as high as 28,000 feet per second have been indicated; and it appears that the greater the distance the vibrations have travelled through the earth's substance the greater their average speed. This agrees with what on other grounds is probable, that the elasticity of the materials forming the earth increases with depth below the earth's surface at a greater rate than does the density.

As will be seen in next chapter, the phenomena of Sound depend mainly upon the waves of condensation and rarefaction in air.

When a wave of pure distortion is advancing in a medium the motions of the particles take place in directions at right angles to the line of advance of the wave. On the other hand, in a wave of pure compression the particles move to and fro in this line of advance. It is motion analogous to the former kind that is believed to take place in the ether, or medium through which light, radiant heat, and electric energy are transmitted. The further discussion of the properties of this kind of wave-motion is best deferred until the phenomena which illustrate them have been described.

When, by means of a force-pump, an intermittent stream

of fluid is sent through a tube formed of extensible and elastic material like india-rubber, waves are propagated along the tube, which react upon the flow of fluid and influence greatly the character of the flow through successive portions of the tube. The wave has its origin in the forcible distension of the tube near the force-pump. In virtue of the elasticity of the material, the tube recovers itself and passes on the distension to the neighbouring section. Thus the speed of propagation of the wave depends primarily upon the elasticity of the material, just as small ripples on water depend on the surface tension. But it will also be influenced by the fluid in the tube; for as the wave passes it imposes upon the fluid a simultaneous wave-motion. Consequently along a given tube the speed of the wave will diminish as the density of the fluid is increased. The energy of the original disturbance is gradually lost because of viscosity of both fluid and tube; hence the amplitude will diminish as the wave progresses. The chief effect on the flow of the fluid is to diminish its intermittent character, so that in long tubes the flow becomes continuous. The most important case of the flow of liquid in extensible elastic tubes is the circulation of the blood in the arteries and veins. The elastic waves just described are, consequently, known as **pulse waves.**

§ 146. **The Wave-Front** is a very important conception in the discussion of the progression of waves. A familiar instance is the ridge or crest of a long ocean swell. It runs at right angles to the direction of motion of the wave. The more exact description of the wave-front is as follows: Through any particle set in motion by the wave, imagine drawn a continuous surface containing all particles which are simultaneously in the same phase. This surface or **Wave-Front** is considered as advancing with the wave—that is, having fixed our attention upon a wave-front passing through contiguous particles in one particular phase, we regard that phase as characteristic of the wave-front. Thus the velocity of the wave-front is the velocity of the wave.

In ordinary cases of media having equal elasticities in all directions, the wave-front at any point is perpendicular to the direction of propagation of the wave. When there is radiation from a point, the wave-front is a sphere. When we are dealing with distant sources, any limited part of the wave-front may be considered a plane.

When an ocean swell impinges upon a breakwater it is reflected, and the reflected wave is thrown off in a direction which is different from that of the incident wave, but which makes the same angle with the breakwater. Let *ae*, *a′b′e′*, *a″c″e″*, &c. (fig. 104), represent successive positions of a

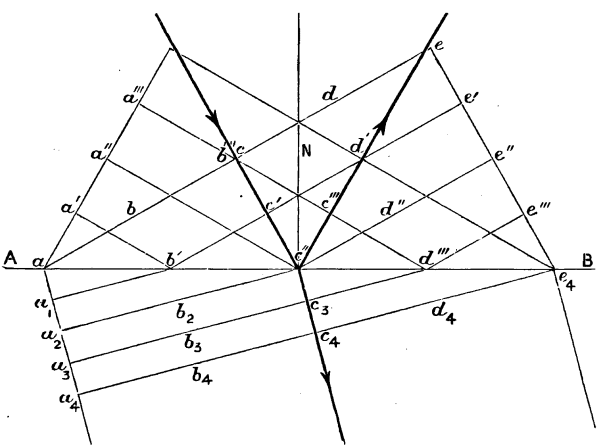


Fig. 104.

portion of a wave-front falling obliquely upon the surface A B. In the position *abcde*, *a* is just on the surface. As *b* advances to *b′*, *a* passes with equal speed backwards to *a′*. When *c* reaches AB in the position *c″*, *a* and *b* are at *a″* and *b″*, and so on. The speed is the same for all parts of the wave-front, and the wave-front is not altered in length. Hence it is a geometrical necessity that the angle *a‴d‴* A which the reflected wave-front makes with

the surface is equal to the angle daB. But the angle which the wave-front makes with the surface is equal to the angle which the direction of propagation makes with c''N, the *normal* to the surface. This direction of propagation (the heavy line in fig. 104) may be called the *ray*, a word specially used in connection with light, but applicable to other kinds of wave-motion. The angle between the incident ray and the normal is called the angle of incidence, and the angle between the reflected ray and the normal, the angle of reflection. Thus the consideration of the propagation of waves leads to the well-known law of reflection, viz. :

The Incident and Reflected Rays are in the same plane with the normal to the reflecting surface, and make equal angles with the normal on opposite sides of it.

When the incident ray falls perpendicularly on the surface, the reflected ray is evidently sent back along the same line.

If the surface at which reflection takes place is not a complete barrier, but separates two media in both of which the wave-motion can take place, then, in general, part of the wave will pass into the second medium. In all probability the speeds of the wave will be different in the two media. Let us suppose the second medium to be that in which the speed is the smaller. Because of the smaller speed in the second medium, the disturbance at a will travel only to a_1, as the disturbance at b moves to b' ; and so on as the wave-front advances. The distance aa_4 will be less than the distance ee_4 in the ratio of the speeds ; and the ray will be bent towards the normal, or *refracted.* Further, if i and r are the angles of incidence and refraction, being the angles which the corresponding *rays* and wave-fronts make with the *normal* and surface respectively, and if v,v' are the corresponding speeds, then,

$$\frac{v}{v'} = \frac{bb'}{aa_1} = \frac{\sin. i}{\sin. r};$$

and this ratio of the sines of the angles of incidence and refraction is called the refractive index of the second medium

relatively to the first. If the second medium is the one in which the wave moves faster, then the ray will be refracted away from the normal. In such a case, if the angle of incidence be too great, there will be no refracted wave at all. Suppose, for example, that v' is $2v$, then sin. $r = 2$ sin. i. But the *sine* of 30° is $\frac{1}{2}$, so that for an incident angle of 30°, the angle of refraction is the angle whose *sine* is unity—that is, 90°. The refracted ray passes off *parallel* to the surface. For any incident angle greater than 30°, there is no value for r, because the sine of an angle is never greater than unity. There is, in fact, no refracted ray. The original ray is totally reflected within the denser medium, and no part of it passes into the less dense medium. The question of reflection and refraction of waves is of supreme importance in the theory of light and (to a less extent) of sound.

The discussion given above applies only to the propagation of waves in *isotropic* media. If the media have different elasticities in different directions, the investigation becomes much more complex.

EXERCISES.

1. In uniform circular motion, the acceleration is of constant amount and acts towards the centre; prove that in its orthogonal projection, Simple Harmonic Motion, the acceleration is towards the centre, but varies in magnitude as the distance from this centre.

2. Show that, for small arcs of oscillation, the tangential acceleration acting on the bob of a simple pendulum is proportional to the displacement from the position of equilibrium, and that, in accordance with the result of Question 1, pendulum motion is, to a close approximation, simple harmonic motion.

3. A vessel, 500 feet long, is steaming through the sea at a rate of twenty knots. The crest of a wave, which is moving in the same direction as the vessel, is observed to take one minute to pass from stern to bow; calculate the speed of the wave and its wave-length.

4. What is the wave-length of the ripple which will advance with the same speed as the wave of last question?

5. The length of a wire is 6 feet and its mass is 1 oz. What tension must be applied so that the wire may vibrate 500 times a second?

6. A compressional wave passes from water into iron. Show how to find the angle of refraction when the angle of incidence is nearly 90°.

7. Why do spring-tides occur twice a month, and why, at any given place, do they occur at about the same hour of day?

8. Change the numerical factors in the formula at the top of p. 258 so that the speed is given in miles per hour instead of inches per second.

9. The surface tensions of mercury and water are as 6·7 to 1, and the densities are as 13·6 to 1. Find the expression for the speeds of a wave and ripple on the mercury surface.

10. At what rate will an oscillatory wave 10 feet long make up on an oscillatory wave 8 feet long?

SUMMARY.

§ 140. Wave-motion. Transmission of Disturbance. Elasticity. Wave-length, Period, Amplitude. Speed of Propagation.

§ 141. Simple Harmonic Motion. Composition of Harmonic Motions.

§ 142. Waves and Ripples on Water.

§ 143. Tides.

§ 144. Waves along Stretched Cord. Their Speed. Opposite Trains of Waves. Nodes. Frequency.

§ 145. Elastic Waves. Their Speed. Compressional, Distortional, and Longitudinal Waves. Pulse Waves in Elastic Tubes.

§ 146. The Wave-front. Laws of Reflection and Refraction.

CHAPTER XVII.

SOUND.

§ 147. **Sound,** in ordinary language, is that which appeals to us through our organs of hearing. Almost every sound can be traced to a source outside of us, and as a rule the sound is characteristic of the source which is its origin. Generally speaking, the air is the vehicle of the energy which produces in us the sensation of sound. This energy is transmitted as a wave-motion, which, as was seen in last chapter, must be of the condensational-rarefactional type. It is a simple matter tracing the transformations of energy from, say, the vibrations of a tuning-fork to the subjective sensation we call its sound. We see with our eye that the prongs of the tuning-fork are in rapid vibration, or we may feel their motion against the fine hairs on the back of our hand. As the prongs vibrate to and fro they push the air first on one side and then on the other; and just as the hand moved slightly to and fro in water starts a series of waves travelling outwards along the surface, so the tuning-fork starts in the air a series of waves of condensation and rarefaction which travel outwards through the air. These waves impinge on the tympanum of the ear, and set it in motion; and this motion is transmitted, and suitably modified in transmission, to the connected parts of the organ of hearing, until the final nerve-centre is stimulated. In physics, we deal only with the transformations which occur outside the ear—that is, with the vibrations of the 'sounding' body, and the corresponding movements in the surrounding medium. The succeeding transformations are studied under Physiology. To avoid circumlocutions, it is customary to

apply the term Sound to those external transformations which correspond to the subjective sensation; and in this sense the word is used here.

Sounds differ greatly in character, and it is our present purpose to find what physically corresponds to these differences. Thus many familiar sounds are pleasant to the ear, and have a certain sustained and definite character. These may be distinguished as **musical sounds.** Others, again, have no such sustained character, but are abrupt, broken, and often disagreeable. These may be distinguished as **noises.** As already stated, a sound is generally characteristic of its source. Different voices are easily recognisable, and there is no difficulty in distinguishing a trumpet-blast from a violin-note. Thus we recognise that sounds differ in *quality* or in *timbre.* Then, obviously, they may differ in *intensity*, from the faintest audible whisper to a thunder-clap. But one of the most remarkable characteristics is that known as the *pitch* of a sound. On it the whole theory and practice of music are based. Even the least musical ear can distinguish between a deep-toned note and a shrill one; between, for example, the extreme notes on a piano or organ. In terms of these three fundamental characteristics, all differences of musical sound can be expressed. They correspond to certain characteristics of wave-motion. Thus *pitch* depends upon the *periodic time*, *intensity* upon the *amplitude*, and *timbre* or *quality* upon the *form* of the wave. The question of intensity need hardly concern us further. Strictly speaking, it depends upon the *energy* of the wave-motion, and this is proportional to the *square* of the amplitude.

§ 148. **Pitch.**—Let the edge of a card be held firmly against the edge of a revolving toothed wheel. At first, when the wheel is revolving slowly, a succession of taps are heard as the card gets loose from each tooth and impinges sharply on the next. As the wheel rotates more rapidly these taps come more and more quickly until they are no longer

distinguishable, but coalesce to form a note of not very pure quality, but still a note of a recognisable pitch. The faster the wheel revolves the higher is the pitch of the note. Thus we easily convince ourselves that pitch depends upon the number of impulses communicated to the air in a given time.

The same truth is well demonstrated by means of the **Siren,** an instrument which produces musical sounds by introducing a regularly recurring discontinuity into an otherwise steady blast of air. Seebeck's siren consists of a large circular disc pierced with small holes at equal intervals apart in the same concentric circle. While the disc is made to rotate more or less rapidly upon its axis, a nozzle, through which a strong blast of air is driven by means of a bellows, is brought so as to bear directly upon any desired circle of holes. When a hole is opposite the nozzle a puff of air escapes; when an unpierced portion of the disc comes in front, the air is checked. Thus the blast of air is broken up into a succession of puffs, whose number per second is exactly equal to the number

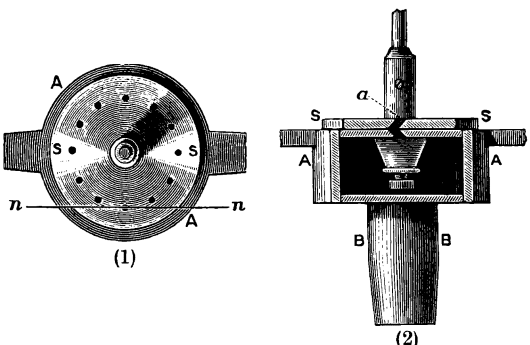


Fig. 105.

of holes which pass in front of the nozzle in one second. This number per second, or *frequency*, is readily calculated when we know the rate of rotation of the disc and the number of holes in the circle that is being used.

In Cagniard de Latour's siren the blast of air whose interruption gives the note also drives the siren. Fig. 105 (1) shows the upper surface of the siren, SS, resting on the cover of the wind-chest, AA. In fig. 105 (2), the instrument is shown in vertical section through

the line *nn* of (2). The blast enters by the pipe BB. The cover of the wind-chest is pierced with exactly the same number of holes as the disc, the only difference being that the holes pierced obliquely in both disc and wind-chest cover are oppositely directed in their obliqueness, as shown at *a* (2). When the corresponding holes are in apposition the blast of air is driven through all the holes, and by its impact on the obliquely cut walls of the holes in the disc forces the disc round. This brings the unpierced part of the disc opposite to the holes in the cover, and the air is cut off until the holes again come into apposition. The stronger the blast the more quickly will the disc be driven, and the higher the pitch of the note produced. This experiment is more instructive than the experiment with the card and tooth-wheel, inasmuch as each individual puff is barely audible, and yet the note produced by the rapid succession of puffs is of considerable power. The best forms of siren are provided with a gearing of toothed wheels and connected pointers which move in front of suitable dials, and measure the rate of rotation of the siren. By adjusting the blast we can get the siren note to be of the same pitch as any given note, and in this way measure the frequency of the latter.

When men and women sing in what musicians call unison, the note sung by the man is not of the same pitch as the note sung by the woman. It is an *octave* lower. Now, when we measure the *frequencies* of these notes, we find their ratio to be as 1 to 2. To go up an octave in pitch means to double the frequency. In the same way we can compare the frequencies of the notes of the ordinary diatonic scale. These we shall represent by the familiar *sol-fa* syllables, *doh, ray, me, fah, soh, la, te, doh,* which have the great merit of not being associated with any set of *fixed* tones. The successive notes of the true diatonic scale are as follows—*doh* being taken to represent the fundamental or key note of frequency n:

note	d	r	m	f	s	l	t	d^1	r^1
frequency	n	$\frac{9}{8}n$	$\frac{5}{4}n$	$\frac{4}{3}n$	$\frac{3}{2}n$	$\frac{5}{3}n$	$\frac{15}{8}n$	$2n$	$\frac{9}{4}n$

and so on to higher octaves. It is a remarkable fact that the most harmonious chords are formed by combinations of notes whose frequencies have simple numerical ratios. Thus

the common chord or major triad is composed of the notes *doh*, *me*, *soh*, whose frequencies are as the numbers 4, 5, 6.

Although every pitch corresponds to a definite frequency, every frequency does not produce an audible note. The ear is sensitive to impulses only when these come faster than 20 or 30 times a second, and more slowly than about 70,000 times a second. The limits vary a good deal with different ears. Some are sensitive to high shrill sounds when others hear nothing.

§ 149. **The Monochord** is a stretched string whose vibrations are rapid enough to produce an audible tone. The violin, harp, pianoforte, guitar, and all forms of stringed instruments consist of monochords, each of which is tuned to a definite pitch. The pitch is determined by the frequency n, which is the reciprocal of the periodic time of the longest wave which can pass to and fro along the string. But (§ 144) this longest wave is twice the length, l, of the string. Hence we have

$$2ln = v = \sqrt{\frac{T}{\rho}};$$

where v is the speed of the wave, T the tension, and ρ the mass per unit length of the string, all measured in terms of the same units.

Clearly, then, by tightening a string we raise the pitch of its note; by slackening it we lower the pitch. This, in fact, is the manner in which all stringed instruments are tuned. In order to raise the pitch an octave—that is, to double the frequency—it is necessary to increase the tension fourfold. The same result may be produced by diminishing ρ in the ratio of 4 to 1. This is done by taking a string of half the diameter. By using thin strings for high notes, and thick strings for low notes, the musician is able to tune his instrument over a considerable range without employing too great a range of tensions.

But the monochord can vibrate with other frequencies than that corresponding to the wave-length, $2l$. For, in accordance with the principles established in § 144, waves of frequencies $2n$, $3n$, $4n$, &c., can be made to pass to and fro along the

stretched string. By gently touching the middle point of the string, and bowing either half, we throw the two halves into vibration, and get from the string a note exactly an octave above the fundamental note which is given by the string vibrating as a whole. Similarly, by gently touching the string at one-third of its length from either end, we can, by suitable bowing, make the string vibrate in three segments, and give forth a note whose frequency is three times that of the fundamental note. And so on, in the other cases.

Another interesting experiment is the following: Attach to one prong of a tuning-fork a light string which passes over a pulley and is kept stretched by an appended weight. If the free length of the

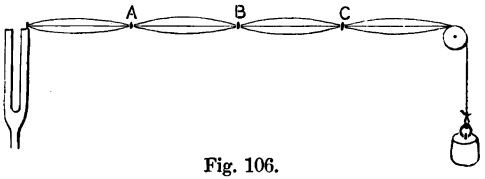


Fig. 106.

string be properly adjusted, the string will be thrown into vibrating segments, 2, 3, 4, or more, as the case may be. Suppose the length and tension to be so adjusted that there are three *nodes*, A, B, C, and four vibrating segments or *loops*. Then, if we shorten the string to the nearest node, C, it will continue to vibrate in three segments; but for any intermediate length, the tuning-fork will not be able to produce a steady vibrating state in the string. Again, by increasing the tension in a certain ratio we can get the given length of string to vibrate in three instead of four segments. It is easy to calculate the exact ratio, according to the formula connecting wave-length, frequency, and tension. The frequency is constant because it is the same tuning-fork which is the source of the vibration; hence at once

$$\text{tension} \propto (\text{wave-length})^2.$$

But the wave-length in the three-looped condition is four-thirds of the wave-length in the four-looped condition; hence the corresponding tensions must be as 16 to 9.

The tension and density being the same, it is evident that the frequency of the fundamental note of a vibrating string is inversely as the length of the string. It is on this principle that the majority of the notes on the violin, guitar, and other similar instruments are obtained. The string is 'stopped' by being pressed down upon the handle or frets, and the note given forth by this shortened string is correspondingly raised in pitch.

§ 150. **Harmonics and Overtones.**—Suppose the fundamental note of a stretched string to have a frequency of 100 —a note in the neighbourhood of G♯ below the bass C on piano or organ. Then, as we have just seen, we may get from this string notes of frequencies, 200, 300, 400, 500, 600, &c., so long as the segment set in vibration is not too short. These possible higher notes, or *overtones*, are said to be harmonically related to the fundamental tone. Each is considered to be a pure tone when every moving part of the string is performing simple harmonic motion. They are then, including the fundamental, called the *Harmonics* of the string.

Not only is each harmonic a possible tone for the string to sound, but several of the harmonics may be given forth at one and the same time. The note given out is then no longer a pure tone, but is built up synthetically of a number of pure tones harmonically related. The wave-form running along the string has no longer the simple symmetrical sweep of the curves shown in fig. 103, but may have a very complex form. In fig. 107 we see how the form of the wave is changed by superposing upon a given vibration the first and second overtones, having frequencies twice and three times the frequency of the fundamental note. What is true in these simple

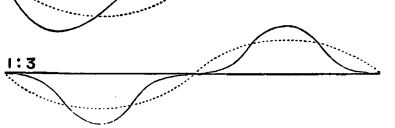


Fig. 107.

cases holds also for the more complex. By plucking or bowing the monochord near one end we throw it into a vibration of a complex form, which can be represented as the resultant of a number of simple harmonic vibrations of frequencies, n, $2n$, $3n$, $4n$. . . . One or other of the harmonics may, of course, be absent, but there always will be some of the series present, and these will give to the note its peculiar quality or character.

§ 151. **Quality, Character, or Timbre.**—Of all instruments the tuning-fork gives forth the simplest sound. The ear recognises at once that its note lacks character. It is pure but colourless, and is *musically* of little account. It is of value because it gives a definite pitch which is practically constant. A tuning-fork, once tuned to a particular pitch, remains at that pitch to the end. When a tuning-fork is sounded, other notes, much higher than the fundamental note, are often heard. But these rapidly die away, while the true fundamental note continues to be heard for a much longer time. By sounding together a set of tuning-forks, whose frequencies are as the natural numbers 1, 2, 3, 4, &c., we get a note of much richer quality than is given by one tuning-fork, thereby proving synthetically that quality depends upon the harmonics present. By suitable combinations of tuning-forks, Helmholtz obtained imitations of vowel-sounds, thus showing that these are due to the relative intensities of the component harmonics.

A close investigation into the character of various musical sounds shows that the all-important thing in determining the quality is the number and relative magnitudes of the simple harmonic tones that build up the given sound. The quality of note of a string in piano, violin, or harp is distinctly changed if we strike, bow, or pluck it at a different point. The characteristics of tone in different pianos depend mainly upon where and how the string is struck. The accomplished violinist instinctively bows the string at such a place and in such a way as to give a rich and full tone.

The same principles find illustration in the action of organ-pipes, the peculiar quality of whose notes is unmistakable. Here an air-column is set in vibration, the air-column, namely, which is bounded by the pipe. To start the vibration, a discontinuity in the blast of air must be produced. This may be done in several ways, as, for example, by making the air impinge upon a suitable obstacle or by having a vibrating tongue intercepting the air. The familiar trick of producing a note from a key by blowing sharply across the end of the hole, or the action of the ordinary tin whistle, sufficiently illustrates this part of the process. Now, just as on the stretched string the vibration consists of a wave passing to and fro along it, and being reflected at the fixed ends, so in the case of a column of air surrounded by a tube or pipe, the vibration consists of waves of condensation and rarefaction passing to and fro along the tubes, and being reflected at the ends. And again, just as the string can be thrown into segmental vibrations with nodes and loops occurring alternately, so a similar phenomenon occurs in the case of the air-column in the organ-pipe. The difference between a stretched string and a pipe open at both ends is, that the ends of the string are nodes, whereas the open ends of the pipe are loops. Except where the nodes are, every particle of air in the tube is vibrating to and fro. In the neighbourhood of a node, and on both sides of it, the layers of air are now moving towards it, now receding from it. The nodes, in fact, are the places where condensation and rarefaction steadily alternate; whereas at the intermediate loops there is no change of pressure, but simply a to-and-fro motion of the layer of air there. If, at any instant, one of two contiguous nodes is in a state of condensation, the other is in a state of rarefaction.

In the open organ-pipe both ends are loops, between which one node at least must exist when a sound is produced. This gives the fundamental vibration, and may be diagrammatically indicated as in fig. 108 *a*. The wave-length is approximately double the length of the tube. The second harmonic is pro-

duced when two nodes intervene, as in *b*; the third when three nodes intervene, as in *c*; and so on. The wave-lengths of these are, respectively, the length of the tube and two-thirds the length of the tube. If the fundamental note has frequency 100, the second has frequency 200, the third 300, and so on. In the closed or stopped organ-pipe, again, one end is a node and the other a loop. In fig. 109 *a*, we have a diagrammatic representation of the fundamental vibration whose wave-length is (approximately) four times the length of the tube. Thus by simply stopping the end of an open organ-pipe we lower the prime tone a whole octave. The next possible mode of vibration is indicated in *b*. Here the wave-length is $1\frac{1}{3}$ times the length of the tube. Again, when there are three nodes (*c*) the wave-length is $\frac{4}{5}$ of the length of the tube, and so on to higher harmonics. If the fundamental tone of a closed organ-pipe has frequency 100, the next possible harmonic will have frequency 300, the next 500, and so on. Thus into any note sounded by an open organ-pipe all the harmonics may enter, but in a closed organ-pipe only those of odd number can be present. This lack of the even harmonics gives a curious nasal quality to the tone of the closed organ-pipe. By overblowing we may so accentuate the second harmonic in the open pipe as to make it sound a note appreciably an octave higher than the fundamental note. By overblowing the closed pipe we raise the pitch of the note an octave and a half. With flutes and whistles similar effects may be produced.

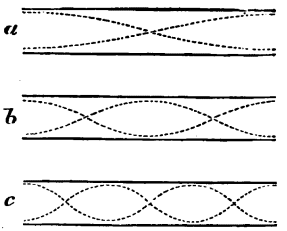


Fig. 108.

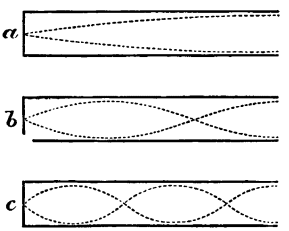


Fig. 109.

Generally speaking, the greater the number and the higher the pitch of the harmonics present, the more brilliant the quality of the note. If, however, the very high harmonics are too much in evidence, the note may become too piercing in quality to be agreeable. By combining the same harmonics in different *phases* we obtain different wave-forms, and in extreme cases this kind of change in the wave-form affects the quality. The quality is also affected, especially in trumpets and similar instruments, by the presence of *anharmonic overtones*—that is, overtones whose frequencies are not harmonically related to the fundamental tone. Except, indeed, in the simpler cases of string and organ-pipe vibrations, the various tones which can be produced from a vibrating body are not harmonically related.

§ 152. **Vibrations of Rods, Plates, Membranes, &c.**—A tuning-fork consists of two rods vibrating side by side. In producing the fundamental note each prong vibrates so that there is one node at the base, as indicated in fig. 110 *a*. The next possible mode of vibration is when a second node exists, as shown in *b*. The frequency of this second vibration has no simple numerical relation to that of the first. For example, if the tuning-fork be tuned to C of the bass clef, this first overtone is two octaves and 7·7736... mean semitones higher than the fundamental note —i.e. a little flatter than G♯ above the treble C. Theoretically, there are higher notes with 3, 4, 5, &c., nodes, but practically they are difficult, if not impossible, to obtain.

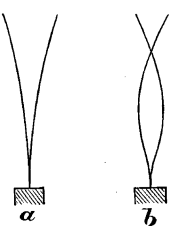


Fig. 110.

If a rod fixed at the one end be rubbed along its length with the exertion of considerable pressure between the rod and rubber, waves of condensation and rarefaction are set up in the rod. These travel to and fro, just as the aerial waves do in the closed organ-pipe. For a rod of ordinary length the note obtained is very high in pitch. Its frequency will be

the ratio of the velocity of the condensational wave along the rod to four times the length of the rod. Thus, in an oak rod, 3 feet long, the frequency of the note is (see § 145)

$$\frac{11000}{3 \times 4} = 917.$$

For the same material the frequency is inversely as the length of the rod.

The vibrations of plates fixed at the centre may be investigated experimentally in the way devised by Chladni, one of the earliest scientific investigators of the phenomena of sound. Suppose the plate to be square, and let sand be sprinkled over the surface. Place the finger on any corner, and draw a violin-bow across the middle point of one of the sides. The plate will be set vibrating in segments with the diagonals as nodal lines. Along these nodal lines the sand will accumulate, being tossed off the vibrating parts of the plate. The appearance is shown in fig. 111 (*a*). By placing the finger at

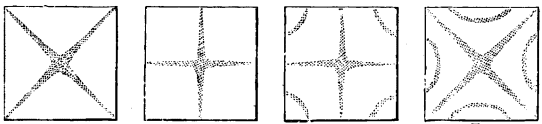


Fig. 111.

the middle point of one of the sides we make the symmetrical diameters become the nodal lines (*b*). The plate vibrates in four square segments, but the pitch of the note is lower than in the first case. Other modes of vibration can easily be obtained by suitable bowing or by fixing other nodal points. Figs. (*c*) and (*d*) represent the arrangements next in order of simplicity to (*b*) and (*a*).

Bells behave in a similar manner, and it is impossible to get from them overtones harmonically related to the fundamental tone and to one another. The characteristic clang of the bell is no doubt due to the presence of these anharmonic overtones; and the art in bell-making is to prevent

them having a pronounced discordant effect on the ear. The manner in which the bell is set in vibration is also an important factor in determining the quality of the tone; whether, for example, it be struck with metal or wood, on the inside or on the outside.

Stretched membranes present another important case, differing from plates and bells in several particulars. For example, the membrane has its margins fixed and its centre free. Then, again, the tension is an important factor, and can be altered through a considerable range, whereas in plates and bells it is the elasticity of the material that is the effective agent in sustaining vibrations. The vibrations of the circular membrane have been very carefully studied experimentally by Savart, Bourget, and Bernard. The vibrations were excited in the membrane by sounding near it an organ-pipe of suitable pitch. When the pitch of the organ-pipe approximated to the pitch of one of the possible vibrations of the membrane, the membrane began to vibrate also, in accordance with the principle of resonance discussed in next paragraph. The vibrations were detected by means of the motion in fine sand sprinkled over the membrane. Nodal lines were found to form either concentric circles or diameters arranged symmetrically, or both in combination. In fig. 112 a few examples of possible forms of vibration

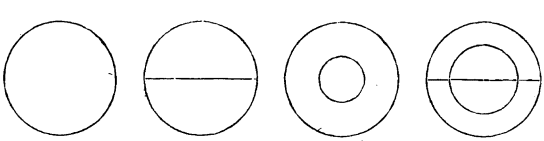


Fig. 112.

are shown, the numbers attached being proportional to the frequencies as calculated for a perfectly flexible membrane freely vibrating in vacuo. The lines represent nodes.

§ 153. **Resonance: Free and Forced Vibrations.**—When a vibration is sustained by the periodic action of an external

agent, the vibration is said to be *forced*. When the vibrating body is left to itself, its vibrations are said to be *free*. The tidal action of the moon imposes a forced oscillation of $12\frac{1}{2}$ hours' period upon the ocean waters which cover the greater part of the surface of our earth, producing at every point the familiar ebb and flow of the tide (§ 143). Similarly, the daily variation of barometric pressure is the direct result of the periodic action of the sun's heat upon our atmosphere—it is a forced vibration. Other examples are given by reed-pipes and sounding-boards and all cases of *resonating* bodies.

The volume and quality of tone given by stringed instruments depend to a very large extent upon the sounding-board to which the strings are attached. If the ends of a vibrating string were attached to two independent blocks of stone, a note of very feeble intensity would be produced. The same string stretched on a guitar or violin will be heard with ease through a large hall. The reason is, that the string, because of its comparatively small surface, is not able to transmit its energy to the air at a sufficiently great rate. But when its ends are fixed to supports on the instrument, the vibrations are transmitted through these supports to the instrument as a whole, which by virtue of its greater surface gives off energy to the air much more rapidly. A small tuning-fork held in the hand at arm's-length is barely audible; but when its end is set hard against a table or wooden panel the note is heard over the whole room.

When the resonating body has a period of free vibration equal to the period of the forced vibration, the resonance becomes very powerful. Just as a pendulum or swing may be made to describe larger and larger arcs by properly timed impulses, so a *resonator* responds to the timed impulses of the note to which it is tuned. A tuning-fork held in front of an organ-pipe, one of whose harmonics has the same pitch, has its tone powerfully reinforced. It is usual to mount large tuning-forks on rectangular boxes with one end

open. The box is made of such dimensions that the air within it has a free vibration whose frequency is equal to the frequency of the tuning-fork. The result is, that the air in the cavity takes up the vibration in great strength and transmits it to the air outside, after the manner of the air-column in an organ-pipe. A hollow open cylinder with a piston that can be set in any position forms an adjustable resonator which may be tuned to any note within a particular range of pitch. The part between the piston and the open end acts the part of a closed organ-pipe. The lowest note it will resound to is one whose wave-length in air is four times the length of the tube. Thus, to produce resonance with a note of frequency 500, the tube must be adjusted to a length of $\frac{1}{4} \times \frac{1100}{500}$ feet $= 6{\cdot}6$ inches, 1100 feet per second being taken as the speed of sound in air.

Von Helmholtz's spherical resonators, tuned to the successive harmonics of a particular note, are hollow metal spheres of proper size, each of which has two apertures diametrically opposite one another (fig. 113). The smaller aperture is in the form of a small projecting tube which can be fitted close into the ear. Through the other and larger aperture the outside disturbance acts, and sets the mass of air inside the sphere into vibration. As an example of their use, suppose the largest resonator of the set to be in tune with the note sounded by an open or closed organ-pipe. By placing in turn each resonator to the ear we readily convince ourselves that the successive harmonics are all present in the sound of the open organ-pipe, but that with the closed pipe the even harmonics are absent. Each resonator resounds to the particular harmonic with which it is in tune; if that harmonic is not present there is no resonance.

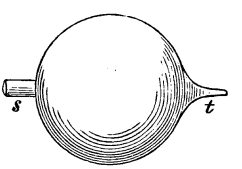


Fig. 113.

The following is a very simple experiment of the same character. Hold down any key on the piano so as to leave

the corresponding strings free, and then strike the key an octave lower or an octave and a fifth or two octaves lower. Release this key of lower pitch so that its strings become damped and its note ceases to sound. At once the note of higher pitch associated with the key that is being held down will be heard distinctly as if it had itself been struck. Its intensity may be reinforced again and again by repeated striking of a lower note of which it is an harmonic. The strings of the note whose key is being held down act as resonators to the corresponding harmonic of the note that is struck.

§ 154. **Interference : Beats.**—When two sets of waves or ripples meet, they produce certain effects which are included under the general term *Interference.* The term is derived primarily from the mutually destructive effect of a crest of the one set meeting an equal but opposite trough of the other. But of equal importance is the mutual reinforcement when crest meets crest or trough meets trough. In the case of sound-waves, it is clear that if two trains of equal waves, one of which is half a wave-length behind the other, fall upon the ear at the same instant, the condensation due to the one will neutralise the rarefaction due to the other, and silence will be the result. If they differ in phase by any other fraction of a wave-length, the resultant intensity of sound will depend upon the difference of phase.

Hold a small vibrating tuning-fork in the fingers near the ear and rotate it gently to and fro. The intensity of sound will change with the position of the plane containing the forks. Each prong is a source of waves of condensation and rarefaction, and the resultant effect of the two sets of waves at any point depends upon the angle which the direction of that point from the tuning-forks makes with the plane of the prongs.

But the most familiar example of interference in sound is the phenomenon known as **Beats.** These occur when two notes differing very slightly in pitch are sounded together. The difference may be so slight that the acutest musical ear

could not distinguish any difference when the notes are sounded successively. But when they are sounded together the least musical ear can at once distinguish a curious rise and fall in the *intensity* of the combined sound, much as if plugs were being inserted into the ears and removed again at periodic intervals. Suppose, for example, that two tuning-forks of frequency 200 and 202 were being sounded together. The wave-lengths differ only by 1 per cent., so that at the beginning of a chosen second, when the waves are, say, in the same phase, there will be for a finite though short time a reinforcement of intensity. But gradually, as the first quarter-second passes, the one set of waves lags behind the other set, and the 50th crest of the one will fall half-way between the 50th and 51st of the other. In other words, crest will fall with trough, and if the amplitudes are equal, silence will be the result. If the amplitudes are not equal, the resultant amplitude at this quarter-second will be the *difference* of the amplitudes, whereas at the beginning the resultant was the *sum* of the amplitudes. In all cases, then, there must be a steady diminution in intensity during the first quarter-second. After this the resultant intensity will begin to increase again, and after half-a-second, when the one set of waves will have lagged one whole wave-length behind the other, the original intensity will be recovered, the 100th crest of the one coinciding with the 101st crest of the other. In the next half-second an exactly similar fall and rise in resultant intensity will take place. Thus the intensity of the combined tone will go through a fall and rise twice a second—there will be two *beats* per second. There is generally no difficulty in getting beats from a pianoforte note, since the two or three strings belonging to a note are rarely in accurate tune.

§ 155. **Combinational Tones.**—In general, if m and n are the vibration numbers of two notes sounding together which give beats, $(m-n)$ will be the number of beats per second. If this beating does not occur oftener than two or three times a second the ear is

not distressed. Rapid beating, however, produces very unpleasant sensations even after it has reached a rapidity too high to be counted. When the difference of frequencies $(m-n)$ is greater than 25 or 30, the note of frequency $(m-n)$ is heard in addition to the two original notes. There is no difficulty in hearing this differential tone, as it is called, when the two notes are sufficiently loud. On instruments giving sustained sounds, such as the organ, harmonium, and concertina, very marked differential tones are produced; and to an ear trained to their perception they are recognisable on the piano. When the difference of the frequencies lies between 30 and 100 the rattling of the beats may often be distinguished from the low hum of the differential tone; so that we are not warranted in regarding these two phenomena as of the same nature. If we bring the two notes by different courses to the two ears we can hear the beats, but cannot hear the differential tone. Moreover, in addition to the differential tone $(m-n)$, there are other differential tones of frequencies $(2m-n)$, $(m-2n)$, &c., and also at times a weak summational tone of frequency $(m+n)$. This last-named tone was discovered by Von Helmholtz, to whom we owe the complete discussion of the origin and significance of these Combinational Tones, as they are collectively termed. Two kinds are distinguished by him. If the vibrations transferred from the vibrating body to the air are very large the simple law of superposition may not hold. A simple pendular vibration, such as is given by a tuning-fork, will when transferred to the air lose to some extent its simple harmonic character, and higher harmonics will enter in. If two simple pendular vibrations act powerfully on the air, combinational tones will be produced in addition to the higher harmonics of the two original notes. These combinational tones existing in the powerfully disturbed air can be reinforced by use of resonators. Combinational tones of the second kind cannot be so reinforced, since they are produced in the ear itself. They are due to the asymmetric character of the drum of the ear, which cannot respond to two coexisting vibrations without producing combinational tones. The frequencies of these combinational tones, whether of the first or second kind, are all included under the general formula $Mm \mp Nn$, where m and n are the frequencies of the original notes, and M and N are integers from zero upwards. As experiment shows, only the first few integers are of any importance, and no summational tone of higher order than $m+n$ (M and N both unity) has ever been heard. As an example, take two notes having frequencies 200 and 315. Their principal combinational tones will have frequencies 115 (315 - 200) and 85

$(2 \times 200 - 315)$. This latter may be regarded as the differential tone between the lower prime and the first combinational tone. These two tones can both be heard if the intensities of the sounds are sufficiently strong. Both theory and experiment show that the comparative intensity of combinational tones grows rapidly as the intensities of the real notes are increased, and also that combinational tones of low pitch are most prominent.

As first brought out clearly by Helmholtz, combinational tones are of peculiar interest when the two notes form a consonant interval. Thus, take any two notes a musical fourth apart. Their vibration numbers may be represented by $3n$ and $4n$. Their principal differential tone will have the frequency n $(=4n-3n)$, and will therefore form the fundamental tone of which the given two are harmonic overtones. Again, take any perfect triad (*doh*, *me*, *soh*) having frequencies $4n$, $5n$, $6n$. Each successive pair gives the same differential tone n; the first and last together give the differential tone $2n$. Thus we hear the low tone which is harmonically fundamental to all, and its octave. In some cases the differential tone becomes so loud that the real notes which are being sounded become merged in it as upper harmonics. If the notes of the triad are not in perfect tune the differential tones will not be harmonically related. On an organ tuned in equal temperament the chord built upon the treble C consists of notes having frequencies 522, 657·7, 782·1. The two lowest differential tones have frequencies 135·7 and 124·4—notes which, sounding together, produce 11·3 beats per second. The ear that has accustomed itself to the pure harmony of the perfect triad will easily recognise a certain dissonance in the triads given by pianos and organs.

Beats always mean the coexistence of two notes of nearly the same frequency. If on any organ or harmonium any note and its fifth are struck together, beats are heard. It is, indeed, by getting rid of these beats that we finally effect a perfect tuning of, say, two contiguous violin-strings. These beats on the tempered instruments are due to the fact that the interval is not a true fifth, so that the higher harmonics which are present cause beating combinations. This is shown by comparing the first three harmonics of notes which form (1) a perfect fifth and (2) a tempered fifth:

Perfect Fifth.			Tempered Fifth.		
C.	G.	Beats.	C.	G.	Beats.
522	783	...	522	782·1	...
1044	...	...	1044	...	...
1566	1566	0	1566	1564·2	1·8
&c.	&c.	...	&c.	&c.	...

Similar results may easily be obtained for all tempered intervals, if the higher harmonics exist in sufficient strength. We may, however, make use of the combinational tones in producing beats. For instance, by the rule given above, we get for the first combinational tones 260·1 and 261·9, which beat also 1·8 times a second. The second combinational tone is intensified by the actual presence of the second harmonic of C; and we can make the beating still stronger by sounding the octave along with C and G.

§ 156. **Influence of the Medium.**—It is usual to think of sound as consisting of waves of compression and rarefaction. And there is no doubt that the waves must be of that character before they reach our ear, for they are transmitted finally through air or other fluid. But before they enter the fluid they usually have their origin in vibrations of solids, and these may be distortional as well as compressional.

Like all wave-motion (§ 146), sound-waves suffer reflection and refraction when they meet the boundary separating two different media. The echo is popularly supposed to be the most familiar example of reflection of sound; but, as we have seen, the phenomenon is essentially involved in steady vibrations of strings and air-columns. Refraction of sound does not appeal to us in the same way that refraction of light does. It can be experimentally demonstrated by concentrating it through a convex lens-shaped bag filled with carbonic acid gas. For in this gas the speed of sound is less than in air. Since elastic waves travel much faster in liquids and solids than in air, a convex lens made of any of these media will not produce concentration at all. For example, as proved by Colladon's experiments on the Lake of Geneva (see also § 145), sound travels fully four times faster in water than in air. A sound-wave in air falling on the surface of water at an angle of incidence greater than 15° will be totally reflected. All compressional waves in water will be powerfully reflected as they pass into air, and will, whatever be their angle of incidence, pass off through air in directions approximately coincident with the normal. Earthquake sounds are waves of compression and rarefaction passing upwards through the air, having been originated as a surface disturbance due to the emergence of elastic waves of high speed in the earth.

When the densities of the two media differ considerably, a very small fraction of the energy of vibration in the one medium passes into the other. It matters not whether it is the less dense or the more dense medium in which the vibration is started. Aerial waves impinging on the surface of a solid yield very little

energy up to that solid; and, on the other hand, the energy of the distortional or compressional waves in a solid which escapes into the air is very small. When vibrating solids give forth strong sounds to the air, it is because of mass-motions of the solid—as in plates, membranes, bells, telephone diaphragms, and so on.

Newton was the first to calculate the speed of sound in air from its elasticity and density (§ 145). He assumed Boyle's Law, and therefore used the compressibility at constant temperature. The value obtained fell short of the true value by about 180 feet per second. The error lay, as Laplace first showed, in assuming Boyle's Law to hold. For Boyle's Law is true only at constant temperature; but in the rapid condensations and rarefactions which constitute audible sounds, the temperature varies, increasing during condensation and decreasing during rarefaction. Hence in applying the formula

$$v = \sqrt{E/\rho}$$

we should use the elasticity of bulk at constant energy or in the adiabatic condition—that condition, namely, in which heat is neither lost nor gained by the substance. In the case of gases well above their critical temperatures, the adiabatic resistance to compression is 1·41 times the isothermal resistance to compression, or 1·41 times the pressure. The formula then becomes $v = \sqrt{1{\cdot}41\, p/\rho}$. Hence the speed of sound in gases varies inversely as the square-root of the density. In other words, a given wave-length will be associated with a higher frequency in the less dense gas. Thus, if an organ-pipe be blown with hydrogen gas in it instead of air, the note will leap up nearly two octaves (accurately $\sqrt{14{\cdot}4}$) in pitch, since the density of air is 14·4 times that of hydrogen.

Since in a gas the pressure is proportional to the density the speed is the same at all densities. It is, however, affected by change of temperature.

EXERCISES.

1. Equal lengths of the same string are taken and tuned to the notes of the triad *doh*, *me*, *soh*. Compare their tensions.

2. What must the relative diameters of eight equally long wires be, so that, with equal tensions, they may sound the notes of the diatonic scale?

3. A stretched string 4 feet long vibrates in 3 segments with a

frequency of 300. Calculate the speed of the transverse waves passing to and fro along it.

4. At what distances apart must the frets be put so that a guitar-string, 27 inches long, may give the successive notes of the true diatonic scale?

5. Two notes, with the first six harmonics present, are tuned to approximately a fourth apart. Their frequencies are 300 and 402·5. What will be the character of the beats heard?

6. A signalman in his box notices that the whistle of the engine is lowered in pitch as the train passes him. Explain this fact.

7. How many vibrations per second will the note have which is sounded by an open organ-pipe 30 feet long?

SUMMARY.

§ 147. Sound. Musical Sound. Noise. Pitch, Intensity, Quality.

§ 148. Pitch. Siren. True Diatonic Scale. Limits of Audibility.

§ 149. Monochord. Tension, Mass, Length, and Pitch.

§ 150. Harmonics and Overtones.

§ 151. Quality, Character, or Timbre. Organ-pipes, open and closed. Brilliancy in quality. Anharmonic constituents.

§ 152. Vibrations of Rods. Chladni's Plates. Bells. Stretched Membranes.

§ 153. Free and Forced Vibrations. Resonance. Helmholtz's Resonators.

§ 154. Interference. Beats.

§ 155. Combination Tones. Beats in Tempered Triad.

§ 156. Influence of the Medium. Speed of Sound in Water and in Air. Newton's Calculation. Laplace's Correction.

END OF PART I.

PHYSICS.

PART II.

MATTER, ETHER, AND ENERGY.

INTRODUCTION.

THE ETHER.

In the present development of science, the branches of Physics to be discussed in Part II. differ in one fundamental characteristic from those discussed in Part I. The dynamics of matter, including Sound and Heat, can be described in terms of matter and energy. In its direct effects, gravitation is so essentially dynamical that we have almost got into the habit of regarding it as an ultimate fact. The extreme simplicity of the law of its action has rendered unnecessary any close inquiry into its actual mechanism; and, as we have seen, no attempt to explain gravitation has been altogether satisfactory. Nevertheless, inasmuch as gravitation is an influence transmitted through regions of space devoid of matter in the ordinary acceptation of the term, it should, strictly speaking, be classed with magnetism, electricity, and light, all of which require for their transmission a substantial medium, having properties analogous to certain properties of matter, but in many other respects differing fundamentally from it. This medium we call the ether.

The ether is an essential part of the modern theory of light and radiant energy generally. Light, we find, passes with greater speed through *vacua* than through regions occupied by matter, so that the most transparent matter, instead of being a vehicle of light, acts really as an obstructive. The fact that the speed of light is finite shows that, if light

is not itself corpuscular but is wave-motion in a medium, this medium must possess properties analogous to elasticity and density. The great speed of light as compared with the speeds of elastic waves in material media is readily explained as being due to the high elasticity or to the extreme tenuity of the ether, or to both combined. Because of this extreme tenuity we might compare the ether to a highly-rarefied gas; but such a comparison would be misleading. For the elasticity of the ether is wholly different from that of a gas. To explain the phenomena of polarisation of light, we must assume the ether to be thrown into vibrations transverse to the direction of propagation of the light. In a gas, on the other hand, longitudinal vibrations only are possible. The gas transmits disturbances because it is compressible and elastic. But, so far as can be inferred from the phenomena of light, the ether is incompressible. Its elasticity is elasticity of form, not of bulk. The strains are distortions or shears, and the stresses correspond. In short, as regards its elasticity, the ether is comparable to an incompressible solid. Nevertheless matter moves freely through it and encounters no appreciable resistance. These properties seem to be at first sight incongruous; but it must be remembered that the ether is not material in the ordinary acceptation of the term. To quote from Sir George Stokes, 'The ether is a substance, if substance it may be called, respecting the very existence of which our senses give us no direct information; it is only through the intellect, by studying the phenomena which nature presents to us, and finding with what admirable simplicity those of light are explained by the supposition of the existence of an ether, that we become convinced that there is such a thing.'

Assuming the ether to possess certain dynamical properties, Fresnel, by pure mathematical reasoning, deduced results which not only accorded with known phenomena, but showed where to search for new truths. Beautiful and far-reaching though his theory proved to be, the basis of

it was dynamically unsatisfactory. Cauchy, Green, Stokes, and others have tried to deduce it as a special case of vibrations in an elastic solid; and in this inquiry, although the subject bristles with difficulties of a formidable character, considerable advance has been made. One outstanding difficulty springs from our ignorance of the nature of the region that separates free ether from ether loaded with particles of matter —in other words, the nature of a material boundary.

Turning now to electric and magnetic phenomena, we note that the possibility of the transmission of electric and magnetic action does not necessarily depend on the presence of matter. All kinds of electric and magnetic induction take place through empty space—that is, through the ether. The ether, then, may be as truly the vehicle for the transmission of electric and magnetic effects as it is for the transmission of light. This, indeed, is the view now widely accepted by physicists. Until the time of Faraday, attention was fixed almost exclusively upon certain obvious *concomitants* of electrical action—upon the so-called charges and the currents of electricity, and the distributions of magnetism. Faraday, on the other hand, imagined the intervening space to be filled with lines of electric and magnetic force, in terms of which the various phenomena could be described. Faraday's conceptions were developed by Maxwell into a complete and connected whole, the essential feature of which was the tracing of electric and magnetic influences as stresses through a medium. The theory involved the remarkable conclusion that an electro-magnetic disturbance is propagated through the ether with a speed which, though expressed in terms of the electric and magnetic properties of the medium, is proved by experiment to be equal in value to the speed of light. In short, according to Maxwell's view, the ether is the medium through which electro-magnetic waves are propagated, and light is itself an electro-magnetic vibration of short wave-length and high frequency. Maxwell's brilliant theory has received remarkable confirmation from the experi-

ments of Hertz (1888), who first devised a method for demonstrating to the sense of sight the existence of electro-magnetic waves propagated through space.

In a homogeneous medium the electro-magnetic disturbance consists of two parts, an electric and a magnetic disturbance, mutually perpendicular to one another, and both perpendicular to the direction of propagation of the disturbance. The theory leads to the same results as Fresnel's theory of light if in a plane polarised ray the electric disturbance is assumed to be parallel to the elastic disturbance imagined by Fresnel. Tait has pointed out that these associated disturbances in Maxwell's theory may not be themselves *primarily* disturbances in the ether, but may be 'necessary concomitants, qualities, or characteristics of one and the same vector-disturbance.' Following out this idea, he has shown how to derive Maxwell's two equations from the equation of motion of an elastic incompressible solid. One of the two associated electro-magnetic disturbances is the time-variation of the elastic displacement, and the other is the rotational part of the accompanying strain. If one of these be the change which affects our sense of sight, then we shall have to regard 'light as an effect produced by the concomitants of the ether-disturbance, and not directly by the ether-disturbance itself.'

Having thus indicated the present tendency of speculation as to the constitution of the ether, which is the vehicle of communication between the farthest limits of the physical universe, we shall confine our attention in the succeeding chapters to the development of each branch along purely experimental lines.

ELECTRICITY AND MAGNETISM.

CHAPTER I.

PERMANENT MAGNETS.

§ 1. A particular iron ore, popularly known as *lodestone*, has a very remarkable attraction for pieces of iron. If plunged into a mass of iron-filings, it will, when taken out, bristle with the tiny iron filaments which attach themselves end on to various parts of its surface.

By stroking a steel knitting-needle or knife blade with such a lodestone, we may develop the same remarkable property in the metal, which then becomes what is known as a Permanent Artificial Magnet.*

A permanent magnet has usually, in addition to its peculiar action on pieces of iron, one other very familiar property. It has the power, when freely suspended, of pointing in a definite direction. This property is utilised in the Mariner's Compass; and our first steps in the study of magnetism will be taken with the aid of such a suspended or pivoted magnet. When of small size, it is conveniently called a *Magnetic Needle.* Except when otherwise stated, a magnetic needle will always be supposed to be so constructed as to have freedom of rotational motion in a horizontal plane.

* The word magnet is derived from Magnesia, in Asia Minor, where the lodestone seems first to have been discovered.

The direction in which a magnetic needle sets itself in this country is, roughly speaking, north-north-west and south-south-east. However much the needle may be disturbed from its natural position, it will, when released, return to the *same position*, with the *same end* pointing northerly. The direction indicated by the northerly-pointing end is called Magnetic North; and the end which so points is usually distinguished from the other end by colour, by mark, or by having the letter N engraved on it. It may be called the north, positive, or north-seeking end; and the other the south, negative, or south-seeking end.

Permanent magnets may be of a variety of shapes, but generally they have the shapes shown in figs. 1 and 2, known respectively as the *Bar Magnet* and *Horseshoe Magnet*.



Fig. 1.

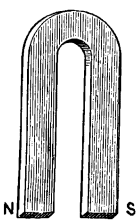


Fig. 2.

To find which is the north end of any magnet, we may suspend the magnet horizontally by an untwisted thread, or float it on a cork on water (see fig. 3). The magnet will set itself (if no other magnet be near) so that the north end will point magnetic north. But there is a much simpler way of discriminating between the ends of a magnet, by studying its action on a freely-suspended magnetic needle.

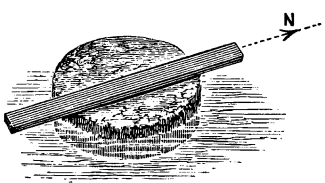


Fig. 3.

§ 2. **Magnetic Polarity.**—The magnetic needle is very sensitive to the near approach of a piece of iron or steel. Still more pronounced is its behaviour when the steel or iron

happens itself to be a magnet. The following experiments are particularly instructive.

Bring a bar magnet (whose ends have been distinguished by the method already given) near the needle, so that one end—say the positive or north end—is fairly close and the other end is as far away as it can be (see fig. 4*a*). The *south end* of the needle will be strongly attracted. Turn the bar magnet suddenly end for end, and the needle will be set into violent oscillations, and will ultimately come to rest with its *north end* strongly attracted (fig. 4*b*). Generally, the north end of any one magnet will attract the south ends of all other magnets, and repel their north ends; while any south end will attract north ends, and repel south ends. We speak of ends of magnets which behave similarly as being *similarly magnetised;* and opposite ends as being *dissimilarly magnetised.* The fundamental law of action between magnets may then be stated concisely thus: *Similarly magnetised ends of two magnets repel each other; dissimilarly magnetised ends attract one another.* Now, unmagnetised iron, when brought near a magnetic needle, simply attracts the end which happens to be the nearer one. If such an iron rod is reversed, the same end of the needle will still be attracted. Thus we can readily distinguish a magnet by the fact that each end repels a particular end of the needle. The two north ends will be mutually repulsive, as also the two south ends. This principle enables us rapidly to determine the north and south ends of magnets.

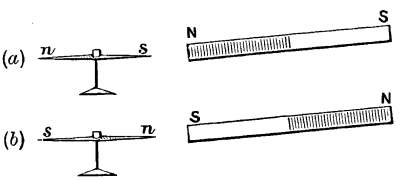


Fig. 4.

Every magnet which has two ends possessing these opposite qualities is called a **bipolar magnet.** It is the normal form of a magnet, the simplest form possible, as we shall see later. What the one end repels, the other will attract.

Polarity is the name given to this attribute of magnets; and it is customary to call the ends of a magnet its *poles*, the more precise significance of which term will become apparent as the subject is developed.

§ 3. **Magnetic Poles and Axis.**—Set a bar magnet on a convenient block of wood, and study the behaviour of a pivoted needle in different positions near it. Fig. 5*a* shows an elevation with the needle near one end; fig. 5*b*, a plan with the needle in various positions. In making the experiment, move the needle gently round the magnet, as indicated by the dotted line in *b*, fig. 5. As it is being so moved, the needle will rotate on its pivot. The various positions taken by the needle in the neighbourhood of the bar magnet may be indicated by arrow-heads, the direction in which the arrow points being the direction in which the north end of the needle points. Such a diagrammatic representation is given in fig. 6. It will be noticed that the arrow-heads seem to diverge from a region near one end, and converge upon a region near the other. These regions are the poles. In many a bar magnet which is long compared to its breadth and thickness, these regions reduce practically to points. Such a magnet approaches the condition of the ideal uniformly-magnetised rod with *point poles*; and its action can be very accurately represented by means

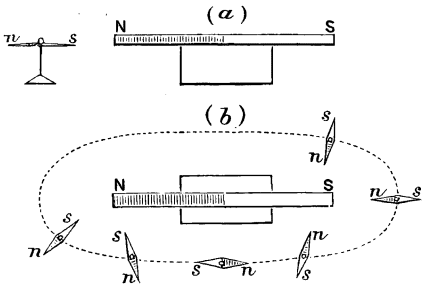


Fig. 5.

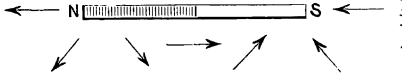


Fig. 6.

of attracting and repelling 'magnetic matter' concentrated at these points. The conception of a magnetic pole is important in the mathematical theory of magnetism. Ordinarily, the term pole is used somewhat vaguely in the sense of a region of concentration of magnetic force.

The *Magnetic Axis* is usually defined as the line joining the poles, and when definite poles exist, this is an accurate definition. For ordinary practical purposes we may assume the magnetic axis of a bipolar bar magnet or magnetic needle to coincide with the axis of figure; but in accurate magnetic measurements it is often necessary to find by experiment the true position, or, at any rate, the true direction of the magnetic axis. When a magnet of any form is freely suspended at its centre of gravity in a region of space where the compass needle points in a definite geographical direction, there is always one definite direction in the magnet which remains invariable in space, whatever position the magnet as a whole may take. This direction is parallel to the direction of the magnetic axis; and the magnetic axis itself we may take as passing through the centre of gravity.

§ 4. **Concentration of Magnetic Power towards the Ends of a Magnet; the Neutral Line.**—From a piece of fairly-stout iron wire cut a number of different lengths, say $\frac{1}{2}$ inch, 1 inch, $1\frac{1}{2}$ inch, and so on. Place the bar magnet with one end projecting well over the edge of a table (fig. 7), a weight being set on the other end to keep it in position. Find by trial where each piece of iron wire will *just hang* from the lower surface of the magnet. When this position is found, the least displacement towards the centre of the magnet will result in the piece of wire falling down to the ground. The longer the piece of wire, the farther it must be placed from the centre, so as just to be held suspended. At or near the centre itself there will be no apparent attraction, even for the

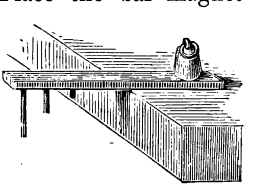


Fig. 7.

smallest piece of wire. If we support the magnet on its two ends, as in fig. 8, we may study the attractive powers of both halves at the same time. The result will be somewhat as shown in the figure, where the wires of different lengths are hanging as near to the centre as it is possible for them to be.

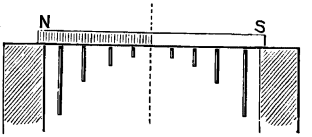


Fig. 8.

The dotted line indicates the place at which the bar magnet exerts no attraction upon a piece of iron brought into contact with it. It is called the *Neutral Line*, and separates the two oppositely-magnetised parts of the magnet.

If, however, we break the magnet into two parts, the magnetic condition of each *separate* part becomes quite changed. Each part becomes, in fact, a bipolar magnet, with two end regions of attractive power separated by a neutral line (see fig. 9).



Fig. 9.

It is impossible to separate a magnet into two parts, so as to have one piece altogether a north end, and the other piece altogether a south end. In other words, a single pole cannot exist isolated in one portion of matter. One opposite pole at least must exist along with it. It matters not into how many or how small fragments a magnet may be broken, each fragment, if it be a magnet, is as truly bipolar as the original unbroken magnet.

The significance of this is well shown by the following experiment with two bar magnets of the same size and as nearly as possible the same strength.* First lay the one magnet pointing north, and place near each end a magnetic needle in such a position that it points at right angles

* The best bar magnets are sold in pairs with opposite poles connected by small armatures or keepers of soft iron. Such a pair gives two nearly equally strong magnets—that is, magnets which produce nearly equal magnetic effects under exactly the same conditions. See below, § 20.

to the axis of the magnet. Place a third magnetic needle in the same line with the first two, and at a distance from one equal to the distance of these two apart (see fig. 10*a*).

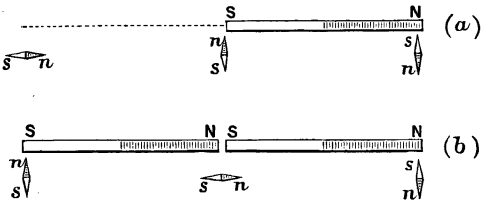


Fig. 10.

This third magnetic needle, being at some distance behind or in front of the bar magnet, will not be greatly deflected from the position it would occupy were the bar magnet removed. Now introduce the second bar magnet, pointing along the same line as the first, as shown in fig. 10*b*. The two bar magnets form a long bar, with dissimilar poles touching, or all but touching, in the middle. The middle needle will be released from the position in which it was held when the one magnet only was present, and will swing into a position parallel, or nearly so, to the double bar magnet. At the same time the needle, which at first was hardly influenced, will be made to point at right angles to the axial line of the bar magnets. Comparing figs. 5, 10*a*, and 10*b*, we see that the double magnet of fig. 10*b* behaves exactly in this respect as would a single magnet of the same length. One way of looking at this phenomenon is to regard the action of the south pole of the one magnet as balancing the action of the contiguous north pole of the other.

§ 5. **Magnetism probably a Molecular Phenomenon.**—The persistence of the bipolar quality in the fragments of a broken magnet suggests the hypothesis that **magnetic polarity is essentially a molecular phenomenon.** The bipolar quality belongs to the particles of the magnet; and what we witness in the action of a finite magnet is the resultant of the actions of the very numerous and exceedingly small mag-

netic molecules or molecular magnets which build it up. How this occurs may be illustrated by the following experiments.

Set the one bar magnet again as in fig. 10*a*, and then place the other bar magnet a few inches distant from the first on the side opposite to the side where the needles are, as in fig. 11*a*. The second magnet is to be put pointing towards the south, so that its north pole is over against the south pole of the first magnet. When this is done, the needles will be observed to move round a little, as shown in fig. 11*a*. Now slide the second magnet gradually up to the first. The little needles will swing round more and more from their original position in fig. 10*a*, until at last, when the two magnets are in close contact one beside the other, the two needles will point north, or nearly so (fig. 11*b*). Thus the action of the one magnet is nearly, if not wholly, annulled by the action of the other.

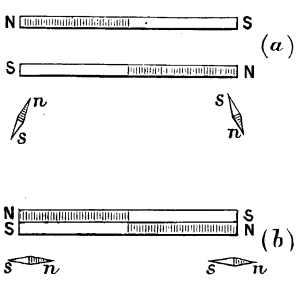


Fig. 11.

In fig. 12, we have another example of the same balancing of actions in which, by pushing the one magnet gradually over the other, directed oppositely, we diminish the magnetic force acting on the key. When the force becomes too weak to support it, the key falls.

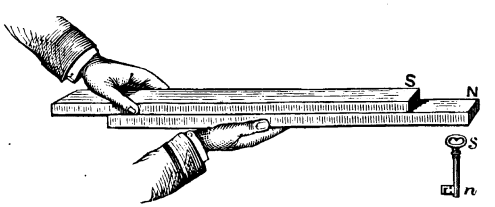


Fig. 12.

It is not always possible to get two bar magnets suffi-

ciently nearly equal in strength, so as to obtain precisely the results indicated in the experiments of figs. 10 and 11. But even with magnets differing appreciably in strength, results differing but slightly from those described will be obtained.

The experiments may be varied by bringing *similarly magnetised* ends of the two magnets into contact. In these cases the action of the one magnet will intensify that of the other. For example, instead of the configuration of fig. 10*b*, we shall get the configuration of fig. 13*a*. Now it is possible to obtain a single bar magnetised in the way here indicated. We may have a magnet with three poles, the middle pole being of opposite character to the two extreme poles, which are of the same name. Or a single bar magnet may have four poles, north and south alternating as we pass along the bar. The existence of these extra or *consequent* poles is easily shown by means of a small needle carried slowly along the side of the magnet. Where the poles occur, the needle will tend to set itself at right angles to the bar, as in fig. 13*b*. For ordinary purposes, good magnets should not have consequent poles. Except when otherwise stated, magnets are always regarded as bipolar.

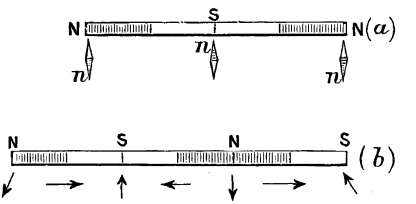


Fig. 13.

§ 6. **Sketch of Molecular Theory.**—Of the experiments that have been described, that of fig. 11 is particularly noteworthy. Even more striking is the same kind of experiment with two horseshoe magnets of equal size placed with their opposite poles in contact. It suggests that a piece of matter with feeble or no magnetic polarity may, by a mere change of position of its component parts, become a body of powerful magnetic quality. We have seen also that mag-

netism is probably essentially molecular. We conclude accordingly that the apparent magnetic action of an ordinary magnet depends probably upon the arrangement of its molecular magnets. These may be arranged so as to have a very small resultant effect on external pieces of iron; or they may be arranged so as to have a very pronounced resultant effect. A piece of iron which has no magnetic polarity may nevertheless be built up of molecules, each of which is as powerfully magnetic as are the molecules of a steel magnet. This, indeed, is the molecular theory of magnetism due to Weber and in universal acceptance. An unmagnetised piece of iron is believed to differ from a steel or iron magnet simply in the different arrangement of the magnetic molecules. What any special arrangement may be, we have no means of discovering. We may, however, represent the difference between magnetised and unmagnetised iron by means of crude diagrams of molecular arrangement. We will represent the molecular magnets by circles with blackened semicircles, to indicate their north pole. In *a*, fig. 14, the molecules are arranged so that about as many have their north poles pointing in one direction as have them pointing in another. In *b*, the majority of the molecules have their north poles facing one way. In *c*, all face one way. These groupings may be taken as representing in order unmagnetised iron, moderately-magnetised iron, powerfully-magnetised iron.

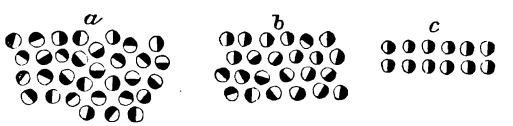


Fig. 14.

EXERCISES.

1. A bar magnet is placed anywhere on the table in the neighbourhood of a compass needle, and is slowly rotated round a vertical axis through its middle point. Describe the behaviour of

the needle (1) when the magnet is very close; (2) when it is a few feet distant.

2. Arrange three bar magnets of equal strength so that there is no magnetic effect on a neighbouring magnetic needle.

3. A piece of steel wire, bent so as to form two sides of a square, is magnetised in such a way that each of its free ends is a north pole, and the bend a south pole. When placed upon a cork floating in water, how will it set? (1890)

4. Two bar magnets of equal length are set on end a few inches apart. A small magnetic needle is carried round the upper poles in a figure-of-eight course. How will it point in the various positions occupied, (1) when the upper poles are *like* poles; (2) when they are *unlike* poles?

SUMMARY.

(1) The Lodestone. Permanent magnets; magnetic needles. Magnetic north. Usual forms of magnets.

(2) Magnetic Polarity. Behaviour of a magnetic needle in the presence of a magnet. Fundamental law of action between magnets.

(3) Magnetic Poles and Axis. Poles, their meaning and position. The magnetic axis.

(4) Concentration of Magnetic Power towards the Ends of a Magnet. The neutral line. One pole cannot be isolated. Two magnets joined so as to have all the apparent properties of a single magnet. Neutralisation of one pole by another, of one magnet by another.

(5) Magnetism probably a Molecular Phenomenon. Consequent poles.

(6) Molecular Theory of Magnetism. Distinction between unmagnetised and magnetised bodies.

CHAPTER II.

THE MAGNETIC FIELD AND LINES OF MAGNETIC FORCE.

§ 7. **The Magnetic Field.**—The magnetic needle, as we have seen, points in a definite geographical direction. When a magnet is brought near, the needle is powerfully affected; and if the magnet is placed at rest in the neighbourhood, the

needle will again settle into a definite position. That is to say, it still retains the property of pointing in a definite direction, but this direction is no longer (in general) the direction of magnetic north.

There is no evidence, therefore, that the neighbouring magnet has brought into play a *different kind* of influence. It has simply modified the influence already present. This leads naturally to the consideration of what is known as the Magnetic Field.

Definition : **A Magnetic Field is a region of space at any and every point of which a freely-suspended magnet points in a definite direction.**

From the experiments described in last chapter it is easy to see that a magnetic field is associated with every magnet. If we make the natural assumption that *every* magnetic field has associated with it some one or more magnets, we must conclude that the earth itself is a magnet. In other words, it is a source of magnetic influence, and the region around it is a magnetic field. We may draw an analogy between the earth's magnetic field, which makes a needle point magnetic north, and the earth's gravitation field, which makes a plumb-line hang vertical. After a slight displacement to right or left, the plumb-line will oscillate pendulum-wise about its position of rest. Similarly, the magnetic needle, when disturbed and left to itself, oscillates to and fro about its position of rest. The force of gravity, acting on the plumb-line, acts downwards in the direction given by the plumb-line as it hangs free and at rest. Similarly, the magnetic force, acting on the needle, acts in the direction in which the needle points—more accurately, in the direction in which the magnetic axis of the needle points. But there is one important difference between the two cases. If we unfasten the upper end of the plumb-line, the whole will fall towards the earth. But if we float the magnetic needle on water so as to leave it free, it will show no tendency at all to move as a whole in the direction of magnetic north. In short, the earth's

magnetic field has no appreciable *translational* effect upon a magnet freely suspended in it.

To determine the true direction of the earth's magnetic field, we must suspend the magnetic needle so that no other turning forces act upon it. We must suspend it at its centre of gravity. Suspend an unmagnetised knitting-needle at a point as near to its centre of gravity as possible, and set it horizontal (fig. 15*a*). Without in any way altering the point of suspension, stroke the needle several times, always in the same direction, with one pole of a strong magnet. The needle being thus magnetised, will tend to set itself in the direction of the earth's field. But not only will its north pole turn towards magnetic north; it will also dip down (fig. 15*b*). Thus we learn that in this country the direction of the earth's magnetic field is inclined downwards to the horizon. The whole subject of the magnetism of the earth will be discussed in Chapter V. Meanwhile, it is important to bear in mind that the horizontally-suspended needle, so familiar in the Mariner's Compass, gives us information concerning the *horizontal component* only of the earth's magnetic force. Such a needle is made to hang horizontally by being suspended at a point well above the centre of gravity, so that the moment (or leverage) of its weight prevents any appreciable tilting. Every permanently-magnetised body suspended at its centre of gravity in a magnetic field will set itself so that its magnetic axis is parallel to this field. This gives us the direction of the field.

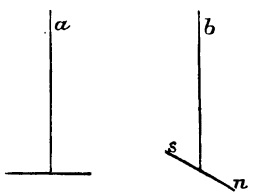


Fig. 15.

§ 8. **Translational and Rotational Effects of a Magnetic Field on a Magnet.**—Hang a magnetic needle by a long untwisted thread, and place on the table a bar magnet, with its centre close beneath the centre of the needle, as in fig. 16*a*. The needle will set itself so as to point more or less

approximately in the direction opposite to that in which the magnet is pointing. Rotate the magnet round a vertical axis through its middle point into any other position, such as that indicated by the dotted line, and the needle will follow after it. Here we have evidence of a magnetic field, which, like that due to the earth, has no translation effect upon the needle when it occupies this particular position in it. The needle points in a definite direction, but it has no tendency to move as a whole in that direction.

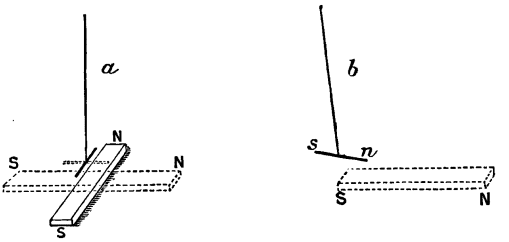


Fig. 16.

But now slide the magnet along in the direction of its length. Then, as its centre moves farther and farther away from its first position, the needle will be drawn slightly to one side, and the suspension thread will no longer hang vertically. When the needle is just over the one end of the magnet, this inclination of the thread to the vertical will have its greatest value (see fig. 16*b*). As the magnet is farther removed, the needle will begin to fall back towards its original position of rest. Here, then, we have a magnetic field, which not only gives direction to a needle placed in it, but has also a translation effect upon it. If we bring the pole of the magnet close up to the pole of the needle, the attraction will of course be very powerful.

This and similar experiments show that the magnetic field of a bar magnet in the neighbourhood of a pole has a powerful translational effect, as well as a directive effect upon a magnetic needle placed in it. On the other hand, the magnetic field at a region not near to either pole has a very small translational effect, quite inappreciable at great distances;

and the magnetic field at regions equidistant from the poles has no translation effect at all in the direction of the field.

§ 9. **Magnetic Field in the Axial Line of a Magnet.**—Set a compass needle on a broad table, and by chalk or otherwise mark out the direction of magnetic north and south, and the direction at right angles thereto. This may be called the line of magnetic east and west. The centre of the needle lies at the intersection of these two lines, which are shown dotted in the diagrams. Now, at a distance of a yard (say), lay a bar magnet along the east-west line, and gradually slide it along this line towards the needle. The needle will be deflected from its original position, and the nearer the bar magnet the greater the deflection. We conclude that the field associated with the magnet is stronger the nearer we are to the magnet. Fig. 17 shows the effect when the bar magnet, placed to the east, has its north pole pointing towards the needle. The direction (*sn*) of the needle gives the direction of the magnetic field in which the needle is situated. This magnetic field is called the resultant of the fields due to the earth and the bar magnet.

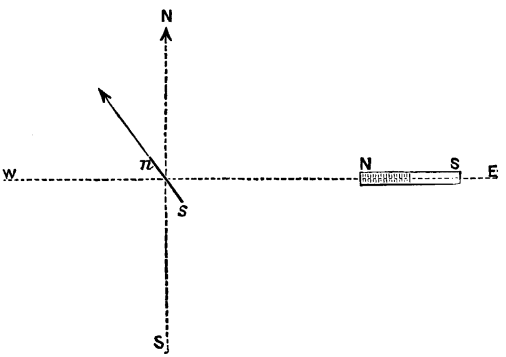


Fig. 17.

Now place the bar magnet along the north-south line, as in fig. 18*a*. The needle will suffer no deflection, and yet we know that to the earth's magnetic field we have added that due to the magnet in the particular position shown. The

reason is that, in this case, the magnetic field due to the bar magnet has, in the position occupied by the needle, *the same direction* as has the magnetic field due to the earth. The same is true for the configuration of fig. 18*b*, where the magnet is to the north of the needle, and has its north pole pointing from it. However close the bar magnet may be to the needle in these two positions, the same holds true. Rotate the bar magnet about its magnetic axis, and the same result is obtained. Thus we learn that, at every point of the line drawn through the axis of a bar magnet, the direction of the magnetic field is along that axial line. Its direction is *from* the north pole outwards and *towards* the south pole inwards.

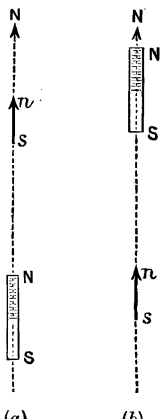


Fig. 18.

§ 10. **Superposition of Magnetic Fields.**—Vary the last experiment by inverting the bar magnet end for end in the positions of fig. 18. First, set the bar magnet at a considerable distance to the south, say, with its south pole pointing towards the needle, and gradually slide it nearer and nearer. So long as the centre of the magnet lies to the south of a certain position, A, as in fig. 19*a*, the needle will continue to point north. But when the centre of the magnet lies to the north of A, the needle will turn round and point south. As the magnet passes through the critical position separating these two distinct cases, the needle *will seem to have lost* altogether its directive property.

This result is readily explained in the light of the last experiment. In all cases, the needle is under the influence of a magnetic field which is the resultant of the fields due to the earth and the magnet. In fig. 18 these two fields are *co-directional*, and simply intensify each other's action. But in fig. 19, they are *anti-directional*, and to a greater or

less extent counteract each other's action. The field due to the earth remains constant throughout; but the field due to the magnet varies in strength with the distance from it, increasing as the distance diminishes. In the configuration of fig. 19*a*, the magnet is so distant that its field in the position occupied by the needle is *less* than the earth's field. The latter accordingly has the preponderance, and the needle continues to point north. But in the configuration of fig. 19*b*, the magnet is sufficiently near to have its field in the position occupied by the needle *greater* than the earth's field. The former accordingly has the preponderance, and the needle points south in the resultant field. As the magnet is moved up gradually from a considerable distance, the resultant field gradually diminishes* until the position A is reached, when the two superposed fields just cancel each other, and the needle is under the influence of *zero* field. There is, in fact, nothing to make it point in a definite direction. As the magnet is moved into positions nearer than A, the resultant field changes sign, and increases* more and more as the magnet is pushed nearer and nearer. In general, the distance from the needle of the critical position A differs with different magnets. We may regard that magnet as the stronger one for which this distance is the greater.

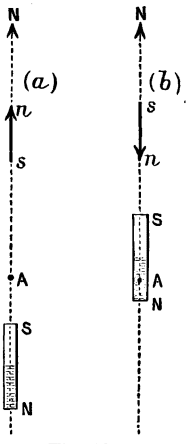


Fig. 19.

Very similar effects are produced if the magnet is put to the east or west of the needle.

In the first place, set the magnet pointing south at some distance to either side of the needle (fig. 20*a*). The needle will be still seen to point north, and will continue so to

* This may be proved by taking oscillations as described below, § 23.

point, however near the magnet may be brought. In the second place, set the magnet pointing north. Then, just as in the previous experiment, there will be for each bar magnet a certain critical position, B, which separates the two cases shown in fig. 20, *b* and *c*. When the magnet is beyond B, the needle points north, because the earth's field preponderates; when the magnet is at B, the two fields exactly balance each other, and the needle has no directive property; when the magnet is nearer than B, the earth's field is overbalanced by the oppositely-directed field due to the magnet, and the needle points south. In these and similar experiments, in which the properties of a magnet are being studied by means of a needle, the needle should be very small, and should have fastened to it a long light pointer of brass or aluminium.

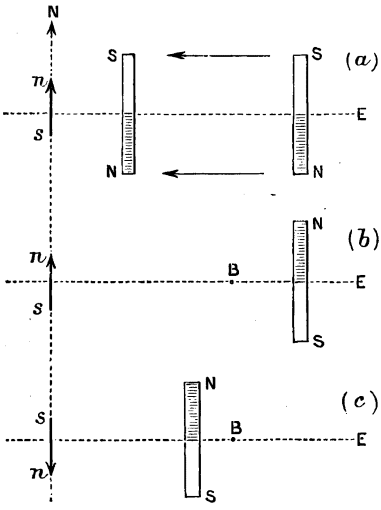


Fig. 20.

§ 11. **Lines of Magnetic Force.**—Gathering up the lessons of these experiments, we find that the field due to a bar magnet has certain important characteristics which may be thus described: (1) The field at all points in the axial line is in the direction of the magnetic axis of the magnet; (2) The field at all external points in a certain plane, which approximately bisects the magnet and is perpendicular to the magnetic axis, is in the direction opposite to that of the magnetic axis; (3) The field, at points in the axial line, diminishes

in strength as the distance from the magnet increases, and similarly for points in the plane defined as above.

These characteristics are represented diagrammatically in fig. 21. Each arrow gives the direction of the field at the

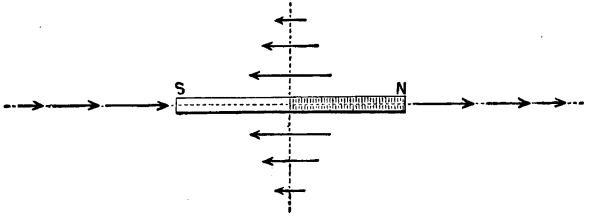


Fig. 21.

position of its middle point, and the stronger field is shown by the longer arrow.

Definition: **A Line of Magnetic Force is a line—in general, a curved line—whose direction at every point of it coincides with the direction of the magnetic field there.**

A small magnetic needle, moved always in the direction in which it points, will describe a line of force. It is easy to see that the axial line of a bar magnet is one of its lines of force; and that all lines of force outside the magnet become parallel, though anti-directional, to the axis as they cut across a certain plane which may conveniently be called the *equatorial plane.* But how do the lines of force run through other points in the neighbourhood of a magnet? This question could be answered at once by experiment, if there were no magnetic field due to the earth. We should simply have to take a small needle and move it in the direction of its axis, and the needle would trace out the line of force required. If, however, we try the experiment in the neighbourhood of a magnet, we get a line of force due to the combined influence of the earth and the magnet, and not a line of force due to the magnet alone. To be certain that we are dealing with a line of force due to the magnet alone, we must get rid of the earth's magnetic

action on the needle. This may be done in the manner suggested by the experiments of § 10, as follows.

§ 12. Place in proximity to the needle a bar magnet, M, such that its magnetic effect upon the needle will just balance the effect of the earth's field. In other words, set M in one of the critical positions A or B of figs. 19 and 20. Take now the magnet K, whose field is to be studied, and bring it into various positions relatively to the needle. The needle will be very sensitive to every movement of the magnet K. Being subject to no other directive force, it will set itself tangential to the particular line of force due to K, which passes through the place occupied by it. In fig. 22, the steady magnet M is shown in dotted outline, and the needle is pointing along a particular line of force due to K.

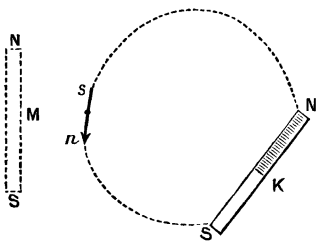


Fig. 22.

When a needle, as in the above experiment, is uninfluenced by the directive force due to the earth's magnetic field, it is said to be astatic.

An astatic needle will point equally readily in all directions; and such a needle becomes very sensitive to any other magnetic influence which may be introduced. The simplest plan of rendering a needle astatic is that just described—namely, by adjustment of an external magnet—or, more generally, by superposing upon the earth's field in the place occupied by the needle an equal but opposite field.

In fig. 23 is shown a very useful combination known as an astatic pair of needles. The two needles are fixed to the same vertical axis, so as to point in opposite directions. In whatever position the pair is set, the two needles will tend to be pulled round in opposite directions, and the directive force acting on the two together will be the difference of the directive forces acting on each, considered singly. To

be truly astatic, the two needles ought to be of exactly the same strength—a condition which is very difficult of realisation. Since, however, the directive force acting on any magnet depends on the strength of the magnet itself, as well as on the magnetic field in which it is placed, it is clear that in such a combination in which the magnets are not quite of the same strength, the directive force acting on the whole will be much smaller than the directive force acting on either magnet taken singly.

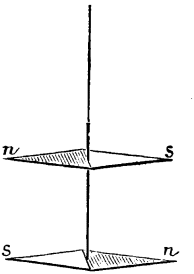


Fig. 23.

EXERCISES.

1. A bar magnet is placed anywhere on the table in the neighbourhood of a magnetic needle, and is slowly rotated round a vertical axis through its middle point. There are two positions of the magnet for which the needle points along the line of its undisturbed position. Explain this.

2. A strong bar magnet is placed on a table with its north pole pointing towards the north. State in what direction a compass needle points (1) when placed immediately over the centre of the bar magnet; (2) when gradually raised vertically upwards. (1888)

3. A bar magnet is brought near an astatic pair of needles. What takes place as the magnet, kept parallel to itself, is gradually raised or lowered? Explain the effect.

4. Describe the behaviour of a magnetic needle when a bar magnet, with its axis vertical, is moved up and down in its axial line, anywhere in the neighbourhood of the needle.

5. Show how to investigate experimentally the direction of the field in the axial line of a bar magnet, and also in the neutral or equatorial plane.

6. When a magnetic needle has no directive quality, but continues to point in any direction in which it may be placed, what do we infer as to the magnetic relations of the place occupied?

SUMMARY.

(7) The Magnetic Field. The earth's magnetic field; its translational effect nil.

(8) Translational and Rotational Effects of a Magnetic Field on a Magnet. Translational effect strongest in the neighbourhood of the poles.

(9) Magnetic Fields in the Axial Line of a Magnet.

(10) Superposition of Magnetic Fields. Annulling of one field by another.

(11) Magnetic Lines of Force.

(12) Lines of Force indicated by means of a small Magnet. Astatic magnets and astatic combinations of magnets.

CHAPTER III.

MAGNETIC INDUCTION.

§ 13. **Magnetisation by Induction.**—We will now consider more carefully the action of a magnetic field upon a piece of iron originally unmagnetised. We know that a magnet attracts iron; but before this attraction can be made evident, we have to introduce the iron into the field associated with the magnet. If at a sufficient distance from the magnet, the iron is not sensibly attracted. The nearer the iron is brought to one pole of the magnet, the stronger is the field in which it lies; and we find that the attraction increases correspondingly. But, in addition, experiment shows that the piece of iron so introduced into the magnetic field *has become itself a magnet.* It has become **magnetised by induction.**

If, instead of iron, we use pieces of nickel or cobalt, similar effects are produced. These three metals are known as the *magnetic metals.* Of the three, iron is most susceptible to magnetic influence; and, meanwhile, we shall continue to confine our attention to it.

As already noted (§ 2), the essential difference between a magnetised and unmagnetised piece of iron is, that neither end of the latter has a repulsive action upon either pole of a magnetic needle. If such a repulsive action exist,

the iron is a magnet. To remove all trace of magnetic polarity, we must heat the iron to a bright red heat in a furnace.*

Lay a rod of soft unmagnetised iron, several inches in length, with its one end near a magnetic needle, as in fig. 24*a*. A slight deflection will probably be produced; but if the bar possesses no magnetic polarity, it will be possible to adjust it so that the needle continues to point very nearly north and south. Now bring up a bar magnet, so that its south pole is close to but does not touch the further end of the iron rod (fig. 24*b*). A strong deflection is at once produced on the needle. Then invert the bar magnet, so that its north pole is near to the iron rod (fig. 24*c*). The needle responds by turning its north pole away. In either of the positions (*a* and *b*), remove the iron rod, but leave the magnet as it was. The deflection of the needle will distinctly diminish. Put the iron rod back into position again, and the deflection will return to its old value. Thus we see that the iron rod and the magnet together exert upon the needle a distinctly stronger influence than does the magnet alone, and a very much stronger influence than did the iron rod before the magnet was laid in position. The rod, so to speak, extends the influence of the magnet in the direction in which it points. But again, as may be shown by moving a small magnetic needle to and fro in the neighbourhood of the region between the magnet and the iron, the end of the iron near

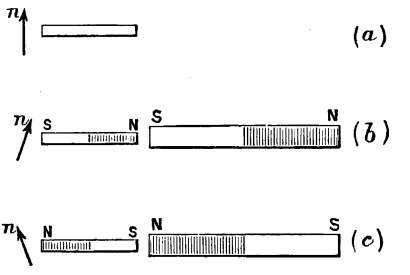


Fig. 24.

* In treating it thus, we should lay the iron with its length in the direction of magnetic east and west; otherwise, on cooling, it will be (feebly) magnetised by the inductive action of the earth's field.

to the magnet has all the characteristics of a magnetic pole. It is a north end in position *b*, and a south end in *c*. Thus the rod is temporarily a magnet, with its magnetic axis co-directional with the magnetic axis of the inducing magnet, as indicated by the letters SN in the figure. Again, place the rod—a good-sized iron nail will do very well—parallel to the magnet and on either side of it. Then, when tested by a small needle, it will be found to be a bipolar magnet, anti-directional to the inducing magnet. By such experiments we find that **iron is magnetised inductively in the direction of the inducing field.**

We readily see that the attraction which exists between a magnet and a neighbouring piece of iron is of exactly the same character as the attraction which exists between the dissimilar poles of two magnets. For the iron is itself a temporary magnet, under the inductive action of the magnetic field associated with the permanent magnet. The nearer the magnet is, the stronger is the field, the more pronounced the influence of the temporary magnet, the more powerful the induction. At length, when the magnet is brought very near, the iron rod becomes so strongly magnetised that it moves into contact with the magnet.

§ 14. **Simple Illustrations of Magnetic Induction.**—Take a few ordinary nails or bits of iron wire, half an inch to an inch long, and lay them in line on the table, so as just to touch each other. Bring the magnet into contact with the end nail, and lift it up. Several of the other nails will be lifted up also, and will hang suspended as a continuous chain (see fig. 25). The number of nails which can be so suspended will depend on the weights of the nails and on the character of the magnet as indicated by its *lifting power*, or weight that can be supported from its end. Now, if we try the experiment, we shall find that the lowest nail

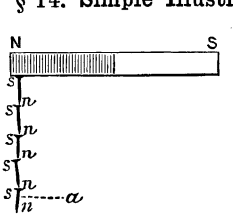


Fig. 25.

(*a*) could not be supported in that position by the magnet, if it were not joined by the connecting chain. Each nail is, in fact, supported by the one immediately above it. All are little magnets with dissimilar ends in contact, as indicated by the letters *sn* in the figure.

While things are in this position, grasp the uppermost nail firmly between finger and thumb, and pull the magnet away from contact with it (fig. 26). If the nails do not at once fall down, a slight shake will suffice to loosen the chain, and cause most of them to drop on the table. When once they are separated, it will be impossible to make even two of them hang together, unless the magnet be again brought near. Thus the magnetic quality is only a temporary possession on the part of the nails. This **temporary magnetism** is developed in them by the inductive action of the magnetic field.

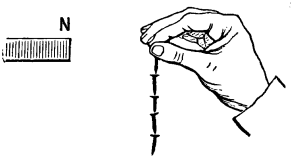


Fig. 26.

If a chain of nails hang from the north pole of a magnet, their lower ends will be north poles (see fig. 25). That the free end of the chain is a north pole may be demonstrated by bringing the north pole of another magnet near it, as in fig. 27*a*. The chain of nails will be repelled, and will probably be loosened because of the neutralising effect of the opposing inductive action of the second magnet. If, on the other hand (fig. 27*b*), we bring the south pole of the second magnet near, attraction

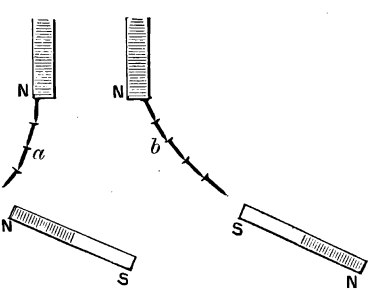


Fig. 27.

will be evident, and the chain will be strengthened all along its length.

By using two magnets with dissimilar poles facing each other at a distance of several inches, we can bridge the interval across by a chain of nails. An ordinary horseshoe magnet, which is simply a bar magnet bent till its opposite poles are brought near each other, can be made to support a chain of considerable length, as in fig. 28.

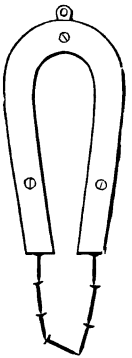


Fig. 28.

Place two fairly-long nails side by side on, say, the north pole of a magnet. They will diverge, because they have become magnets with their farther ends both north poles, and these will repel each other (fig. 29). The same phenomenon is shown in a striking way by dipping the end of the magnet into a heap of iron-filings, or of short bits of iron wire. On removing the magnet, we find its end bristling with the tiny pieces of iron. These accumulate along the edges where, consequently, we infer the magnetic field to be strongest.

§ 15. **Pictorial Representation of the Magnetic Field.**—Iron-filings are a very useful aid in giving us a graphic picture of the magnetic field associated with a magnet, or with special combinations of different magnets.



Fig. 29.

Place over a strong magnet a thin sheet of cardboard or glass, and sprinkle over this surface as uniformly as possible a quantity of iron-filings. A tendency is at once apparent for these filings to accumulate just over the edges, and especially the ends, of the magnet. The magnetic field at once makes its influence felt—first, by magnetising the filings inductively; and second, by impelling them to get into line, so to speak, with the lines of force. To aid this latter tendency, tap the cardboard or glass gently. With the momentary freedom so acquired, the magnetised filings set

themselves along the lines of force passing through them, and at the same time accumulate still more towards the ends of the magnet. The general appearance is shown in fig. 30. Groups of filings arrange themselves end to end, forming a beautiful series of curved lines, which diverge from the one pole of the magnet and converge upon the other. These curved lines give a very good idea of the way in which the lines of force in a particular plane pass from pole to pole of a magnet.

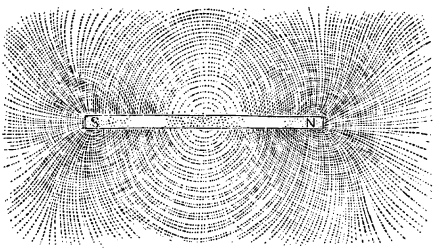


Fig. 30.

Similar experiments may be made with various combinations of magnetic poles. A few of these are shown in figs. 31, 32, 33. In fig. 31 we have a picture of the field produced by two dissimilar poles face to face; in fig. 32 a picture of how the filings set themselves when the contiguous poles are of the same name; and in fig. 33 we have the appearance presented when a magnet

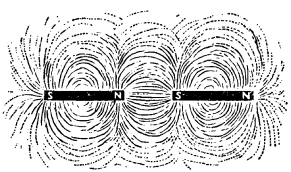


Fig. 31.

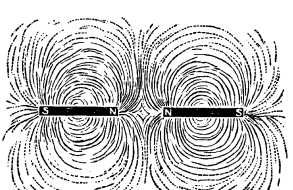


Fig. 32.

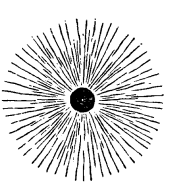


Fig. 33.

of circular section has its one pole turned up immediately below the sheet bearing the filings.

§ 16. **Induction on the Molecular Theory.**—Take a horseshoe magnet (such a magnet is generally sold with a piece of soft iron known as the **Keeper** or **Armature** set across the ends) and study the lines of force as indicated by the configurations of iron-filings, (1) when the armature is removed altogether, (2) when it is placed at various distances from the ends of the horseshoe magnet. Such experiments will do more than pages of description to impress upon the mind the general features of a magnetic field, and the manner in which a piece of iron, when introduced into it, alters its configuration. In fig. 34, examples of two such experiments

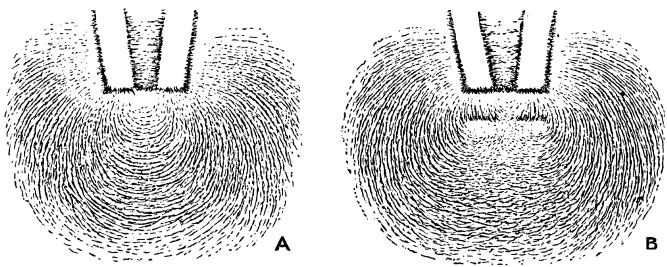


Fig. 34.

are shown. As we pass from A to B, we notice a distinct concentration of field in the region between the poles and the armature, and an indefiniteness in the region beyond the armature. When the armature is in contact with the magnet, the external field is reduced to a minimum. We know, however, that the magnet is still acting powerfully, so that the field must be somewhere. What has happened is that the lines of force are to a large extent included *in the armature.* We may follow them in imagination through the armature, across the surface of contact into the negatively magnetised end of the magnet, through the horseshoe itself, and out from the positively magnetised end till they complete their course in the armature.

Similar results will be obtained with two bar magnets set side by side, with unlike poles opposing each other in pairs, but not touching. A keeper of soft iron brought up to either pair will concentrate the field into itself. Two such keepers joining the pairs of poles are useful in preserving the strengths of the two magnets. A like function is performed by the single keeper of a horseshoe magnet. The keeper reacts inductively upon the magnet, and the lines of induction pass through both, and sustain the set of the magnetic molecules.

According to the molecular theory of magnetism briefly sketched in Chapter I., the magnetic quality of a bar magnet is explained as the result of an average pointing in one particular direction of the magnetic molecules that build it up; while the unmagnetised condition of a piece of iron is referred to the magnetic molecules forming closed chains or magnetically-balanced groups, and so producing no external magnetic action. But when a magnetic field is brought to bear upon the iron, the magnetic molecules are acted upon, some of the balanced groups are broken up, and an average set of molecules is produced, just as in the permanent magnet. These permanent molecular magnets tend to set along the lines of force, just like the inductively magnetised iron-filings. When the field is removed, the little molecular magnets recover more or less their mutually-balanced groupings; and the induced magnetism in the iron largely diminishes.

§ 17. **Magnetic Induction** may be regarded as a **fundamental property** of the magnetic field. Not only do previously-unmagnetised pieces of iron become magnetised when placed in such a field, but permanent magnets are themselves influenced in exactly the same way. It can be proved by careful experiment that a magnet is slightly more powerful when it is pointing magnetic north than when it is pointing south. In both positions it is under the inductive action of the earth's horizontal field. This inductive action gives to the

magnet a small extra magnetisation in the direction of the horizontal field. Hence, when the magnet has its axis co-directional with the earth's field, its power is increased by a small quantity; but its power is similarly decreased when the magnet is turned round so as to point the other way. If placed east and west, the magnet is not acted upon *lengthwise* by any magnetic field, and the induction acting across the magnet is altogether inappreciable in its effects.

Induction may be shown to take place even in such a weak field as that of the earth. A poker or rod of soft iron, if held vertically (in this country) over a magnetic needle, will attract the south end and repel the north end; if held beneath the needle, so that the upper end is brought close, it will attract the north end and repel the south end. The poker is a magnet with north end pointing downwards. The best effects are obtained when the poker is held in the true direction of the earth's field, that is, along the line of Dip, which will be fully explained in Chapter V.

By making the field powerful enough, we may reverse the polarity of a permanent magnet placed in it. This phenomenon is often observed when we bring the one pole of a very small magnetic needle near the similar pole of a strong bar magnet. If not brought too near, it is repelled in accordance with the fundamental law that like poles repel each other. But, if the poles are very close, the powerful induction due to the strong field in the neighbourhood of the bar magnet reverses, temporarily, if not permanently, the polarity of the needle. There will, consequently, be attraction between two ends which, when far apart, were similar magnetic poles. In fact, the needle is compelled to behave like a piece of unmagnetised iron.

The inductive action of a bar magnet is most powerful where the associated field is strongest—that is, near the poles. In order to magnetise a piece of iron with the aid of a magnet, it is not necessary that the two should come into direct contact. But it is evident that by direct contact we insure the

iron getting into as strong a field as possible. Thus we readily magnetise the blade of a knife by stroking it, always in the same direction, from end to end with either pole of a magnet. The blade will be magnetised in such a way that the end which the magnet leaves after each stroke is of opposite character to the pole that has just been drawn along it. In the same way, by drawing, say, the north pole of a bar magnet along a magnetic needle from its south to its north end, we may permanently reverse the polarity of the needle, so that what was at first the south end becomes the north, and *vice versâ*. The jarring or vibration produced by contact and friction of the metals are found to assist materially the operation of magnetising.

§ 18. **Coercive Force, or Retentivity.**—Every piece of iron which is introduced into a magnetic field becomes for the time a magnet. When it is taken out of the field again, it may retain in large measure the magnetic polarity induced in it, or it may return practically to its original unmagnetised condition. In the former case it is left a permanent magnet.

The power any piece of iron has of retaining a high magnetic polarity depends upon its structure and temper. Soft iron, which has been cooled slowly from a high temperature, is altogether unsuitable for the manufacture of permanent magnets. All such must be made of steel or hard iron; and experience is the only guide as to the best mode of tempering the iron or steel that is to be made into a magnet.

Let us contrast the behaviour of two equal-sized pieces of soft and hard iron under identically similar magnetic treatment. Let both be unmagnetised to begin with. Introduce each into a strong magnetic field. Both will become magnetised by induction. Tap each vigorously, so that all the possible molecules may be well shaken into their new positions. Then remove both from the inductive influence of the strong magnetic field, and again tap vigorously. It will be found that the soft iron has lost nearly all its magnetic polarity,

whereas the hard iron has retained, and will continue to retain, a very marked magnetic polarity. In short, by this treatment, the hard iron has been made a permanent magnet.

By substituting a steel or hard iron rod for the soft iron rod in the experiment of § 13, fig. 24, we may show that the steel or hard iron responds less readily than the soft iron to the inductive influence of the magnetic field. The soft iron is easily magnetised; but the magnetisation is almost wholly temporary, and is mostly lost when the magnetising force ceases to act upon it. On the other hand, the hard iron responds less readily to the magnetising force. So long as the magnetic field acts upon it, it shows less magnetic power than the soft iron in the same circumstances. But when it is taken out of the influence of the inducing field, its magnetisation persists in great measure. It is usual to speak of steel and hard iron as possessing *great* **coercive force,** and of soft iron as possessing *small* **coercive force.** The term is not a well-chosen one. As emphasising the one aspect, a much better term is **Retentivity,** or power of retaining whatever magnetic change has been induced; and the greater the retentivity of any substance, the less pronounced is its response to a magnetising force acting upon it.

Magnetic needles, such as are used for the mariner's compass, and all magnets whose *permanency* is the chief desideratum, must be constructed of material of high retentivity.

Material of low retentivity is necessary in instruments—such as dynamos, transformers, &c.—in which magnetisation of a temporary and often rapidly-changing character is required.

§ 19. **Methods of making Magnets.**—Before the discovery of electro-magnetism (§ 83), the only way of developing magnetic polarity in a piece of metal was by treating it by means of permanent magnets. Some of the simpler modes of doing this have been already touched upon (as in § 17).

A simple and effectual way of magnetising a rod of steel or iron is to place it in the usual position of the armature of a strong horseshoe magnet, as in fig. 35, and then slide it to and fro in the line of the poles. Each portion of the rod passes successively into the strongest field, and a fairly-powerful magnetisation is induced in the rod as a whole.

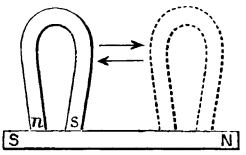


Fig. 35.

By laying the rod to be magnetised with its ends resting on the dissimilar poles of two bar magnets, and by moving the horseshoe magnet to and fro along it, we obtain still better results. Instead of the horseshoe magnet, two separate bar magnets may be used, their opposite poles being kept slightly apart by a small piece of wood, and then moved to and fro as a whole along the rod. This method is called **Magnetising by Double Touch.**

Magnetising by Separate Touch is probably the best of all methods for magnetising by means of permanent magnets. The rod to be magnetised is laid with its ends resting on the dissimilar poles of two bar magnets (see fig. 36). Two other bar magnets, N′ S′, are then laid with their opposite poles resting on the middle of the rod, each pole being the same as the pole of the supporting magnet nearest it. The magnets are then steadily and simultaneously drawn in opposite directions to the ends of the rod. After being lifted away, the same poles are brought back again to the centre of the rod, and the same operation is gone through. A few steady double strokings made in this manner, first on one side of the rod, and then on the other, quickly induce in it a fairly uniform permanent magnetisation. For ordinary purposes, the supporting magnets may be dispensed with.

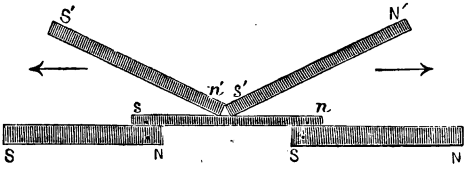


Fig. 36.

It should be remembered that, in all such operations, **induction** is the fundamental principle involved. It is not by *transmission* of magnetic power that the permanent magnet magnetises the metal. The magnet simply brings its magnetic field to act upon the molecules of the rod. These are compelled to take up new positions; and if the retentivity is sufficiently strong, these new positions are in great measure retained.

Other important effects of magnetic induction, in other substances as well as in iron, will be discussed later under Electro-magnetism. At this stage it is well to warn the reader that magnets need not necessarily be steel or iron. The one evidence of a magnet is the magnetic field associated with it; in this sense, as we shall see later, every electric current is magnetic.

EXERCISES.

1. Three precisely similar magnets are placed vertically, with their lower ends on a horizontal table. Iron-filings are scattered over a plate of glass, which rests on their upper ends, two of which are north poles and the third a south pole. Give a diagram showing the forms of the lines of force mapped out by the filings. (1889)

2. It is suspected that a magnetised bar of steel has consequent poles. How would you ascertain whether this is so or not? (1892)

3. Two bar magnets are laid on the table at right angles to each other, so that the axis of one passes through the middle point of the other. They do not touch. A sheet of cardboard, over which iron-filings are uniformly scattered, is placed above them and tapped. Draw a picture showing the appearance which the filings present. (1892)

4. An iron ball is held over a pole of a horseshoe magnet. Will the attraction exerted on the ball be altered if the poles of the magnet are connected by a soft iron keeper, and if so, in what way, and why? (1889)

5. A compass needle is deflected 15° from the meridian, when a bar magnet is placed on the table some distance away. Will the deflection be altered if the poles of the magnet are connected by a bent iron rod? Give reasons. (1890)

6. A test tube is filled with steel-filings, and then stroked several times in the same direction with one pole of a strong magnet. How will the filings behave?

7. A key hangs at the end of a bar magnet which is held horizontally. A piece of iron is set to the end, so as to continue the bar magnet beyond the key, and the key falls off. Explain the phenomenon.

8. A magnet can support a chain consisting of three pieces of soft iron set end to end, but cannot support three equal-sized pieces of steel similarly placed. Why?

9. A steel knitting-needle is stroked from the middle to the ends by the *similarly magnetised ends* of two bar magnets. How will it be magnetised?

10. A rod of iron, when brought near to a compass needle, attracts one pole and repels the other. How will you ascertain whether its magnetism is permanent, or is due to temporary induction from the earth? (1891)

SUMMARY.

(13) Magnetisation by Induction. Attraction between a magnet and a piece of iron fundamentally identical with attraction between two magnets. Iron magnetised inductively in the direction of the field.

(14) Simple Illustrations of Magnetic Induction.

(15) Pictorial Representation of the Magnetic Field by means of Iron-filings.

(16) Induction on the Molecular Theory. Lines of force imagined as continuing through the magnetised iron.

(17) Magnetic Induction a fundamental property of the Magnetic Field. Induction in the earth's field. Induction may reverse the natural polarity of a permanent magnet. Induction greatest near the poles of a magnet.

(18) Coercive Force or Retentivity. Comparison of hard and soft iron.

(19) Methods of making Magnets.

CHAPTER IV.

COMPARISON OF MAGNETIC FIELDS AND OF MAGNETIC MOMENTS.

§ 20. **Comparison of Magnetic Fields.**—A very limited experience suffices to show that magnets differ greatly in the degree in which they produce magnetic effects.

As in the first experiment of § 9 (see fig. 17), set a bar magnet, A, to the east of a compass needle in the end-on position. The needle will set itself in the direction of the resultant magnetic field. Let us adjust the distance of the magnet, so that the needle points in a direction making 45° with magnetic north, as shown in fig. 37. Now we know (§ 11) that, in the configuration shown, the field due to the magnet acts in the axial line—that is, in the direction OW. The field due to the earth acts constantly in the direction ON. Since the magnetic needle sets itself so as to bisect the angle between these directions, we conclude that the two component fields must be equal. We thus find a definite point in the axial line of the magnet, at which the magnetic field associated with the magnet is of the same intensity as the horizontal field due to the earth. Measure the distance of this point from the centre of the magnet.

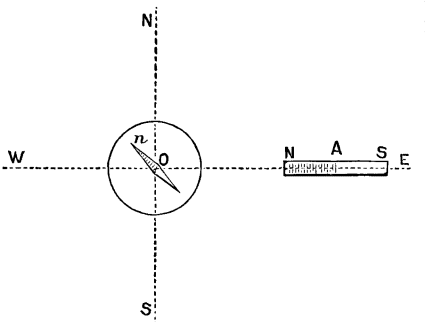


Fig. 37.

Remove the magnet A, and make exactly the same experiment with another magnet, B, adjusting its position so as to have the same deflection of needle—namely, 45°. Measure, similarly, the distance of its centre from the needle. Proceed in this manner with as many more magnets as may be convenient. The distances at which the different magnets must be placed so as to produce this deflection of 45° in the needle will be found to vary considerably. What is called the *weaker* magnet must be placed *nearer*, and the *stronger* magnet at a *greater* distance. Thus we can easily arrange a number of magnets in the order of their strengths. It

should be mentioned that any other angle of deflection may be chosen besides that of 45°. But when this angle is chosen, we not only compare the magnets amongst themselves, but we find for each magnet a definite point in its neighbourhood at which its field is equal in intensity to the horizontal field of the earth.

A more rapid way of simply comparing the magnets amongst themselves is to set each in succession at exactly the same distance from the needle. The deflections will then be found to vary; the *greater* deflection being produced by the *stronger* magnet.

As in the preceding experiment, find the position *east* of the needle, at which the magnet A, set end on, will produce a deflection of 45°. Keeping A in this position, introduce the magnet B to the *west* of the needle, and adjust it so that the needle is brought back to zero (see fig. 38). Then it is clear that the fields due to A and B, acting as they do at right angles to the direction in which the needle is now pointing, must be *equal and opposite*, so as to annul each other's effect. If A is now removed, the needle will again make an angle of 45° with the direction of magnetic north, but on the opposite side of it, as compared with the first deflection produced by A acting alone. If B is reversed so that its poles interchange positions, the needle will be deflected to the same amount in the other direction.

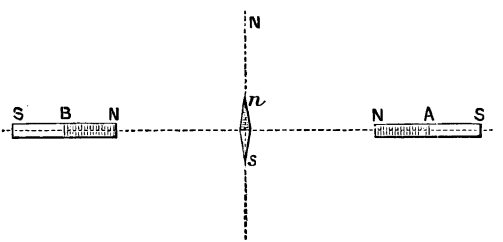


Fig. 38.

§ 21. **The Tangent Law.**—Having found the positions *on opposite sides* of the needle, for which the magnets A and B, each acting singly, produce the same deflection of 45°, let them

act in those positions both at the same time. The arrangement is shown in fig. 39, in which, for clearness, the graduated circle is shown greatly enlarged. In this case the magnets supplement each other's action, each producing at O, and in direction OW, a field equal in intensity to the earth's horizontal field. Thus the *total field* in direction OW is **twice** the *earth's field*, which acts in direction ON. Now let OP be the direction in which the needle points. The angle NOP is found to be nearly $63\frac{1}{2}°$. Draw P*n*, P*w*, perpendiculars to the lines ON, OW. Then it will be found that O*w* is **twice** O*n*. Now O*w*/O*n* is the same as P*n*/O*n*, a ratio which is called the tangent of the angle PO*n*. In this case, when the ratio of the fields is 2, the tangent of the angle of deflection is also 2. In the case considered in § 20 (fig. 37), the two fields were equal, and the angle of deflection was 45°, whose tangent is unity.

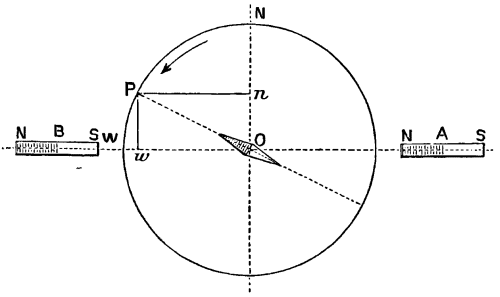


Fig. 39.

This law is general, and may be illustrated by further experiment. For example, let NO*a* (fig. 40) be the deflection due to the action of A alone, and NO*b* that due to the action of B alone, the sole condition being that the field due to each magnet acts in direction OW. Through N draw a line parallel to W, meeting O*a*, O*b*, produced in the points P and Q. Take PR equal to NQ, so that

$$NR = NP + NQ.$$

and therefore

$$NR/ON = NP/ON + NQ/ON.$$

Draw O*c* through the point R; then the angle NO*c* will be found to be the deflection produced when A and B in the given positions act simultaneously upon the needle. Now the ratios just given

are the tangents of the angles NO*c*, NO*a*, and NO*b* respectively. Hence the following very important rule, enabling us at once to compare magnetic fields quantitatively one with another.

Under the combined influence of the earth's horizontal field and any other horizontal field whose direction is perpendicular to the direction of magnetic north, a magnetic needle is deflected from the line of magnetic north through an angle whose tangent is equal to the ratio of this field to the earth's field.

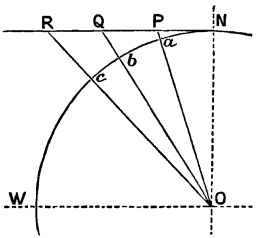


Fig. 40.

Suppose, for example, that we adjust the magnet A in fig. 37 so as to give in succession deflections 30°, 45°, 60°. Then we at once conclude that the fields due to A in these several positions are respectively 0·577, 1, and 1·732 times the earth's horizontal field; for these numbers are the tangents of the angles named.

The law just given is of great importance in the study of electromagnetic phenomena, and especially in the measurement of electric currents.

§ 22. **Variation of Field in the Neighbourhood of a Magnet.**—In comparing magnetic fields in the manner just described, we must know that the *directions* of the fields to be compared are perpendicular to the direction of the earth's field. This is an all-important condition; otherwise the simple tangent law is inapplicable.

Now (see Chap. II., § 11, fig. 21) the field associated with any bar magnet is, at every point in what has been called the equatorial plane, anti-directional to the magnetic axis. Hence, if we place a magnet to the south or north of the magnetic needle, and with its axis pointing east or west, the additional field so made to act on the needle will be perpendicular to the direction of magnetic north. One configuration is shown in fig. 41. Here the magnet produces in the position occupied by the needle a field directed towards E. The needle sets itself in the direction of the resultant of the two fields; and the tangent of the angle of deflection measures the ratio of the superposed field to the earth's horizontal field. In this position the deflecting magnet is said to be set 'broadside on' to the needle.

In the 'end-on' position the magnet lies to the east or west of the needle, and the axis passes through the needle. In the 'broad-

side-on' position the magnet lies to the north or south of the needle. In both the magnet points either east or west.

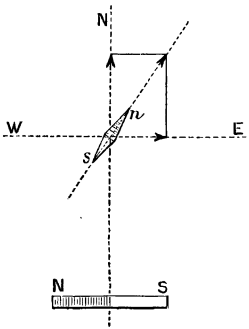


Fig. 41.

Set a good magnet, about eight inches long, *end on*, with its centre at a distance of two or three feet from the needle. Note the deflection. Next set it *broadside on*, with its centre at the same distance from the needle. Again note the deflection. The second deflection will be found to be distinctly smaller than the first. If we take the tangents of the deflections, these will be in the ratio of two to one. Hence we draw the following conclusion:

The intensity of the magnetic field at any external point in the axial line of a good bar magnet is very accurately double that at an equidistant point in the equatorial plane, provided the distance of the points be not too small.

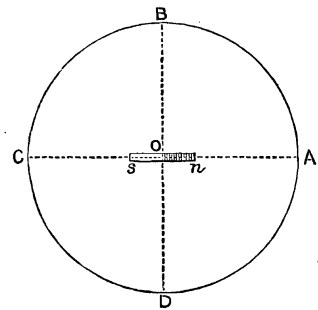


Fig. 42.

Thus let O (fig. 42) be the middle point of the bar magnet *sn*. With centre O and any radius greater than (say) twice the length of the magnet, draw the circle ABCD. The magnetic field at A or C is double that at B or D.

It is a simple matter experimentally to determine how the intensity of field varies from point to point in the axial line of a magnet. We have merely to place the magnet 'end on' at different distances from the needle, and note the deflections produced. If the magnet were uniformly magnetised, and if the real position of the axis of the needle were accurately known, one measurement at each distance would suffice. But such an ideal magnet is not realisable. It is expedient, therefore, to place the magnet first to the east and then to the west at the same distance from the needle, and in each such position to make two observations, with the magnet pointing, (1) towards the needle, and (2) from it. The mean of the four slightly different deflections so obtained will give the most probable value of the deflection for that particular

distance. By experiments of the kind just indicated, the following law may be easily established.

When the distances are not too small, the intensity of the field at any point in the axial line is inversely as the cube of the distance from the centre of the magnet.

For example, set a good strong magnet end on, at a distance of (say) thirty inches from the needle, and note the deflection. Next move it out to a distance of forty inches, and again note the deflection. The tangents of the angles of deflection in these two cases will be in the ratio of $(\frac{1}{30})^3$ to $(\frac{1}{40})^3$, or as 64 : 27.

§ 23. **The Law of the Inverse Square.**—The law of the inverse cube, just enunciated, holds for distances, as measured *from the centre of the magnet*, which are greater than three or four times the length of the magnet. At nearer distances the field does not increase quite so rapidly as the distance is diminished; and the law becomes very approximately that of the inverse square *of the distance from the nearer pole* when the distance is made distinctly smaller than half the length of the bar magnet. In this case the action is reduced almost entirely to that of the nearer pole, the action of the farther pole being in comparison unimportant.

One simple way of showing that, in the vicinity of one pole of a magnet, the field varies inversely as the square of the distance, is by means of oscillations, the significance of which may be illustrated by the following experiment.

Suppose we have discovered by end-on experiments with a magnet, A (as in fig. 37), the distance (measured from the centre of the magnet) at which it exerts on the needle a magnetic force equal to that of the earth. Then set the magnet at this distance to the south of the needle (fig. 43), and pointing towards it. Since the field due to the magnet is co-directional with that due to the earth, the needle will not be deflected. *But we know* that the field *now* acting on the needle is twice that of the earth. Let the needle be set oscillating about its position of rest, and count the number of oscillations in one minute,* or, better, five minutes. Remove the magnet to a far distance, and again count the number of oscillations made by the needle in the same interval of time. In the second count, the needle is influenced only by the earth's field. Now if we square the number of oscillations made in each count, we shall find that the ratio of these squared numbers is as 2 to 1.

* For satisfactory work, it is necessary to suspend the needle by a long silk fibre; even with the best agate and steel point pivoting, the friction is considerable, and soon destroys the oscillatory motion.

For example, with the magnet in position, suppose that there were 34 oscillations per minute; then, with the magnet removed, there will be almost exactly 24 per minute. The corresponding fields are in the ratio of $34^2 : 24^2 = 17^2 : 12^2 = 289 : 144$, which is almost exactly 2 : 1. By putting a second magnet at the proper distance to the north of the needle, we increase the field threefold, and the number of oscillations in the ratio of $\sqrt{3}$ to 1.

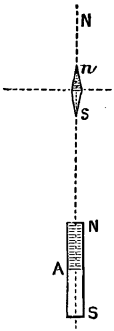


Fig. 43.

The truth here illustrated is that when a given magnetic needle oscillates in a magnetic field, the strength of the field is proportional to the square of the number of oscillations made by the needle in a chosen interval of time.

Take now any bar magnet, and mark the position of its poles as indicated by the convergence of lines of iron-filings sprinkled over it, as in the experiments of § 15 (fig. 30). Set the magnet vertical with one pole at the same level as the magnetic needle, and to the north or south of it. Adjust it at different distances, and count the number of oscillations made by the needle for each position of the magnet. One position is shown in fig. 44. Subtract from the square of each number of oscillations the square of the number of oscillations obtained when the magnet is quite removed. The differences so found will be proportional to the magnetic fields due to the magnet in the corresponding positions. For small distances these fields will be found to be inversely as the squares of the distances. Suppose, for example, that the distances, measured from the middle point of the needle, are 2, 3, and 4 inches respectively. Then the experiment will bring out the fields as nearly proportional to $\frac{1}{2^2}, \frac{1}{3^2}, \frac{1}{4^2}$, or as 36 : 16 : 9.

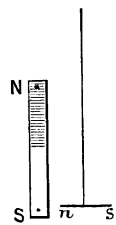


Fig. 44.

Coulomb established the laws of attraction and repulsion between magnetic poles by experiments with the Torsion Balance, the principle of which is explained in its application to the corresponding electrostatic problem (§ 48). The essence of the method consists (1) in the use of two long narrow bipolar magnets, with well-marked polar regions so small that any probable error in the estimation of their positions is insignificant; (2) in the arrangement of these, so that their mutual action may be regarded as almost wholly due to the action between two of the poles. This latter

condition is fulfilled by placing the magnets in the positions of the insulating rods and balls in the corresponding electrostatic experiment, and by having the distance between the effective pair of poles in each observation small compared to the distance between any other pair.

§ 24. **Simpler Results of the Mathematical Theory.**—These experiments suggest that in any magnet each pole, considered as acting alone, will attract or repel another pole with a force which varies inversely as the square of the distance. This law, properly interpreted, satisfactorily explains all the ordinary phenomena of action between magnets.

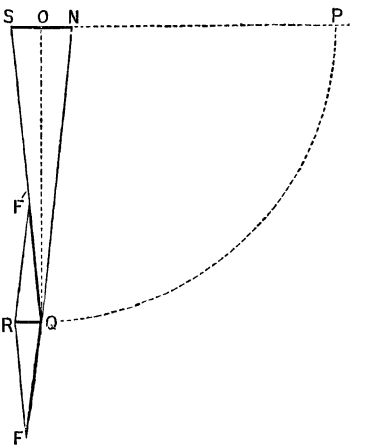


Fig. 45.

Thus the two experimental laws of § 22 are readily deduced.

Let P be a point in the axial line at a distance great compared with the length of the magnet, or, more accurately, the distance between its poles. By hypothesis, the pole N exerts a force proportional to $1/NP^2$, and the pole S exerts an oppositely-directed force proportional to $1/SP^2$. The resultant of these forces is therefore proportional to

$$\frac{1}{NP^2} - \frac{1}{SP^2} = \frac{SP^2 - NP^2}{(NP.SP)^2} = \frac{(SP-NP)(SP+NP)}{(NP.SP)^2}$$
$$= \frac{2SN.OP}{OP^4} = \frac{2SN}{OP^3}$$

approximately, if SN is so small compared to OP, that we may regard $OP^2 = NP.SP$. Thus we deduce the law of the inverse cube.

Again, take Q in the equatorial plane, so that OQ = OP. Now N and S are equidistant from Q. Hence they will exert *numerically* equal forces acting along the lines QN and SQ. If QF represents the force due to N, QF′ will represent the force due to S. Complete the parallelogram of which QF and QF′ are sides, and draw the

diagonal QR. QR represents the resultant force. But by similar triangles

$$QR : QF = SN : NQ, \text{ or } QR = \frac{SN}{NQ} . QF.$$

Now, by hypothesis, QF is proportional to $1/NQ^2$. Hence QR is proportional to $\frac{SN}{NQ^3} = \frac{SN}{OQ^3}$ approximately. Thus the magnetic field at Q is approximately half that at P.

If the point P be near the magnet, we cannot put $OP^2 = SP.NP$. But in this case, experiment shows that the field at P is proportional to the quantity $\frac{1}{NP^2} - \frac{1}{SP^2}$.

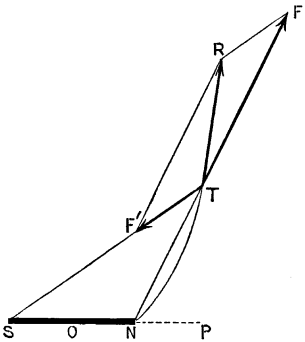


Fig. 46.

To find the field at any point T, draw NT and ST, and take in these directions TF and TF′ equal to the inverse squares of the distances NT and ST—that is,

$$TF = \frac{1}{NT^2}, \quad TF' = \frac{1}{ST^2}.$$

These are proportional to the forces exerted by N and T respectively. Complete the parallelogram of which they are the sides, and the diagonal TR will represent to the same scale the field both in direction and magnitude at the point T.

§ 25. **Magnetic Moment.**—We have seen that at equal distances from two given magnets, the fields in general differ. The magnets themselves differ in power, or, as it is more definitely expressed, in **Magnetic Moment.**

The best conception of the meaning of magnetic moment is got in this way. Let a magnet, freely suspended, be pointing in the direction of the earth's field. By application of suitable forces, turn the magnet round till it points at right angles to the earth's field. To do this, a certain amount of work must be done; and to sustain the magnet in this position, a definite combination of forces must be made to act upon it.

In fig. 47, let A be the magnet, pointing at right angles to magnetic north, and let S, N be the poles. We may represent the action upon the magnet as compounded of the force, + F,

acting on N, and the equal and opposite force, – F, acting on S. Such a combination of equal and opposite forces forms a *Couple*. To balance them, an equal and opposite couple must act upon the magnet. This may be produced by the torsion of the suspension, or by the tension of strings pulling horizontally on the magnet. However produced, it must have a definite moment, measured by the product of F into the distance SN. The moment of this couple depends on the strength of the field and on the quantity known as the magnetic moment of the magnet, and is proportional to each.

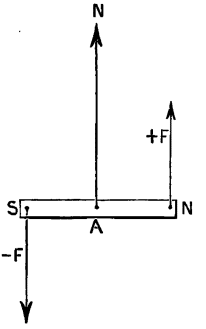


Fig. 47.

If two magnets of exactly the same weight, size, and shape are set oscillating in the earth's field, the magnetic moments are proportional to the *squares* of the number of oscillations executed in a given time. The greater the magnetic moment, the quicker the oscillations, other things being the same.

Again, if any two magnets are similar and similarly magnetised, the fields at equal distances from each are directly as the magnetic moments. Thus, in the experiment suggested in § 20, in which magnets are compared by setting each in succession end on at exactly the same distance from a magnetic needle, a direct comparison is instituted between their magnetic moments. For example, if the one magnet produce a deflection of 12°, and the other a deflection of 23°, the magnetic moments of these magnets are in the ratio of the tangents of these angles—namely, as 1 to 2 approximately.

The measurement of magnetic quantities depends on the dynamic definition of the **Unit Pole**, which is implied in the following statement. The ideal bar magnet, long, narrow, and uniformly magnetised, may be regarded as equivalent to a certain quantity of positive magnetism $+ m$ concentrated at the north pole, and an equal quantity of negative magnetism $- m$ concentrated at the south pole, each quantity being supposed to repel any like quantity,

or attract any unlike quantity, with a force equal to the product of the quantities divided by the square of the distance. Thus, two unit north poles at unit distance apart will repel each other with unit force. Again, any *unit pole* placed in a magnetic field of *unit intensity* will be acted upon by *unit force.* Otherwise, the force acting on unit pole in any field is the measure of the intensity of the field; and in general the force acting on the pole $+ m$ in the field H is measured by the product Hm. This is the quantity represented by F above (see fig. 47). Consequently the couple acting on the magnet when it is set perpendicular to the field H, is measured by the quantity Hml, where l is the distance between the poles. Thus, we recognise in the product ml a quantity proportional to the magnetic moment. We define this product as being equal to the magnetic moment. Hence, if K be the magnetic moment of a magnet, and H the field in which it is, KH is the dynamic couple acting on the magnet when it is set at right angles to the field. By a simple extension of the investigation of § 24, we find for the field at a point in the axial line whose distance, r, from the centre of the magnet is great compared with its length, the expression

$$H' = \frac{m}{(r - \frac{1}{2}l)^2} - \frac{m}{(r + \frac{1}{2}l)^2} = \frac{2lm}{r^3} \text{ approximately } = \frac{2K}{r^3}.$$

EXERCISES.

1. An astatic combination of two magnets is injured, so that the magnets are at right angles instead of parallel to each other. If it be suspended as usual, what position will it assume with regard to the magnetic meridian? Illustrate your answer with a diagram showing the forces which act upon the magnets. (1888)

2. A bar of soft iron, AB, is placed horizontally east and west, the east end A being about four inches to the west of the north-seeking pole of a compass needle. The end A being fixed, B is raised until the bar is vertical. How is the needle affected by the bar when in its original and final positions? (1889)

3. A bar magnet, 10 inches long, is placed in the position of AB in Exercise 2, and raised in a similar manner to the vertical position. For which position of the bar magnet will the needle show the greater deflection? If the near end of the bar magnet be placed at a distance of 10 inches from the needle, will a similar result be obtained?

4. Show how, with four magnets set round a magnetic needle, we may produce a field at the position occupied by the needle equal to five times the earth's field. What objection is there to placing any

two of these magnets in positions having the same direction relatively to the needle—for example, both to the north of the needle?

SUMMARY.

(20) Comparison of Magnetic Fields. Field due to one magnet balanced by field due to another.

(21) The Tangent Law. Experimental illustrations.

(22) Variation of Field in the neighbourhood of a Magnet. 'End-on' and 'Broadside-on' positions. Conditions under which the Law of the Inverse Cube holds.

(23) The Law of the Inverse Square. Comparison of fields by oscillations.

(24) Simpler Results of the Mathematical Theory.

(25) Magnetic Moment. Magnetic moments compared by oscillations or by deflections. The unit magnetic pole. Measurement of magnetic quantities.

CHAPTER V.

TERRESTRIAL MAGNETISM.

§ 26. **The Earth is a Magnet.**—This truth was clearly enunciated so long ago as 1600 by Dr Gilbert, whose book, *De Magnete*, constitutes the first really scientific treatise on magnetic phenomena.

Consider first the behaviour of an ideal magnet, the position of whose magnetic axis (§ 3) is accurately known, and which is suspended or supported at its centre of gravity. If far removed from the neighbourhood of any other magnetic influence, such as magnets or large masses of iron or steel, this ideal magnet will set its axis *accurately* in the direction of the earth's magnetic force. This magnetic force has a definite intensity and a definite direction. To determine the latter, imagine a vertical plane drawn through the magnetic axis of the magnet, and let *sn* be the trace of this plane on the floor. Draw also SN, the trace on the floor of the geographical meridian plane—that vertical plane, namely,

which passes through the true geographical north and south points of the horizon. By analogy the former plane is known as the plane of the Magnetic Meridian. The direction *sn* is the direction of **Magnetic North**; and the angle between *sn* the direction of *magnetic north* and SN the direction of *geographical north* is called the **Magnetic Declination.** By mariners it is usually called the **Magnetic Variation.**

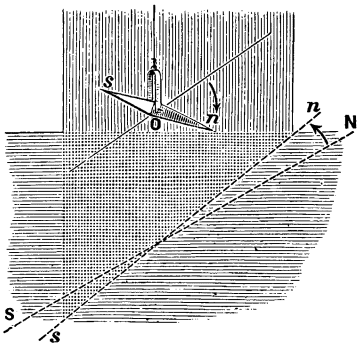


Fig. 48.

Again, draw through O, a point in the magnetic axis, a horizontal line in the plane of the magnetic meridian. The axis of the ideal magnet will make a certain angle with this horizontal line. This angle is called the **Magnetic Dip** or **Inclination.** These two angles, the Declination and the Dip, completely determine the *direction* of the magnetic force at any locality. They constitute two of the three **magnetic elements.** The third magnetic element must give us directly or indirectly the other feature of the magnetic force—namely, its **Intensity.** No completely satisfactory method has been discovered for measuring with sufficient accuracy the intensity of the **Total Force** of the earth's field at places where the dip has a value differing from zero. On the other hand we can, except in high latitudes, measure with great accuracy the **Horizontal Force**—that is, the horizontal component of the Total Force. Consequently, the intensity of the *Horizontal Force*, or simply the *Horizontal Force*, has been chosen as the *third magnetic element.* From it we may calculate, by trigonometry, the Total Force when the dip is known.

There are other ways of representing the properties of the earth's magnetic field. These have their special advantages;

but, practically, the method just described is the most serviceable. We proceed to consider in some detail: (1) The Declination; (2) The Dip; (3) The Horizontal Force.

§ 27. **The Magnetic Declination** (or **Variation**).—This has been virtually defined as the angle between the meridian plane and the vertical plane parallel to the magnetic axis of a free-hanging magnet. Practically, this angle is determined by a magnet suspended, or pivoted, so as to have freedom of motion in a horizontal plane. It is given more or less accurately by every mariner's compass, provided the direction of true geographical north be known.

The direction of true north is obtained very readily by observation of the Pole-star. A glance at a pocket-compass will then suffice to show that in this country the magnetic needle deviates considerably from the direction of true north. Its mean value over the British Islands is at present about 20° west of north (N. by W.$\frac{3}{4}$W., or nearly NNW.).

To the navigator, a knowledge of the declination at the locality where he happens to be is indispensable; for it is by compass that he steers. The necessary information he gathers from Variation Charts, which show in sufficient detail the value of the declination at all accessible localities.

The accompanying chart of the world (fig. 49), which is simply a reduced variation chart, will serve to give a broad idea as to the manner in which this element varies from point to point of the earth's surface.

The curved lines represented on it are called **Isogonic Lines,** or **Lines of Equal Magnetic Declination.** Each line passes through points at which the declination has the same value. Thus one isogone of 20° W. passes through Great Britain. Another isogone of 20° W. passes across Hudson Bay, meets the first named off the west coast of Africa, and continues its south-easterly trend across South Africa. Thus the declination has the value 20° W., not only in Great Britain, but

also in Hudson Bay, Nova Scotia, and certain parts of Africa.

The heavy isogonic lines are the Lines of Zero Declination, or **Agonic Lines,** as they are called. At all points in these lines the magnetic needle points towards geographical north. It will be seen that these agonic lines divide the surface of the globe into two main regions. The one region, including the Atlantic, with the whole of Africa and a large part of the Indian Ocean, is characterised by a westerly declination; and the other (with an interesting exception) by an easterly declination. The American Agonic Line ends at what is called the magnetic north pole* of the earth. This pole, lying in the vicinity of the region *towards* which the lines of force, as well as the isogonic lines, converge, is really a south magnetic region. Between the magnetic and geographical poles, and, as it were, in continuation of the agonic line, is the isogone of 180°. Any magnet set there will turn its marked end towards the geographical south instead of towards the geographical north.

The region of westerly declination is more contracted than the other; but, as if to balance this, there is an isolated region of westerly declination situated in the midst of the region of easterly declination. This isolated region lies on the east of Asia, and is enclosed in an oval-shaped agonic line.

§ 28. **The Magnetic Dip.**—This element is measured by a special form of apparatus known as the Dipping Needle, which consists essentially of a needle suspended at its centre of gravity, so as to be capable of motion in the plane of the magnetic meridian. A vertical graduated circle, in front of which the points of the needle move, completes the apparatus. One form of the instrument is shown in fig. 50. The needle is made of a thin flat piece of steel, pointed at both ends. It is supported by a cylindrical axis which rolls on two agate edges, EE. These agate edges

* In terrestrial magnetism the word *pole* is not used in quite the same sense as defined in former sections. The earth's magnetic poles are regions on the surface at which the dip is 90°, and the horizontal force zero.

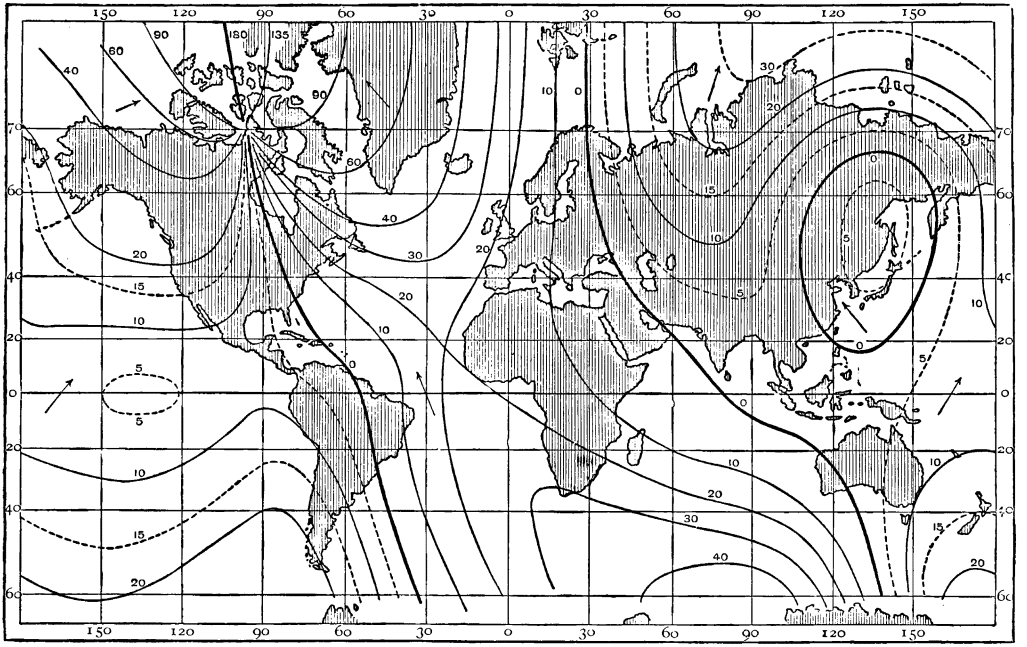


Fig. 49.—Lines of Equal Magnetic Declination, 1885.

must be parallel to the magnetic meridian, and the axis of the needle should be truly centred with respect to the graduated circle.

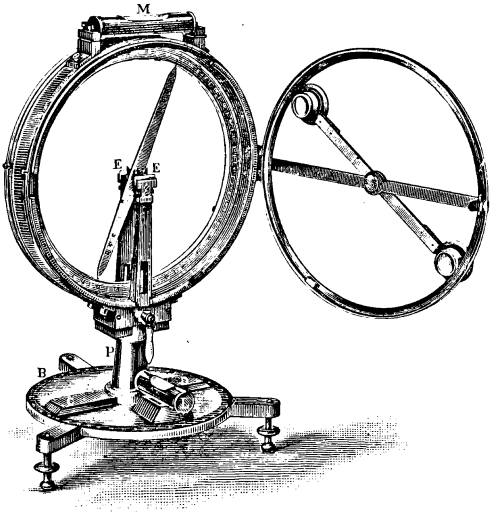


Fig. 50.

The vertical pillar P, which supports the whole apparatus, is movable about a vertical axis, so that the plane of the graduated circle may be made to take any azimuth.*

In making the measurement of the dip, we must take several precautions. The base B must be horizontal, and the vertical circle truly vertical. This is indicated by the levels L, M. This effected, two important operations must be carried out.

(1) *To find the Magnetic Meridian.*—Rotate the circle about the vertical axis until the needle is vertical. This shows that the plane of the circle is *perpendicular* to the magnetic meridian; for in any other azimuth the needle would be influenced by the horizontal component of the earth's field, and would be inclined to the vertical. Note this position on the graduated scale of the base B, and then rotate the whole through a right angle to one side or the other. The graduated circle is now parallel to the plane of the magnetic meridian.

(2) *To measure the Dip.*—Read the scale at the positions occupied by the points of the needle.

If the needle were suspended accurately at its centre of gravity, and had its magnetic axis passing accurately from end to end,

* The azimuth is the angle between any vertical plane and the vertical plane which passes through the north and south points of the horizon.

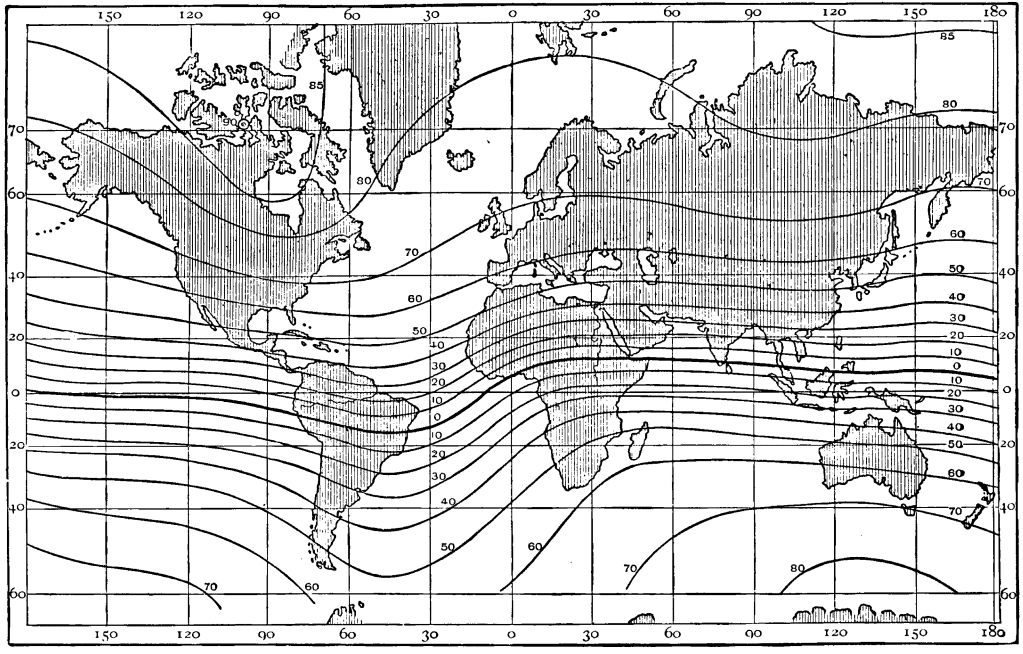


Fig. 51.—Lines of Equal Magnetic Dip, 1885.

and if in addition we were certain that the graduated circle was accurately in the plane of the magnetic meridian, and that all mechanical adjustments were perfect, one pair of readings would suffice. But such perfection of adjustment is impossible; and any one determination of the dip is made to depend on eight distinct pairs of readings. The second pair is taken after the vertical circle has been turned round through 180°. The needle is next turned side for side on its supports, so that the side that was facing east is made to face west. The third pair of readings is then taken; and after another rotation of the vertical circle through 180°, the fourth pair of readings is taken. Finally, the polarity of the needle is reversed by stroking it with the opposite poles of two similar magnets, according to the method of separate touch (§ 19). Four other pairs of readings are taken exactly as before. The mean of these sixteen readings gives the most probable value of the dip.

The manner in which the magnetic dip varies from point to point is shown in the preceding chart (fig. 51). The curved lines are **Isoclinic Lines**, or Lines of Equal Magnetic Dip. Each line passes through points at which the dip has the same value. Thus the dip is 70° in the north of Ireland, the south of Scotland, and the central parts of Russia, Asia, and North America. The thick line marked zero, which intersects the equator to the west of Africa, is the line of Zero Dip, also called the Magnetic Equator, or **Aclinic Line.**

The maximum dip is got when the needle points vertical with either pole down. In the north of Canada, in 97° W. long. and $70\frac{1}{2}$° N. lat., there is such a region. Here the north pole of the magnet points down. It is called the **Magnetic North Pole,** and is indicated in the chart by a circle with a dot inside it. As already noted, the magnetic north pole of the earth is really a *south magnetic region.* The other magnetic pole in the southern hemisphere is inaccessible, but is believed to lie somewhere near 150° E. long. and 73° S. lat. The magnetic poles do not therefore lie at the extremities of a diameter.

The dip has been defined as the angle between the line of force at a given locality and the horizontal plane there—that is, the dips in different latitudes are referred to different planes.

The approximately relative directions in space of the lines of force are shown in fig. 52. This represents the section of the earth along the longitude line which passes through the geographical and magnetic north poles. The directions of the lines of force at latitudes 0°, 30°, and 60° are indicated by arrows, the dotted lines giving the directions of the true vertical at the different points. The angle between any arrow and the corresponding dotted line is the difference between 90° and the dip. AB is the geographical polar axis; S, the 'magnetic north pole,' really a south magnetic pole; and O, O′ are points of zero dip.

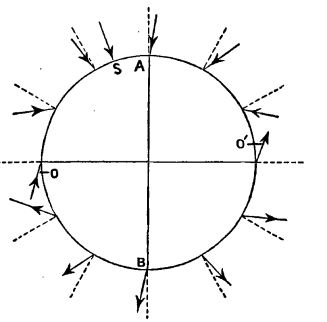


Fig. 52.

§ 29. **The Horizontal Force.**—Suspend a magnet horizontally at the end of a silk fibre, and count the number of small oscillations made in a given interval. Carry the magnet to a different locality, considerably to the north or south of where the experiment was first made, and repeat the experiment. Finally, bring the magnet back to the original station, and repeat the experiment still another time. If the magnetic moment be constant throughout, the number of oscillations will be the same in the first and third experiments; while the number of oscillations in the second experiment will be appreciably different. Now, other things being the same, the intensity of the magnetic force is proportional to the couple acting on the magnet, and this is proportional to the square of the number of oscillations in a given interval (§ 23). Thus we are able to compare the horizontal forces at different localities. We may take the horizontal force at any locality, say Greenwich, as the unit or standard, and express other magnetic forces in terms of it.

For scientific purposes the method described is not sufficiently accurate, and that chiefly for two reasons. In the first place, we cannot be sure that the magnetic moment of the magnet has remained constant. Indeed it is known to be appreciably affected by change of temperature. Then, again, the horizontal force at Greenwich itself is subject to change (§ 30); and it is essentially unscientific to measure quantities in terms of a varying unit.

The measure of the horizontal force in definite units is made by means of a special form of apparatus known as the **Magnetometer.** The essence of the method is to make two distinct experiments as nearly simultaneous as possible.

(1) By means of deflections produced by a magnet of unknown magnetic moment placed end on (§ 22) at different distances from a magnetic needle, we find the ratio of H, the earth's horizontal force, to K, the magnetic moment of the magnet. The approximate formula, given by theory, is

$$K/H = \tfrac{1}{2}r^3 \tan.\ \delta,$$

where δ is the deflection of the needle, and r the distance between the centres of the magnet and the needle (§§ 21, 22, 24, 25).

(2) By means of oscillations of the magnet K in the earth's field, we obtain a measure of the couple KH in terms of the units used in measuring the lengths, masses, and times involved.

Having thus determined the ratio H/K and the product HK of two unknown quantities, we can readily calculate the values of H and K. And thus we determine the horizontal force, H, of the earth, and the magnetic moment, K, of any magnet.

The numerical value of the horizontal force so determined depends on the fundamental units of length, time, and mass which are chosen. If we take the foot, grain, and second as the fundamental units of length, mass, and time, the value of the earth's horizontal force in Great Britain is about 3·7. Since the average dip is about 69°, the total force is a little greater than 10. Again, if we take the centimetre, gramme, and second as the fundamental units, the value of the average horizontal force in Great Britain is about 0·17, and the total force about 0·47.

The variation of the horizontal force over the surface of the globe is shown in the accompanying chart (fig. 53) of **Isodynamic Lines, or Lines of Equal Horizontal Force.** Each line passes through points at which the horizontal force is the same.

The numbers are expressed in terms of the centimetre-gramme-second system of units. The maximum value is in the vicinity of

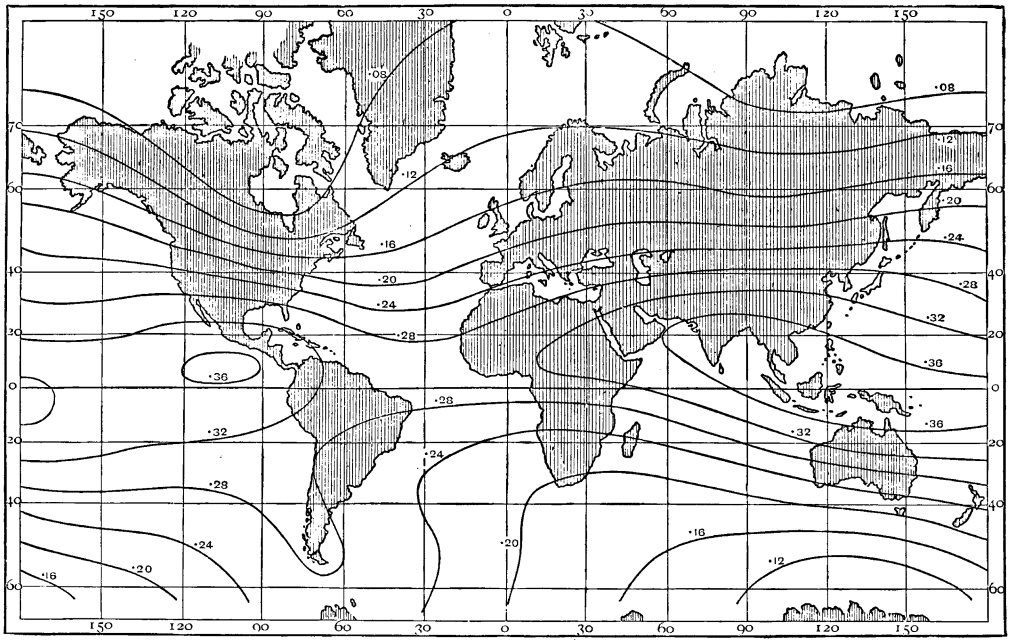


Fig. 53.—Lines of Equal Horizontal Force, 1885.

the magnetic equator, but the points at which the maximum horizontal force is found do not lie on this equator. Zero horizontal force is of course at the poles, where the force is wholly vertical.

The values of the total force may be calculated from those of the horizontal by multiplying the latter by the secant of the angle of dip. The total force is found to increase from ·26 in the vicinity of the magnetic equator to ·7 in the vicinity of the magnetic poles. The maximum values, however, are not exactly at the poles.

§ 30. **Variations of the Magnetic Elements in Time.**—The magnetic force, both as regards magnitude and direction, varies in time; and this is shown by time changes in all the elements.

Compare, for example, the declination charts just given, which are constructed for the year 1885, with the corresponding charts for 1840.* During the lapse of forty-five years, the long agonic lines have, for the greater part of their lengths, moved westward, and the agonic oval has changed form and moved a little eastward.

There is evidence of a periodic variation in the declination extending over some centuries. Thus in 1600 the agonic passed to the *west* of England and through the Cape of Good Hope, the declination in England then being about 8° *east* of north. In 1652 the needle pointed true north in England. In 1700 the *westerly* declination in England had become 6° or 7°, and that at the Cape about 12°. In 1800 the declinations had increased to 23° or 24° west of north at the two places. All this indicates an *eastward* motion of the agonic line. Since 1818 the westerly declination in England has been slowly diminishing, and the agonic line is now slowly drifting westwards. At Greenwich the present annual decrease of the declination is about 7 minutes of arc.

The secular changes in the dip and horizontal force are very slight, and generally take place in opposite directions, so that the change in the total force is still smaller.

The Solar Diurnal Variation is the most easily recognised periodic variation of the magnetic force, and is especially evident in the case of the declination. In all but tropical regions, the declination needle oscillates slightly but distinctly about its mean position for the day, attaining its maximum deviation an hour or two after noon. In the northern hemisphere, the maximum deviation is to the *west* of the mean position; in the southern hemi-

* These are given in Ferguson's *Electricity*, or Chambers's *Encyclopædia*, first edition.

sphere, it is to the east. The range is greater in higher latitudes, and is greater in summer than in winter. The average diurnal variation during a year at Kew is about 10 minutes of arc.

§ 31. The most familiar magnetic instrument is the ordinary compass needle. In the mariner's compass, the needle supports a circular card marked with the 'thirty-two points,' the line joining the south and north points (S and N) being parallel to the magnetic axis of the needle. The pivot on which the magnet and card are balanced stands up from the

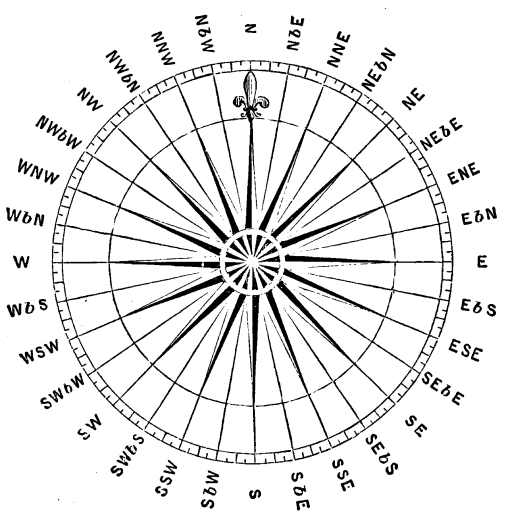


Fig. 54.

bottom of a hemispherical bowl, which the circular card nearly closes. The bowl swings by means of two pivots in a ring, which is similarly supported by two pivots to the outside case. The line joining the one pair of pivots is at right angles to the line joining the other pair. The effect of this double pivoting is to keep the compass-card always horizontal or nearly so, however much the ship may be rolling or pitching. The compass points are shown in fig. 54. There

are evidently 8 points to the quadrant, so that each point corresponds to an angle of $11\frac{1}{4}°$. Sailors measure to quarter-points. Thus NW. by W$\frac{1}{4}$W. means $5\frac{1}{4}$ points, or 59° 3′ 45″ west of north.

The almost universal use of iron in shipbuilding has introduced a further complication into the manipulation of the compass. For, in addition to the variation or declination of the needle, there is a further **Deviation** which is produced by the influence of the ship's iron. This deviation must be found experimentally for every vessel. Its value changes with the course the vessel is pursuing. For, besides the permanent magnetic condition of the ship, there is a temporary magnetisation due to induction in the earth's field. All masses of iron at the earth's surface are, indeed, subject to this inductive action; and although the induction is small, it is not always negligible.

EXERCISES.

1. An iron rod, held vertically, is tapped with a mallet. The upper end is found to repel the south pole and attract the north pole of a compass needle. The rod is now quietly inverted, and the same end (which is now the lower) is tested again. It is then tapped and once more tested. State what results you would expect, and explain them. (1892)

2. A dipping needle can oscillate in the magnetic meridian. A long bar of soft iron, held horizontally in a north and south direction, is brought near to it from the south. How is the inclination of the needle to the horizon affected as the distance between it and the bar is gradually diminished? (1892)

3. A tall iron mast is situated a little in front of the compass in a wooden ship. Explain the nature of the compass error when the ship is sailing in an easterly direction (1) in the northern, (2) in the southern hemisphere. (1891)

4. Given a magnet and the means of suspending it. How will you determine (1) the magnetic meridian, (2) in which direction *north* lies? It is assumed that you do not know which end of your magnet is a north and which a south pole. (1891)

5. The beam of a balance is made of soft iron. When it is placed at right angles to the magnetic meridian, two equal weights, placed in the opposite pans, just balance. Will the weights still appear to

be equal when the balance is turned, so that the beam swings in the magnetic meridian? Give reasons for your answer. (1889)

6. What is meant by saying that the magnetic dip at London is 67° 30′? State in general terms at what places on the earth's surface the magnetic dip is least. (1888)

7. Suppose a magnetic needle to be carried in a circle of a few miles radius round the geographical north pole of the earth. How will the magnetic declination change during one complete circuit? How will it change if the needle be similarly carried round the magnetic pole?

8. Where on the earth's surface does the so-called north pole, or (as some prefer to call it) north-seeking pole, of a magnetic needle point in a generally southerly direction?

9. Consider the changes in the magnetic declination in a voyage from England to Vancouver by way of Suez and Japan.

10. Consider the changes in the dip as we pass along a meridian line from the Cape of Good Hope to the north of Europe.

SUMMARY.

(26) The Earth a Magnet. The general properties of the earth's field at any one locality. Declination, dip, total force, horizontal force.

(27) The Magnetic Declination (or Variation). Isogonic lines.

(28) The Magnetic Dip. The dipping needle. Isoclinic lines.

(29) The Horizontal Force. Magnetometer. Isodynamic lines.

(30) Variation of the Magnetic Elements in Time. Solar diurnal variation.

(31) The Mariner's Compass. The deviation in iron ships.

CHAPTER VI.

ELECTRIFICATION IN ITS SIMPLER ASPECTS.

§ 32. **Electrification by Friction.**—A piece of sealing-wax, or resin, rubbed with woollen cloth, has the power of drawing towards it small shreds of paper or other light bodies. This is the simplest experiment in electricity. It dates from the time of Thales (600 B.C.), a famous Greek philosopher, who observed the phenomenon with rubbed amber or ἤλεκτρον (*elektron*), whence the name electricity. Other sub-

stances, such as vulcanite or glass, act equally well if they are rubbed with a suitable material. Cat's fur, slightly warmed, is very efficient in electrifying vulcanite, or any resinous substance; while silk, amongst simple substances, is often used to excite glass.

If the electrification produced in this way be powerful, a crackling noise is frequently heard, and if the experiment be made in the dark, sparks may be seen. In a dry frosty atmosphere some striking effects are sometimes produced by quite simple means. For example, a vulcanite comb drawn through the hair may become strongly electrified. If passed to and fro just above the back of the hand, it will lift up the small hairs and produce a very peculiar sensation. By stroking a cat's back with the hand, we may electrify both the fur and the hand. The hand, if dry, will attract small shreds; and if any object be brought near the cat's fur, the hairs will be attracted. Here the evidence of the attraction is the motion of the electrified bodies towards the apparently unelectrified body.

When one substance is electrified by friction on another, this other is also electrified. In certain cases this may be shown by the attraction of each for small bodies; but, in general, it can be shown only by an appeal to other properties of electrified bodies. These we proceed to study.

§ 33. **The Two Kinds of Electrification, Positive and Negative.**—Electrify a stick of sealing-wax by rubbing it with cat's fur or flannel, and rest it horizontally in a wire stirrup at the end of a thread, as in fig. 55. Bring any object—the hand, for example—near to the electrified end. This end will swing gently round towards the object, showing that there is *attraction* between the two. But now electrify similarly, by friction with the flannel or fur, the end of a second stick of sealing-wax, and bring it near the end of the suspended stick. The suspended stick will swing away from the other, showing that there is *repulsion* between the two ends.

Instead of the two sticks of sealing-wax, use two glass rods, and electrify them by rubbing them with silk. Put one in the stirrup and bring the other near it, just as in the previous case. Again a distinct repulsion will be observed.

If, however, we bring one of the sticks of rubbed sealing-wax near the suspended glass rod, a very pronounced attraction will be apparent. Similarly, the silk-rubbed glass will strongly attract the suspended and fur-rubbed sealing-wax.

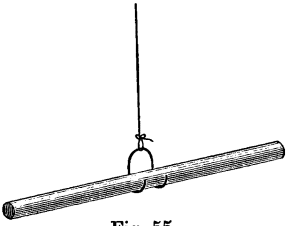


Fig. 55.

We recognise by such experiments that there are two kinds of electrification. When repulsion takes place between two electrified bodies, we say that they are *similarly* electrified. When, however, the bodies are *dissimilarly* electrified, then attraction takes place between them.

It is usual to take resin rubbed with flannel or fur as the type of the one kind of electrified body, and smooth glass rubbed with silk as the type of the other. In the early days of the science these opposite kinds of electrification were accordingly distinguished as *resinous* and *vitreous*. Such nomenclature is misleading, however, for it is possible to get glass to display so-called *resinous* electrification by rubbing it with fur or flannel. Rough or ground glass behaves, indeed, almost exactly like resin when rubbed with either flannel or silk.

The words now used to distinguish these kinds of electrification are *positive* and *negative*.* Thus resin or sealing-wax rubbed with fur becomes *negatively* electrified, while the fur becomes *positively* electrified. Similarly, when smooth glass and silk are rubbed together, the glass becomes *positively*, and the silk *negatively* electrified. We may arrange substances in

* This terminology is peculiarly appropriate, for, as will be proved later, the production of so much positive electrification always implies the production of an equal amount of negative electrification.

order, so that, when any two are rubbed together, the one that comes earlier in the list is positively electrified, and the other negatively electrified. Such lists, which vary considerably in detail, begin usually with fur, flannel, ivory, glass, and end with wax, resin, gutta-percha, while cotton and silk come in between these extremes. As pointed out by Faraday, one piece of cat-skin will differ appreciably from another, while different kinds of flannel occupy very different positions on the list.

§ 34. **Electrification by Contact; Conductors and Non-conductors.**—Suspend a small pith-ball (elder pith is the best) by a dry silk thread from an ordinary retort stand, or as shown in fig. 56. Bring near to it a rod of sealing-wax which has been electrified negatively by friction. The pith-ball is first briskly attracted, but after thorough contact is more briskly repelled. That is to say, it is now in a condition similar to that in which another negatively electrified rod of sealing-wax would be. We infer that the pith-ball has by contact also become negatively electrified. With the pith-ball in this condition, bring a *positively* electrified glass rod near it. It is now strongly attracted; but if allowed to touch the glass, will again be strongly repelled. Contact with the positively electrified glass has brought the pith-ball also into a state of positive electrification.

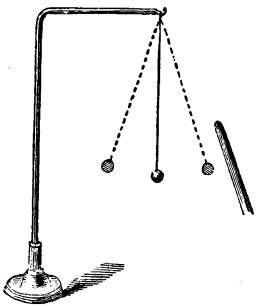


Fig. 56.

Again, suppose we have two pith-balls suspended by separate silk threads (or by cotton threads attached to a dry glass rod) side by side, so as to be in contact. On approach of the electrified rod, they will be attracted, and after contact repelled. Being both now similarly electrified, they will repel one another and diverge as in fig. 57.

Suppose now that we *moisten* the silk thread suspending

the pith-ball (fig. 56), or substitute for it a wire or cotton thread. On repeating the experiment, we shall find attraction as before; but after contact of the ball and rod, *there will be no repulsion.* The ball will either cling to the rod, or drop from it because of gravity.

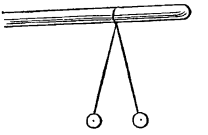


Fig. 57.

The reason of this lies in the character of the suspension. With the dry silk thread, the balls retain the electrical condition into which they have been brought. With the other suspensions named, it is impossible to give to the balls any permanent state of electrification. The dry silk thread is an **Insulator**, or non-conductor; the wire, cotton thread, or moist silk thread, and the wood or metal of the stand, are non-insulators, or **Conductors.**

We may, however, reproduce the original effects of attraction and repulsion by attaching the conducting thread to a dry glass rod. The pith-ball and thread together will form a conductor, and will become electrified as a whole; the glass rod will serve as insulator. If we take hold of any part of the thread with the finger, the pith-ball will soon lose its electrification.

In short, **no body can retain for an appreciable time its electrified condition unless it be separated from the earth by insulating material.**

Thus the glass or resin, when electrified by friction, retains its electrification, being itself an insulator. If, on the other hand, we grasp a metal rod by one end, it is impossible to produce permanent electrification on the other end, either by friction or by contact. The metal is a conductor, and not an insulator, and any electrical condition momentarily produced in the rod vanishes ere we have time to recognise it. But if we attach to the metal rod a glass handle, and hold it by grasping only the glass, then we may electrify the insulated metal rod either by friction with a non-conducting surface or by contact with an electrified body.

All substances which are susceptible of electrification by friction, without any special precautions being taken, other than seeing that they are dry and clean, are insulators to some extent. Such substances used to be called **Electrics;** and the many other substances which had not this property were called **Non-electrics.** The one group, however, is as susceptible of electrification as the other, *if only proper precautions be taken.* In fact, no progress can be made in the study of electricity without the use of electrified conductors supported on or suspended from insulating material.

Silk thread, vulcanite, mica, dry glass, paraffin, and dry gas are all good insulators. As we have seen, the presence of a little moisture causes great deterioration in insulating power. Hence the necessity for thoroughly warming all the glass rods and stands which form so large an item in all electrical apparatus. They must not, however, be made too warm. In a clear, dry, frosty air, electrical experiments can be carried out most effectively. When, on the contrary, the air is very humid, such experiments, if not positive failures, are apt to be unsatisfactory.

In a few of the experiments presently to be described, details as to insulation will be given; but it will in general be understood that electrified conductors are insulated as perfectly as possible.

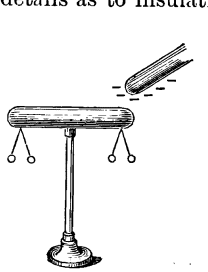


Fig. 58.

§ 35. **Electrification by Induction; the Two-fluid Theory.**—Set an elongated conductor, say a brass rod with rounded ends, on an insulating stand—a well-dried glass bottle will serve the purpose. Under both ends attach by *cotton* threads pairs of pith-balls. Bring an electrified rod of sealing-wax over one end, but not in contact with it (fig. 58). The pith-balls will diverge, thus showing that, by the *mere contiguity* of the electrified sealing-wax, the conductor, consisting of rod and balls, has been brought into an electrical condition.

Remove the sealing-wax without having touched the conductor in any way, and the pith-balls will collapse. Bring back the sealing-wax to its former position, and the pith-balls will again diverge.

While the sealing-wax is held in position, touch the conductor for a moment with the finger. The balls will collapse; but will once more diverge when the sealing-wax is removed. The conductor has been, in fact, *permanently electrified*, without having been brought into contact with the sealing-wax. It has become electrified by **Induction.**

To bring out more clearly the process of induction, place two insulated conductors in contact end to end. Attach pairs of pith-balls to the extremities, and bring an electrified stick of sealing-wax close above the one end (the left in the figure). Without in any way touching the conductors *themselves*, separate the one from the other, and then remove the sealing-wax (fig. 59). The pith-balls will collapse *slightly*, but will remain still divergent. Thus the two conductors are left electrified. Each is said to be *charged* with electricity.

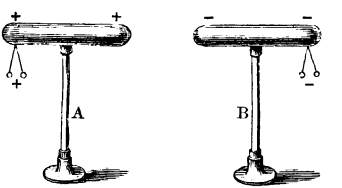


Fig. 59.

Now bring them into contact again, the sealing-wax being still kept at a distance. The balls will completely collapse, and the double conductor will recover its originally unelectrified or uncharged condition.

Go through the same series of experiments again, and bring the two conductors once more into the condition of fig. 59. How shall we test the nature of the electrification on each? To facilitate reference, let A be the one which was nearer to the *negatively* electrified sealing-wax, and let B be the other. Then, if the electrified sealing-wax is brought near to the pith-balls on A, they will be attracted; but if it is brought near to the pith-balls on B, these will be repelled.

These facts, and others presently to be given, are very well co-ordinated in terms of the Two-fluid Theory of Electricity.* It is assumed that there are two kinds of electricity, which may be compared to fluids. These are self-repellent, but mutually attractive. In an unelectrified body they coexist in equal quantities. Electrification is the act of separating these fluids, which are distinguished as *positive* and *negative*, and are represented by the algebraic signs + and −.

In terms of this theory, electrification by friction may be described as an interchange of electricities between the two substances. In the particular case of sealing-wax and flannel, a certain quantity of + electricity is transferred from the sealing-wax to the flannel, and an equal quantity of − electricity from the flannel to the sealing-wax.

When the negatively electrified sealing-wax is brought near any insulated conductor, it attracts + electricity to the nearer end, and repels − electricity to the further end. Both ends become electrified; and thus we explain the divergence of the pith-balls in the experiment of fig. 58. Again, in fig. 59, the electricities having been separated, the conductors A and B are pulled apart. In A, therefore, there is an excess of + electricity, and in B there is an *equal* excess of − electricity. When the sealing-wax is removed to a distance, and A and B are again brought into contact, the opposite electricities combine and completely neutralise one another.

Finally, take the case in which the conductor, in the presence of the negatively charged sealing-wax, is touched with the finger. The conductor is now made a part of the large conductor, the earth. The + electricity is attracted towards the end near the sealing-wax, while the − electricity is repelled to the earth. Removal of the finger disconnects

* This theory was originated by Symmer. Franklin explained all the ordinary facts with equal facility by means of the *One-fluid Theory*. In this the state of electrification was determined by the excess or deficiency of the so-called electric fluid. Both theories fail in taking proper account of the action of the medium separating the conductors.

the conductor from the earth. In the conductor, accordingly, is left a certain quantity of + electricity. On removal of the sealing-wax, this positive electricity, because of the mutual repulsion of its parts, spreads itself over the surface of the conductor, and partly on the pith-balls hanging from it. The distributions of electricities are represented by the algebraic signs + and − in the figures.

§ 36. **The Gold-leaf Electroscope.**—Any instrument for detecting the presence and kind of electrification on a body is called an **Electroscope.** The pith-balls described in the last paragraph constitute such an instrument. A convenient form is the **Gold-leaf Electroscope.**

Two light strips of gold-leaf (EE, fig. 60) hang from the lower end of a metal rod, C, which passes through the cork or stopper, B, of a glass bottle A, and ends above in a plate or knob D. The plate form is the more serviceable. The glass serves partly to prevent air-currents, but chiefly as an insulator. It should have its surface clean and dry before it is used for experiments. The essential part of the electroscope is represented in the T-shaped diagram (*a*).

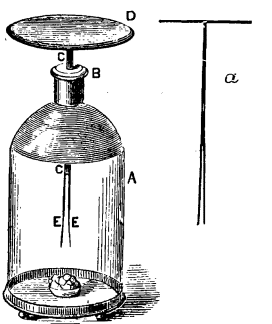


Fig. 60.

Before the instrument can be used as an electroscope, it must be electrified or charged either positively or negatively.

Beat the plate briskly with a piece of flannel, cat-skin, or silk. The gold leaves will gradually diverge, showing a mutual repulsion. This repulsion we explain by saying that the electroscope has become electrified by friction. Again, place a sheet of mica on the plate, and rub it briskly to and fro. Then, without touching the plate, lift away the mica. The leaves diverge. This divergence of the leaves occurs *after the rubbing substance has been removed.* Thus we demonstrate

that conductors as well as non-conductors are electrified by friction; and also that equal and opposite charges are produced on the rubbed surfaces (see below, § 45).

The gold-leaf electroscope is specially useful in the study of induction. Take, for example, the following sequence of experiments, diagrammatically represented in fig. 61.

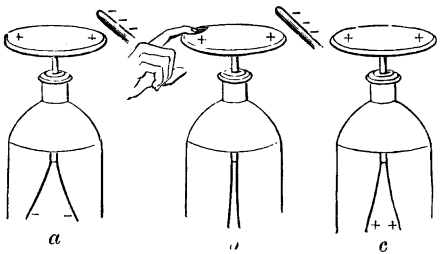


Fig. 61.

(1) Bring negatively electrified sealing-wax above the plate. Induction takes place; the upper part of the electroscope becomes positively, and the lower part negatively, electrified. The gold leaves, being similarly electrified, repel one another and diverge (fig. 61*a*).

(2) Touch the electroscope with the finger, so as to make it part of the huge conductor, the earth. The gold leaves collapse (fig. 61*b*).

(3) Remove the finger, and then remove the sealing-wax. The gold leaves diverge again, and when tested (as below) are found to be positively electrified (fig. 61*c*).

In this condition the electroscope may be used for testing the kind of electrification on any other electrified body.

Thus let a *positively* electrified body—for example, a glass rod rubbed with silk—be brought near the plate. This will act inductively, very much as it would do were the electroscope not charged. That is, *negative* electricity will be induced in the plate, and *positive* electricity in the leaves. But since the whole was at first positively electrified, the first effect will be to diminish the positive electrification of the plate, and increase that of the leaves. Hence, increased divergence will result (fig. 62*a*).

Again, let a negatively electrified body be brought near. Its inductive effect will be just the reverse. Positive electrification will increase in the plate, and diminish in the gold leaves. These will consequently begin to collapse (fig. 62*b*).

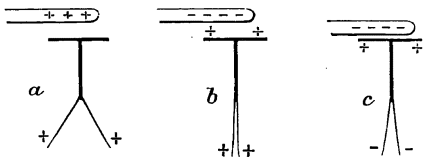
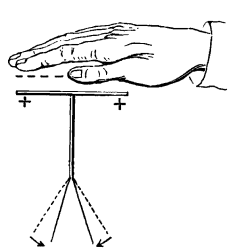


Fig. 62.

As the electrified body is brought nearer and nearer, the gold leaves will collapse more and more; and, if strongly enough electrified, it may be brought so near as to cause the gold leaves to collapse altogether, and then *to diverge again with the opposite kind of electrification* (fig. 62*c*).

What effect will be produced on the approach of any non-electrified body? This question is answered by simply holding the hand over the plate. The leaves will begin to collapse, exactly as if the hand had been negatively electrified. And the hand is indeed negatively electrified on its lower surface—charged inductively because of the presence of the positive electrification in the electroscope. This induced negative charge reacts inductively upon the electroscope, and diminishes the electrification in the leaves (fig. 63). Any non-electrified conductor will produce the same effect; but a non-conductor, devoid of all electrification, will have appreciably no effect.


Fig. 63.

At first, then, an originally unelectrified conductor, brought near the positively charged electroscope, acts very similarly to a negatively charged body. But there is this difference. The uncharged body will never be able to produce the effect shown in fig. 62*c*. It matters not how near it may be brought; it will never cause the gold leaves to fall together and diverge again with the opposite

kind of charge. This experiment shows that there is an inductive reaction upon the charged body. In fact, induction is a mutual action, and affects the electrical condition of both bodies, whether both be originally charged or only one be so.

To charge the electroscope *negatively*, we have merely to substitute a positively electrified glass rod for the sealing-wax, and go through identically the same series of operations as are indicated in fig. 61. Put *positive* for *negative* and *negative* for *positive* all through, and the same description will apply.

§ 37. First rough Ideas as to Degree of Electrification or Amount of Charge.—By experiments of the kind just described, we soon learn that the electrical effect of a charged body is greater at nearer distances. The gold leaves will diverge more and more the nearer the charged body is brought. Or, to return to the simple experiment with the pith-ball, it will be deflected more from its naturally plumb position the nearer we bring the similarly electrified body. Thus the nearer two charged bodies are, the greater will be the attractive or repulsive force between them; and the nearer a charged body is to a second uncharged body, the greater is the inductive effect produced.

A very little experience also shows that a given conductor may produce, under different conditions, electrical effects differing very greatly in degree.

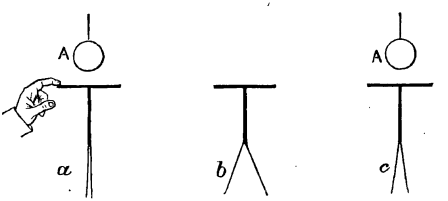


Fig. 64.

Suppose, for example, that we charge an electroscope inductively by bringing a charged conductor, A, to a definite position near it (fig. 64*a*), and then touching the electroscope, insulating it, and finally removing the conductor A (fig. 64*b*). Now let A touch a second insulated conductor, and then bring it back again to

exactly the same position it had relatively to the electroscope. The gold leaves will fall together to a certain extent, but will not collapse completely (fig. 64*c*). The conductor A in its second condition has clearly a smaller inductive effect than it had in the first condition. We know that it has shared its original charge partly with another conductor. Thus we conclude that the conductor A is really less electrified, or has a smaller charge on it in the second condition than it had at first.

Thus, by the inductive action exerted by a charged body on an electroscope under given conditions, we may judge as to the amount of charge on the charged body. Another way of judging of the amount of charge is by means of the attractive or repulsive force exerted by the charged body upon another charged body, as in the simple pith-ball experiment. It is in this way that we get the accurate measurement of charge, as we shall see later (§ 48), when we come to study Coulomb's classical experiment.

§ 38. **Induction the Fundamental Fact of Electrostatic Action.**—In § 34 we discussed electrification by contact. As it appears first to the mind, this mode of electrification consists in the transference of a certain quantity of electricity from one body to another. But before the bodies come into contact at all, there is already electrical action taking place between them. Consider, for example, the simplest case of the pith-ball touched with sealing-wax. As soon as the negatively electrified sealing-wax is brought near enough, separation of electricities takes place in the pith-ball. On the nearer half, positive electricity accumulates; on the farther half, negative. The induced positive charge is nearer to the sealing-wax than the equal induced negative charge, so that there is no difficulty in understanding why there should be, on the whole, attraction. When the two bodies come into

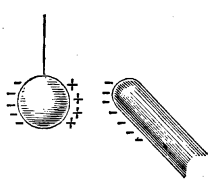


Fig. 65.

contact, positive electricity on the ball combines with as much negative on the sealing-wax. The repulsion between the negative electricities has then its full sway, and the ball is driven off. The negative charge on the sealing-wax is diminished by the amount needed to neutralise the positive electricity induced on the ball just before the two came into contact—that is, by an amount exactly equal to the negative electricity induced on the ball just at the instant of contact, and finally carried off with it. The final result is *as if a certain amount of negative electricity had simply been transferred.* But, if we wish to apply consistently the two-fluid theory, we should consider the negative charge finally developed on the pith-ball as due essentially to induction.

The rapidity with which inductive effects take place when any conductor is brought under the influence of an electrified



Fig. 66.

body is very great. For ordinary-sized conductors of metal the final condition is reached in an extremely short interval of time, practically instantaneously. For example, set an insulated ball as far as possible from the electroscope, and connect it with the plate of the electroscope by means of a long wire also insulated. Bring near the ball an electrified body. At once the gold leaves will diverge (fig. 66). Touch the ball with the finger, and the gold leaves will fall together. Remove the finger, then remove the electrified body, and the gold leaves will again diverge, charged with electricity of the opposite kind to that on the original charged body.

Again, consider the following experiment. First charge the electroscope with (say) + electricity. Then adjust a lath of

wood set on an insulating stand, so that the one end lies just above the electroscope plate, but without touching it. Consequently the positive charge on the electroscope will act inductively on the lath, producing a certain separation of electricities; and the inductively electrified lath will react on the electroscope, and draw a little of the positive charge out of the gold leaves. A slight falling together of the gold leaves will be observed as the lath is placed in position.

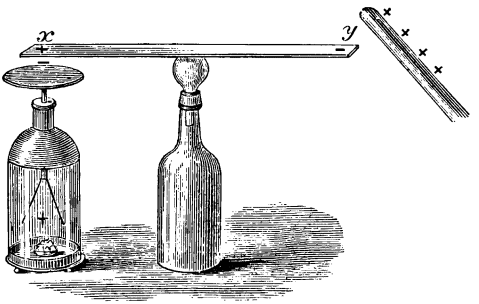


Fig. 67.

Now bring near to the farther end of the lath an electrified body, say a positively charged glass rod. This will act inductively on the lath. On the already existing distribution of electricities, a new distribution will be superposed. Negative electricity will accumulate on the end, *y*, near the glass rod, and positive electricity on the end, *x*, over the electroscope. This in its turn will act inductively on the electroscope, and positive charge will be increasingly developed on the leaves, which will diverge still more (fig. 67).

If, instead of the positively charged body, a negatively charged body were brought near the end of the lath farther from the electroscope, an exactly opposite series of inductive changes would occur, and the gold leaves would, in the first instance, collapse.

Inductive effects may be produced on the human body, which in this respect acts as a conductor. In fig. 68, a boy is shown insulated on two well-warmed tumblers. The one

hand is held over a positively charged electroscope, and the other is presented to the positively charged rod. The gold leaves will diverge because of the presence of the induced positive charge in the hand near it.



Fig. 68.

Numerous experiments of the same character may easily be devised; but enough has been given to bring out the broad features of electrostatic induction.

§ 39. **The Electrophorus.**—There is, theoretically, no limit to the number of bodies which may be charged with one kind of electricity by induction from one and the same originally charged body. The original charge used for producing the induction is not in any way diminished because it is so used. It is only when contact with conductors takes place, or bad insulation exists, that some of the original electrification is lost. One of the most useful instruments for the easy production of electricity is the **Electrophorus,** whose action very well exemplifies what has just been stated. It consists, generally, of three parts—a flat **metallic dish**; a cake of some **resinous material**, or other non-conducting substance easily electrified by friction; a **metallic plate**, provided with an insulating handle. The non-conductor fills the dish or **sole.** One form is shown in fig. 69. The manner of using it is as follows. First, warm the non-conducting cake, which fills the sole D. Secondly, beat it briskly with warm cat-skin, or flannel, until it becomes well electrified. Thirdly, place the conducting plate, P, on the electrified surface, and touch it for an instant with the finger (as shown in the figure). Fourthly, remove the finger, and lift away the

plate by means of its insulating handle, H. The plate will be found to be charged. Present the knuckle to it, and a sharp shock may be felt, and a distinct spark observed. The plate may be charged again and again, after the non-conducting cake has been once excited. The point to be borne in mind is, that although the plate *touches* the resinous surface, it carries away a charge opposite to that which resides on that surface. The reason of this may be fairly well explained on the two-fluid theory. So far as the *kind* of electricity developed is concerned, it matters not whether the plate touches or does not touch the electrified surface, provided only it be held near enough. As simplifying the discussion somewhat, the plate and resin will be supposed not to touch.

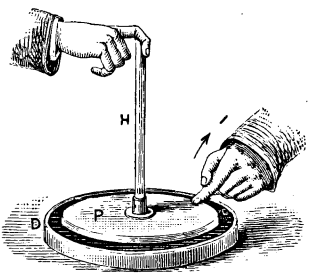


Fig. 69.

While the resinous surface is being electrified negatively, the metal sole rests on the table, and is in connection with the earth. Accordingly, a positive induced charge accumulates on the upper surface of the sole. By its attraction for the negative electricity on the resin, this positive electricity prevents the negative from being removed so easily from the surface as it would otherwise be. The two opposite kinds hold one another partially *bound*. When the movable plate is brought near, inductive effects take place in it also; and when it is also connected to earth, the positive induced charge is left bound on the surface nearer to the resin (see fig. 70). Both sole and plate, being to earth, are part of the same electrical conductor. The plate, however, may be brought much nearer to the electrified surface, so that the induction on it is the greater. When the plate is insulated by removing the finger, and then lifted away so as to be far removed from the influence of the inducing charge, the positive electricity is

no longer held bound, but spreads itself over the whole surface of the plate. The plate becomes an active positively charged conductor.

Practically, the plate is brought down upon the resinous surface. This insures that the distance between them is the smallest possible, so that the inductive effect is the greatest possible. The contact between the two will occur only at a few points. From these a small amount of negative electricity will no doubt be removed; but, because of the bad conducting power of the resin, the negative charge on the parts not touched will not be drawn off. *All parts* of the electrified surface will act inductively, and only a very small fraction of the induced positive charge will be needed to neutralise the negative charges at the points of contact.

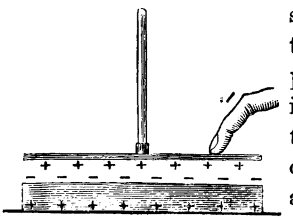


Fig. 70.

The essential conditions for the efficient working of the electrophorus are:

(1) Both sole and plate must be put to earth.

(2) The plate must be insulated just before or just at the instant of being lifted away.

To facilitate the action, it is usual to have a metallic pin passing through the resin and flush with the surface of it, or to have a small tongue of tinfoil attached to the sole, and brought over on to the upper surface of the resin. By either of these means the plate comes at once into metallic contact with the sole as soon as it rests in position on the resin surface. It is not necessary, in these circumstances, to touch the plate with the finger.

A moderately-sized electrophorus, say, six inches in diameter, will give sparks more than an inch long.

§ 40. **Distribution of Electricity on Conductors.**—We have seen how sensitive the gold-leaf electroscope is to the approach of any electrified body, or, indeed, to the approach

of any body if the electroscope itself be charged. The electricity redistributes itself under the influence of the approaching body.

The very separation of the electricities in the originally uncharged body, when an electrified body is brought near, is a redistribution. This redistribution, is, in fact, the first essential to any electric action. The simple primitive experiment described at the beginning of the chapter implies induction. Not till the electricities are separated in the small shreds of paper does attraction show itself.

Then, again, in the case of the electrophorus, it is only when the plate is lifted away that it becomes electrically active. So long as it rests on the resin surface, we can draw no electricity from the arrangement. Yet a positive charge is there on the plate the whole time, but it is held bound by the action of the negative charge which induced it.

These and other experiments which have been described all exemplify the great truth, that the distribution of electricity on any conductor depends to a very great extent upon the presence of neighbouring conductors. But, however the distribution may be made to vary on a conductor, the following law is always satisfied.

In an electrified system the electricity is found only on the outer surface of conductors, whether solid or hollow, provided that in the latter case no insulated charges are within it.

To demonstrate this experimentally, we may make use of what is called the 'proof-plane.' This is simply a small conducting disc fixed to the end of an insulating handle. Its use is to test the charge on any conductor, or rather the concentration of charge on any part of a conductor. The proof-plane is first brought into close contact with the conductor, and then, with the charge so acquired, it is carried to an electroscope, and the amount and kind of charge tested.

The ideal proof-plane is a small and very thin flexible disc, which can be applied to any portion of a conductor, so as to fit close to the surface. The charge formerly distributed

on the area now covered by the proof-plane may be imagined transferred to the outer surface of the close-fitting disc. On removing the disc, we carry this charge with it. Such is a fairly accurate statement of the way in which the proof-plane may be supposed to act. By its means we may readily demonstrate that an insulated uncharged conductor brought into the neighbourhood of a charged conductor (as in fig. 65) has opposite charges distributed on its sides.

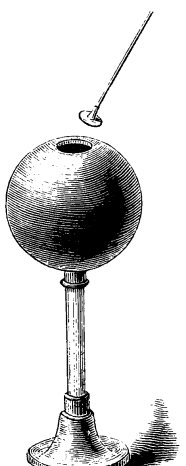


Fig. 71.

To demonstrate the surface distribution of electrical charge, set a hollow conductor with a small hole in it (fig. 71) upon an insulating stand. Electrify it, and then test with the proof-plane. Let the proof-plane be introduced into the interior, and made to touch the conductor on its inner surface. On being withdrawn and presented to the electroscope, it will have absolutely no effect upon the gold leaves. It will acquire no charge whatever. But let it touch the conductor anywhere on the outside, and it will be found to have acquired a considerable charge. In order that there may be an entire absence of charge inside, the hollow conductor should be completely closed. But, even with a hole of considerable size, the charge on the inside is practically inappreciable. In removing the proof-plane, we must take care that it does not touch the rim of the hole. Instead of a hollow vessel with solid walls, we may use one constructed of fairly open meshwork—for example, a piece of coarse wire gauze rolled into the form of a cylinder.

Faraday's 'butterfly-net' experiment shows the same truth in a very pretty way. The butterfly-net, which is a conical bag of cotton gauze, is fastened to a brass ring, and set on an insulating stand (fig. 72). It is kept distended by a silk thread attached to the apex; and the net may be drawn in-

side out by means of a second silk thread, as indicated in the figure. Now, electrify the bag, and test it by means of the proof-plane both outside and inside. Electricity will be found only on the outside; appreciably none on the inside. And now, with the bag always electrified, draw it inside out, and test again. The result will be *exactly as before*—no charge on the inside, the whole charge on the outside. But the surface that is now outside and charged was originally inside and devoid of charge. Thus, during the 'flyping' process, the electricity has changed from the one surface to the other, so as always to keep on the outside.

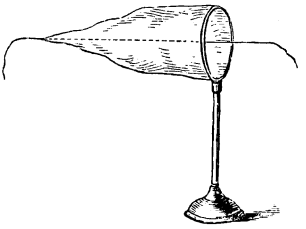


Fig. 72.

Many other experiments may be made to illustrate this very important law.

For example, introduce a charged ball inside an insulated conducting vessel, and let it touch the inside of the vessel. Remove it carefully, and test with the proof-plane. It will be found to have lost its charge completely.

Again, insulate a wire cage, and suspend within it gold leaves or pith-balls by means of cotton or other conducting threads. It matters not how highly charged the cage may be, there is no motion in the gold leaves or pith-balls.

The same truth is illustrated by the experiment first made by Cavendish. An insulated spherical conductor is enclosed in two hemispherical conducting caps, also insulated. The sphere is charged and then connected to the enclosing shell built up of the two hemispheres. These are then removed, and the exposed sphere tested. Not the slightest indication of charge can be detected. By means of this experiment, Cavendish proved with remarkable accuracy that the law of attraction and repulsion between electricities was that of the inverse square of the distance (§ 48).

A second very important characteristic of electrical distribution is the following:

On any elongated conductor in free space—that is, removed from the influence of other conductors—electricity accumulates most towards the ends.

Another mode of expressing the same truth is this:

Electricity tends to accumulate towards exposed points or regions of greatest convex curvature.

Consider, for example, the elongated conductor shown in fig. 73. Let it be insulated and charged by means of the electrophorus. Touch it with the proof-plane at the point A or B, and then measure the divergence produced on the electroscope when the proof-plane is brought into contact with it. Repeat the same experiment after touching the conductor with the proof-plane at the point C. The divergence produced in the latter case will be much smaller than in the former. Thus the proof-plane acquires a greater charge by contact with the end than by contact with the middle of the elongated conductor. If we consider small areas of equal size surrounding the points A and C, and consider the relative quantities of electricity which must be distributed on these areas, we must conclude that there is a greater quantity on the area at A or B than on the equal area at C. In other words, the **electrical density**

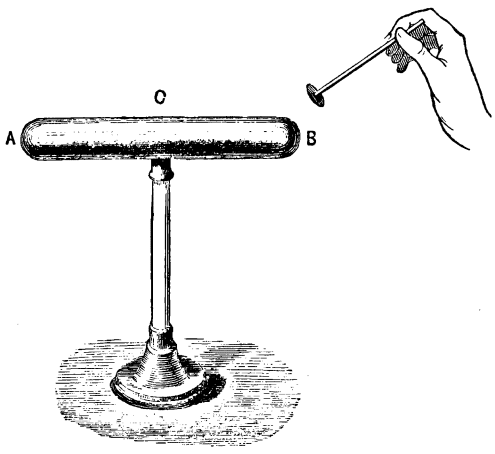


Fig. 73.

or charge on unit area **increases towards the ends** of an elongated conductor.

§ 41. **Magnetic and Electrostatic Inductions compared and contrasted.**—In the duality which both possess, there is a certain similarity between magnetism and statical electricity. We distinguish two kinds of each, and the laws of attraction and repulsion which hold in the two cases are identical.

Like poles repel each other, unlike poles attract. Similarly electrified bodies repel each other, oppositely electrified bodies attract. Also when the one pole or electrified body is much more powerfully magnetised or electrified than the other of similar nature (see § 17), the inductive effect of the stronger pole or of the more powerfully electrified body may mask the original condition of the other, and produce attraction.

Then, again, a magnetic pole brought near to a piece of iron magnetises it inductively with the opposite kind of pole at the nearer end. Similarly, an electrified body brought near a conductor electrifies it inductively with the opposite kind of electrification at the nearer end.

The contrasts, however, are as important as the similarities.

All kinds of metals, indeed all kinds of matter, may be made to show strong electrical action. We can obtain strong magnetic effects in three metals only—iron, nickel, and cobalt.

We cannot separate the poles of a magnet so as to have them on detached pieces of metal (§ 4); but we can readily separate the different kinds of electricity, so that each is distributed on its own separate conductor (§ 35). Electrostatic induction is something which affects the conductor as a whole; magnetic induction acts on the individual molecules.

In developing the subject of magnetism, we considered the properties of the **Magnetic Field.** We propose, after a somewhat similar fashion, to consider the properties of the **Electric Field.**

EXERCISES.

1. How would you prove to a class that there are two kinds of electricity? (1888)

2. A rod of sealing-wax is rubbed with dry flannel. An uncharged pith-ball, suspended by a silk thread, is attracted when the sealing-wax is brought near it, but is unaffected by the flannel. Would you conclude from this experiment that when sealing-wax and flannel are rubbed together, the sealing-wax only is electrified? Give reasons for your answer. (1888)

3. Two pith-balls are attached to the ends of a thread, the middle of which is tied round a glass rod. The balls are electrified, and repel each other. How would you test whether the rod or the thread was the effective insulator?

4. A sheet of hot paper placed on a hot board is electrified by brisk rubbing with india-rubber. How will strips cut from the paper, and allowed to hang close together, act upon one another? A small glass rod, rubbed with silk, repels the strips. What may we infer?

5. The plate of an electroscope is beat with a handkerchief, and the gold leaves diverge. How would you test the kind of electrification? Given that the electroscope is negatively electrified, describe what happens when it is being tested.

6. Sulphur powdered in a mortar is dropped on the plate of an electroscope. The gold leaves diverge, and become more divergent when fur-rubbed sealing-wax is brought near. What does the experiment teach us?

SUMMARY.

(32) Electrification by Friction. Tested by power of attracting small bodies.

(33) The two kinds of Electrification, Positive and Negative. Like kinds repel, unlike attract.

(34) Electrification by Contact. Conductors and non-conductors or insulators.

(35) Electrification by Induction. The two-fluid theory.

(36) The Gold-leaf Electroscope. Method of charging and using it. Inductive reaction of conductor brought near the charged electroscope.

(37) First rough Idea as to Degree of Electrification or Amount of Charge.

(38) Induction the Fundamental Fact of Electrostatic Action. Inductive effects in long conductors and in non-metallic conductors.

(39) The Electrophorus.

(40) Distribution of Electricity on Conductors. The proof-plane. Hollow conductor's charge found only on the external surface.

Accumulation of charge towards exposed parts of great curvature. Electric density.

(41) Magnetic and Electrostatic Inductions compared and contrasted.

CHAPTER VII.

THE ELECTRIC FIELD.

§ 42. An **Electric Field** is any region in which electric force is exhibited, or across which electrostatic induction takes place. It is the region bounded by conductors on which electrical charges are found. For example, if we hold a charged electrophorus plate in the middle of a room, we have an electric field all around whose boundaries are the charged plate and the surfaces of all the nearest objects, whether walls, ceiling, floor, or furniture. Any insulator or non-conducting substance may fill an electric field; and many of the properties of the electric field may be partially realised through substances of low conducting power, such as paper or wood. In ordinary cases, the electric field traverses such substances as air, glass, vulcanite, mica, &c.; and when, in accordance with Faraday's view, such insulators are regarded as *transmitters* of electric induction, they are called **Dielectrics.**

We investigate the properties of a magnetic field by bringing into it a portion of matter susceptible to its influence. We investigate the properties of an electric field in a similar manner, generally by bringing into it a conductor. Such, in fact, is the real significance of the experiments of §§ 35–37, although in these the field associated with the electrified body was brought round the conductor, rather than the conductor brought into it.

Charge an insulated spherical conductor by a series of contacts with the electrophorus plate. If the electrophorus be in good condition, actual contact is not necessary. For, be-

fore the plate and sphere are close enough to touch, a spark will be seen passing between them. This spark means a readjustment of electrical charge, and the net result is that a certain proportion of the original charge on the electrophorus plate has been virtually transferred to the sphere. Now, recharge the electrophorus plate to the full, and again bring it into contact with the sphere. Again recharge the plate, and once more touch the sphere with it, and so on indefinitely. It will be found that each spark is less powerful than the previous one; and that after a few operations, no sparking will occur when the freshly charged electrophorus plate is made to touch the sphere. We have, in fact, electrified the sphere as much as it is possible to do with the means at our disposal.

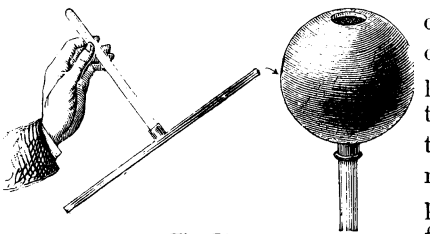


Fig. 74.

This gives us the first inkling as to the meaning of the term **Capacity.** Under given conditions we cannot charge a conductor with more than a certain amount of electricity.

Having thus charged the sphere A, bring into its vicinity a second insulated sphere, B. This sphere, being brought into the electric field, becomes electrified with positive and negative distributions, as shown in fig. 75*a*. In the same way, another insulated sphere, C, brought into the electrical field, becomes electrified (fig. 75*b*).

Now join B to C by an insulated wire. What will happen? Possibly a spark may pass, in which case we always find that B and C have altered their electrical states. Disconnect them, and take each (always insulated) to an electroscope. The one will show a negative charge, and the other a positive charge. Even if no perceptible spark should pass when the two are made one conductor,

they will in all probability show opposite charges when tested singly.

If B is the nearer to A (as in fig. 75 *b*), the final charge on B, after the operations just described, will be negative, and the charge on C positive. But if C is the nearer, it will have the negative charge, and B will have the positive charge. All this is a particular case of an experiment already described (§ 35, fig. 59).

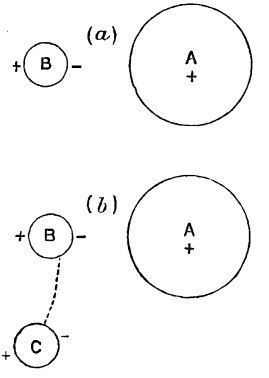


Fig. 75.

Thus the character of the charges left on B and C depends on their positions with regard to the inducing body A. The charges will always be of opposite sign. They will be large, if one body is much nearer than the other to A; and small, if the conductors are nearly the same distance from A. Let C_1, C_2 (fig. 76), be two positions of C, such that when C is placed at C_1, joined to B, disconnected, and carried to the electroscope, its charge is positive; whereas, when C is placed at C_2, joined to B, disconnected, and carried to the electroscope (all other conditions being the same), its charge is negative. Then there will be an intermediate position C_0, such that if C is placed there, joined to B, disconnected, and carried to the electroscope (all other conditions being the same), there will be no charge found on C. The qualification 'all other conditions being the same,' means that, at the beginning of each distinct test, B and C must be in exactly the same electrical state. Since after the first and second tests B would be left with an opposite charge to that on C, both B and C must be removed quite out of the electric field, discharged, insulated, and brought back again. The simplest way to discharge a conductor is to introduce it within a hollow conductor, and allow it to touch the inside (§ 40).

The point to be emphasised is that two insulated conductors (B and C) may be so placed in an electric field, that when they are joined together, *no change of electrical state will take place*, although we know that each may be capable of giving rise to electrical effects. Two conductors, fulfilling this condition, are said to be at the **same potential.**

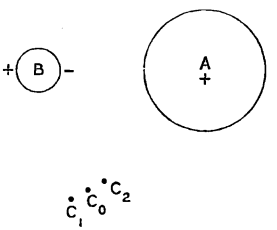


Fig. 76.

We may imagine the one conductor to be kept fixed in position, and the other to be moved about, so that the condition of no transference of charge from one to the other is always fulfilled. Then the second sphere will always be moving from one position in which it has a given potential to another in which it has the same potential. Such an experiment would not be easy to make with only a gold-leaf electroscope at our disposal. The nature of the experiment is, however, sufficiently obvious; and with certain forms of electrometer the experiment could be made. An electrometer is an instrument capable of *measuring* certain electrical quantities, not merely giving a rough indication of their nature and amount. If to such an instrument two small conductors, like B and C just described, are properly connected, the indication given by the instrument will show whether or not B and C are at the same potential. If we fix B, and move C, so that the indication on the electrometer does not change, then C is moving from one position to another in which it has the same potential.

§ 43. **The Potential.**—The conception of the potential here introduced is one of great value. In electricity, it has much the same import as temperature has in heat, or pressure in hydrodynamics. When a channel exists between two quantities of fluid at different pressures, fluid will flow from where the pressure is higher to where it is lower. If the pressures are equal, no flow will take place. In like manner,

if two conductors of the same material* are at the *same potential*, no electrical change occurs when they are joined together; neither parts with electricity to the other. On the other hand, if, after two conductors of the same material* have been connected by a wire, then disconnected, and tested on an electroscope, one of them is found to have undergone some electrical change, then these conductors were originally at *different potentials*. By convention, we regard that body which has lost positive charge or gained negative charge as having been at the higher potential.

Let there be three equal insulated conductors, A, B, and C, and let A be charged, say, positively. Let B and C, both uncharged, be placed at such distances from A as to show



Fig. 77.

appreciably no electrification—that is, in positions where the electric field associated with A is practically powerless.

First, let A and B be connected by a long thin wire; then electric charge passes between A and B, so that the potentials of A and B were originally different.

Second, disconnect A and B, and connect A and C. C will be charged, but not so much as B was at the first contact. Hence, in this second experiment, the difference of potential between A and C was less than was the difference of potential between A and B in the first experiment. Now positive charge has passed from A to the others, so that the potential of A was the *higher* to begin with, and A must have *lost* potential by being connected with B.

* The phrase 'of the same material' is an important qualification; for, as we shall see later (§ 73), by mere contact of two conductors of different material, we produce an apparent difference of potential between them. This is the so-called Volta effect. It is very small compared to the differences of potential with which we work in electrostatic experiments, and may generally be neglected even if the conductors are not of the same material. Throughout the present discussion we shall assume the conductors to be always of the same material.

Third, discharge C and join it to B, and C will again be found charged exactly as it was in the second experiment. Now, at the very beginning, B and C were at the same potential, being both uncharged and practically beyond the inductive influence of A. Hence we conclude that, by connection with A, in the first experiment, B must have had its potential *raised.*

Suppose, however, that A was at the beginning *negatively* charged, B and C being as before uncharged. Then, on going through the same series of operations, we shall get very similar results, the only difference being in the description. To suit this to the new conditions, for the words *higher*, *lost*, and *raised*, put *lower*, *gained*, and *lowered*, respectively.

Further, if we connect any charged body like A to the earth, it loses all charge. If the body be originally positively charged, it *loses positive charge;* if negatively charged, it loses negative charge—in other words, *gains positive charge.* Hence, electric charge passes from or to a charged conductor connected to earth, according as its charge is positive or negative. But the transference of charge shows that the body and the earth were originally at different potentials; and since the charge passes from the body of higher potential to the body of lower, we see that the body is at a higher or lower potential than the earth, according as it is charged positively or negatively.

Now, it is with reference to the earth that the electrical condition of all charged bodies is usually, because most readily, determined. It is convenient, therefore, to regard the potential of the earth as being zero, so that the *difference of potential* between any body and the earth is the measure of the potential of the body. This convention has the further advantage that a single positively charged body in the presence of the earth has a positive potential, and a single negatively charged body a negative potential. It must ever be borne in mind that it is the difference of potential between

two bodies which determines the direction of flow of charge from the one to the other.

Suppose, again, that A (fig. 78) is a *positively* charged body in presence of the earth. Let E be an electroscope, originally uncharged, and quite beyond the inductive influence of A. Let B be an insulated conductor, originally uncharged, which is placed anywhere in the neighbourhood of A. The potential of E is supposed to be zero, while that of A has a *positive* value. Now, join E and B by a thin long wire. At once, as we know, *positive* charge passes into E from B.

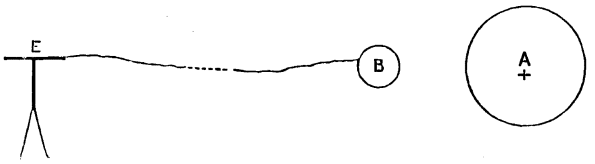


Fig. 78.

Hence, just before connection, B must have been at a *higher* potential than E. The nearer B is to A at the first, the greater will be the charge that passes into E, and the greater, therefore, the difference of potential between E and B just before connection.

Or, keeping E and B connected, move B nearer and nearer to A. The gold leaves will diverge the more, so that more and more *positive* charge is always passing from B to E. Hence we conclude that B is being moved into regions in which its potential tends to become *higher* and *higher*. As we have seen, this connection of B to E produces not only a *fall* in potential of B, but also a *rise* of potential of E. Hence the common potential finally assumed by the connected conductors E and B must be *higher* the nearer B is to A.

To suit the preceding description to the case of A negatively charged, substitute for the words *positive*, *higher*, *fall*, and *rise*, the words *negative*, *lower*, *rise*, and *fall*, respectively.

In these experiments, we see that the potential of the

uncharged conductor, B, depends upon its position in the electric field associated with A. If B be so small that its effect upon the field is inappreciable, we may assume that the potential acquired by B was, before the introduction of B, the original property of the region ultimately occupied by it. In other words, the small uncharged conductor, when placed anywhere in an electric field, acquires a potential which is the original average potential of the region ultimately occupied by the conductor. Thus the potential of any point in an electrical field is to be measured by the potential of a small uncharged spherical conductor placed with its centre at that point. In this way we arrive at the conception of potential as the property of any point in an electric field, as well as the property of any charged or uncharged conductor. Similarly we regard the potential of the conductor itself as the potential of any and every point in it.

Let any conductor be introduced into an electric field. Induction takes place, so that opposite charges become distributed over opposite sides. We may regard this as a transference of positive charge in a definite direction from the side A to the side B (fig. 79). At once we infer that, before the introduction of the conductor, the region A must have been at a higher potential than the region B. In fact, as we pass through an electric field in the direction in which induction is taking place, we are passing along a line of diminishing or increasing potential. If we pass in the direction in which potential increases, we shall ultimately come to a positively charged surface; and if we pass in the direction in which potential diminishes, we shall ultimately come to a negatively charged surface.

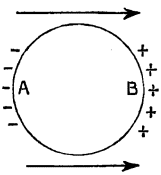


Fig. 79.

It is important to notice that a *positively charged* surface need not be at a higher potential than a *negatively charged* surface. For example, a single conductor cannot but be at

the same potential throughout, yet in an electric field it may easily have positive and negative charges distributed on different parts of its surface.

§ 44. **Equipotential Surfaces.**—All points of a conductor, charged or uncharged, are at the same potential. The surface of such a conductor is consequently an *equipotential surface.* Since in the case of conductors (§ 40) electrical charges exist only at the surface, this surface is all we need consider. In electrostatic experiments, a thin shell or sheet of metal is as efficient as a solid mass.

But equipotential surfaces exist also throughout the electric field. If we join any two points of such a surface by a conducting channel, no transfer of electricity takes place from the one point to the other. We may, in fact, suppose any part of an equipotential surface to become a thin conducting sheet, and yet nothing of the nature of electrical flow along the sheet will take place. We may imagine any one equipotential surface to be traced out in the manner indicated at the end of § 42. A small conductor, F, connected by an insulated wire to a delicate electroscope, E, is moved about so that no change occurs in the electroscope. Having traced one such surface, move the conductor or *feeler* in a little, so as to produce a slight change on the electroscope E, and then proceed, as in the first case, to trace out the equipotential surface S_2 passing through this second point. And so on, step by step, we may suppose the whole electrical field mapped out by a series of equipotential surfaces. Each surface, so determined, will completely enclose the charged conductor. No two surfaces having different potential values can intersect, for no point can have two different potentials

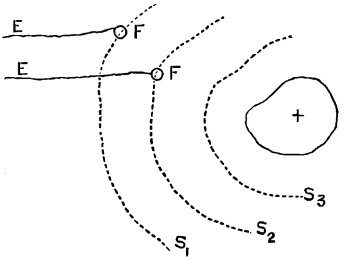


Fig. 80.

at one and the same instant. The nearer we get to the charged conductor, the more like in form will the equipotential surface be to the surface of the conductor, which is itself an equipotential surface.

§ 45. **Induction in the Space within a closed hollow Conductor.**—Set a cylindrical metal vessel, B, on the electroscope; and let down into it, without coming into contact with it, a positively charged conductor, A (fig. 81). As soon as the ball is brought inside the vessel, the gold leaves will diverge, and the vessel B will seem to be positively electrified. Now connect B for an instant with the earth; in other words, reduce it to zero potential. The gold leaves will fall together, and all trace of electrification will be destroyed. But we know from previous experiments that both A and B are charged. For if A be removed, the gold leaves will at once diverge, showing that B is charged; and A, if separately tested, will still be found to have retained its positive charge.

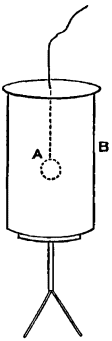


Fig. 81.

But instead of removing A, bring it into connection with B. *Absolutely no change will be observed on the electroscope.* And yet an electrical change has taken place. If A be strongly enough electrified, a distinct discharge will be heard, a distinct spark will be seen through any small opening that may be made in B. Electricity must have passed from A to B, for A was at a higher potential than B. The charge on A has, in fact, been completely destroyed by combining with *the equal and opposite charge induced* on B. To make this experiment accurately, B should completely enclose A. This may be done by hanging A by a silk thread from the lid of B. But with the ordinary gold-leaf electroscope it is sufficient to use a fairly tall cylinder, and to lower A well down into it.

Again, let us introduce the charged body A within the conductor B, which we suppose to remain insulated on

the electroscope. The gold leaves will be divergent, as in fig. 81. And now, without in any way destroying the insulation of B, let A come into contact with B. Electrical discharge takes place between A and B, but *not the slightest change is observed on the electroscope.* The gold leaves remain steadily divergent. Finally, let A be carefully removed from B without again touching it. Then, as we already know (§ 40), no appreciable charge will be found on A when it is tested far from the influence of other conductors.

Thus we see that A has completely lost its charge, but that, in so losing it, it has caused no electrical change to take place *outside the inner surface of B.* Its charge is virtually transferred to B; but B, as tested by the electroscope, has not changed in the very least. In other words, the apparent external electrification of B is exactly the same, whether A is or is not in connection with B. The inductive action of A on B is to develop an equal and opposite charge on the inner surface of B, and an equal and similar charge on the outer surface of B. It is this last-named charge which in all circumstances is effective as regards external regions. The inducing charge and the induced opposite charge mutually annul each other's effects on *all external points.* When they are allowed to unite, they annul each other's effects on all points, both *internal* and *external*—in short, they annul each other altogether.

The truth established by these experiments* is a very important one. **When a charged body is completely surrounded by a conductor, there is induced on the internal surface of the encompassing conductor a charge exactly equal and opposite to that on the enclosed body.**

Let two insulated bodies, known to be charged, be introduced into the closed conductor, itself uncharged; and suppose that absolutely no effect is produced on the electro-

* In making experiments of the kind just described, Faraday used an ice-pail for his cylindrical conductor. Consequently the experiments are often spoken of as the Ice-pail Experiments.

scope. What may we infer? Clearly that the charges on the two bodies are equal and opposite, since they produce equal and opposite inductive effects on the surrounding conductor.

By such an experiment we may prove with great accuracy that there are always equal and opposite electrifications produced by friction of one substance against another. We simply have to conduct the operation within an insulated closed conductor, when we shall find that no charge is induced on the conductor.

The same conclusion may be drawn from the fact that the rubbed surfaces must be *separated* before electrical effects can be obtained (see § 36). Faraday's experiment of rubbing the end of a shellac rod by rotation of a close-fitting cap of flannel, so that the rubbed surfaces always remain in contact, demonstrates this truth very prettily. No electrical effects are obtained until the flannel cap is slipped off.

§ 46. **Electric Screens.**—Consider the electrical system represented diagrammatically in fig. 82. A is an insulated charged conductor wholly enclosed in a second insulated conductor, B. The whole is enclosed within a third conducting surface, at zero potential, say, the walls, ceiling, and floor. Let B be originally uncharged; then the potential of B will be intermediate to the potentials of A and the earth. The region Y will constitute an electric field, with potential changing gradually as we pass from B outwards to the walls. The region X will constitute a second electric field, with potential changing gradually as we pass from A to B.

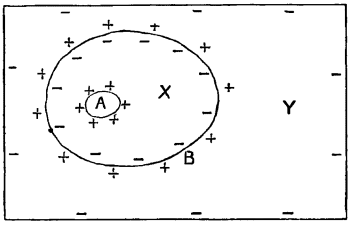


Fig. 82.

The experiments of last paragraph show that B quite separates the fields in X and Y. The field in X may be

destroyed by connecting A with B; no effect will be observed in Y. Similarly, the field in Y may be destroyed by connecting B with the walls of the room; not the least change will occur in X. The closed conducting surface acts as a perfect screen, so that the electric fields within and without it are quite independent as regards any action in either.

By connecting A and B, we reduce them to the same potential; and no electric field exists in the region separating them. Hence the important truth that a region wholly bounded by conductors at the same potential, and otherwise devoid of electrified bodies, cannot be an electric field. Thus, as a particular case, no electric effects can take place within a charged conductor containing no charges (see § 40).

The following experiment illustrates very well the screening effect of closed or nearly closed conductors. Suspend or support close over the electroscope plate E a large flat conductor, B, quite insulated. Bring any positively charged body, A, over it (fig. 83*a*). The conductor B and the electroscope E will now be in the electric field associated with A. Induction will take place, and positive charge will flow in the direction of diminishing potential, so as to bring each conductor to the same potential throughout. Much the same effect will be produced as if the insulated conductor B were not present.

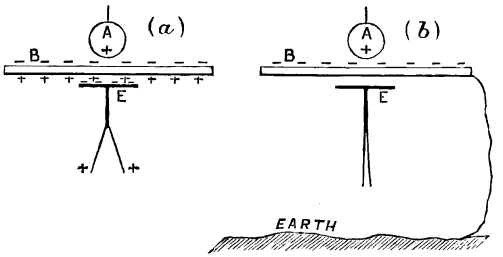


Fig. 83.

But now connect B with the earth, and keep it so connected (fig. 83*b*)—that is, *keep it steadily at zero potential.* Then the electroscope E will be found to be practically screened from the influence of A. The region between B and the

earth, though not completely closed, is sufficiently so in the circumstances; and so long as A is kept above B, its influence on the electroscope will be quite inappreciable.

It will be easily seen that all the usual experiments with the gold-leaf electroscope illustrate the same important principle. For when, after bringing a charged body near the electroscope, we touch the plate to earth, we sufficiently surround the region immediately beneath the plate with conductors at zero potential, so as largely to destroy its character as an electric field.

§ 47. **Electric Force.**—At the beginning of the chapter we defined the electric field as a region in which electric force is exhibited. This is the property which is in evidence in the simplest of all experiments—namely, the attraction between small bodies and electrified sealing-wax. We have, however, chosen to develop the conception of the electric field from the fundamental fact of induction as it shows itself in its effect on conductors placed in the field. We have dealt rather with the motion of electricity along conducting channels, than with the motion of the charged conductor itself. And yet the action of the electroscope depends on this mechanical effect of the electric field upon charged bodies. It is evident, in fact, that the two effects are closely related. Positive charge will pass along conducting channels in a way that suggests repulsion between like charges and attraction between unlike charges. It is in the same general direction that the positively charged body tends to move.

The behaviour of a pith-ball in the neighbourhood of a similarly charged conductor (§ 34), gives us a rough idea as to the meaning of electric force. On it depends the force urging the pith-ball away from the conductor. If the conductor be highly charged, the force acting on the pith-ball is correspondingly greater, other things being the same. Evidently also the force increases as the distance between the pith-ball and conductor is diminished.

That the mechanical force between two charged bodies is

an indication of the amount of charge on these bodies, is a conclusion that is irresistibly borne in upon the mind of the experimenter. There is no eluding it. Consequently, as a first step in the accurate measurement of charge, it is necessary to study this law of force. The earliest direct measurements of this kind are associated with the name of Coulomb.

§ 48. **Coulomb's Torsion Balance Experiment.**—In Coulomb's Torsion Balance, which is historically the first electrometer worthy of the name, the electric force acting on one of the charged bodies is balanced by the torsion of a wire. One form of the instrument is shown in fig. 84 as it is arranged for the electrical experiment. The torsion wire down the centre supports at its lower extremity a horizontal rod of shellac, carrying at one end a gilt pith-ball, B, at the other a counterpoise. The interior of the glass case, within which the ball is suspended, is kept dry by means of sulphuric acid or chloride of lime. The ball A, at the extremity of an insulating rod, is introduced through a hole in the cover. The position which A then occupies is the position which B should at first be made, by adjustment, to occupy with the wire free of torsion. This adjustment is made by means of the nut at the top, to which the wire is attached. The position of this nut is marked by a graduated scale of degrees, over which it moves when it is rotated about a vertical axis. Suppose that when B is adjusted properly the nut indicates zero on its scale. Now take A out, charge it, and re-introduce it into the interior of the case. It comes into contact with B, and electrifies it similarly, so that there is immediate repulsion. After some oscillations, B comes to rest at a certain distance from A. This distance can be calculated from the angle through which the shellac rod bearing B has moved. In this position the electric force acting on B is balanced by the torsion in the wire, which is proportional to the angle through which the wire is twisted. This angle of twist is the angle through which the shellac rod has moved.

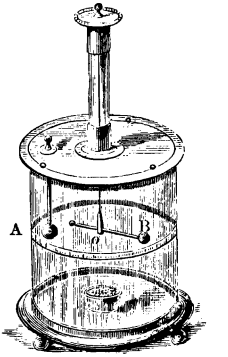


Fig. 84.

Now turn the nut round through a definite angle. This will bring into play a different torsion, and B will adjust itself until the electric repulsion again balances the torsion. For example, take a

particular experiment made by Coulomb in 1785. The first angular displacement of B was 36°, which also measured the twist. The nut was then turned back through 126°, and the angular displacement of B from its first position of rest became 18°; hence the twist on the wire was 126 + 18 = 144°. The force of torsion in the second observation was greater than the force of torsion in the first observation in the ratio of 144 : 36, or 4 : 1. The corresponding angular displacements are as 1 : 2. This suggests the law that *the forces are proportional to the inverse squares of the distances.* Now the angular displacement which will make B's distance from A exactly half the distance they are apart when the angular displacement is 36°, is not 18° but 18° 6′. But this difference of 6′ is smaller than probable errors of observation; and thus the law is established to the degree of accuracy attainable.

Experiments with the torsion balance, even when conducted with the greatest skill, are not capable of giving very accurate results. The great difficulty is to measure the real effective distance between the conductors, especially when these are not small compared to their distance apart.

By an indirect experiment, first made by Cavendish, we can, however, establish the inverse square law with great accuracy. In § 40, Cavendish's result has already been mentioned. It is a mathematical inference from the broad fact that there is no electrical force in the heart of a charged sphere taken in conjunction with other experimental facts concerning electrical distribution, and the assumption that every charged element of surface acts as it would act were no other charged elements present.

§ 49. **Quantity of Electricity or Charge.**—We are now able to obtain an accurate measure of electric charge. This is implied in the enunciation of the great law of electric action, namely:

The force between two small charged bodies is directly as the product of the charges, and inversely as the square of the distance between them.

Up to this point all that we have been able to determine is whether or not two charges are equal. For example, let A and B be two charged bodies. Introduce A inside a closed vessel in connection with an electroscope. Touch the vessel to earth, disconnect, and remove the charged body A. A negative charge equal and opposite to the charge on A is left on the electroscope. Now bring B up, and put it in contact

with the electroscope. If the electroscope is completely discharged, then we know that the charges on A and B were originally equal. But if a charge remains on the electroscope, then A and B must have been differently charged.

It is a simple matter in this way to show that when two equal spheres, exactly similarly situated, are placed in contact and charged, the charges on the spheres will be equal.

Suppose the gilt balls in the torsion balance to be exactly equal in size. Let A be charged and introduced as in fig. 81. By contact B will receive a charge equal to that left on A. Measure the deflection of B. Take out A and discharge it, then introduce it uncharged into its former position. By contact with B, A will receive a charge equal to that left on B. The deflection of B will be considerably less than it was in the first experiment. Turn the nut round so as to allow the deflection of B to be the same as it was at first. Then a comparison of the twists in the wire in the two cases will show that the repulsion between A and B has been diminished in the ratio of 4 to 1. This agrees with the assumption that the charged sphere parts with half its charge to the uncharged equal-sized sphere. Thus if the original charge on A is represented by 4, the charges on A and B in the first experiment are equal, and each will be represented by 2. In the second experiment the charges will each be represented by 1. Applying the law of electric action, we find (the distance being the same in both) that the repulsions should be in the ratio of 2×2 to 1×1, or as 4 to 1.

The complete definition of unit quantity of electricity is as follows: If two very small equally charged conductors, placed at unit distance apart, repel with unit force, each is charged with unit quantity of electricity. Otherwise, if e is the charge on each of two conductors, at unit distance apart, the repulsive force between them is ee or e^2. The unit of charge defined by this law is called the Electrostatic Unit.

To illustrate the precise meaning of the law of electric action, suppose we have two small equal-sized conductors with equal charges, at a distance of 10 centimetres from one another. At this distance the force between them is found to be 1600 dynes.* What are the charges?

* The *dyne* is the name given to the unit of force in the centimetre-gramme-second system of units, now almost universally employed in

Here we have $ee/10^2 = 1600$, or $e^2 = 160{,}000$, and hence $e = \pm 400$.

If we *halve* the distance, the force becomes

$$\frac{400 \times 400}{25} = 6400 \text{ dynes} = 1600 \times 2^2.$$

If we *double* the distance, the force becomes

$$\frac{400 \times 400}{20^2} = 400 \text{ dynes} = 1600 \times \frac{1}{2^2}.$$

EXERCISES.

1. An electrified ball is introduced into a dry glass tube closed at one end, and then, the tube being held in the hand, is brought near to the cap of an electroscope. What will the effect on the electroscope be if the exterior of the tube (1) is, (2) is not, covered with tinfoil? (1889)

2. A muslin bag, containing sulphur and red lead finely powdered, is suspended by a silk ribbon so that it hangs within a metal vessel which stands on the cap of an electroscope. When the bag is jerked, the powders are shaken out through the muslin into the vessel, and become electrified by friction. State and explain what effect (if any) is produced upon the electroscope. (1889)

3. An electroscope is surrounded by a cylinder of wire gauze which is put to earth. If an electrified body is brought near to it, how will the leaves behave? Give reasons for your answer. (1890)

4. Two equal insulated uncharged spheres, B and C, are placed on opposite sides of and at equal distances from a charged sphere, A. What is the electric state of B and of C, and what will happen if the part of B nearest to A is connected by a fine wire with the part of C farthest from A? (1890)

5. Describe an experiment to prove that two points may have the same potential, though one is charged with positive electricity and the other is either uncharged or charged with negative electricity. (1891)

6. Describe a method by which a negatively charged body may be brought to a higher potential than a positively charged body.

7. A pad of flannel is placed at the bottom of a metal vessel which is insulated and connected by a wire with the cap of a gold-

scientific measurement. The dyne is that force which, acting for one second on one gramme, will give it a speed of one centimetre per second. About 445,000 dynes = 1 pound weight.

leaf electroscope. One end of a long rod of sealing-wax is now rubbed against the flannel. What indications will the electroscope give (1) while the rubbing is going on; (2) when the sealing-wax is withdrawn? (1892)

8. A positively electrified body is brought in turn into the neighbourhood of (*a*) an insulated unelectrified conductor; (*b*) an earth-connected conductor. How is the potential of the unelectrified conductor affected in each case? (1892)

9. A piece of silk, which has been used to rub and electrify a glass rod, is dropped on the plate of an electroscope. The plate is then touched with the finger, and the silk is lifted away. Describe the effect produced on the electroscope after each operation.

10. The gilt balls A and B used in the torsion balance are of the same size. A is introduced charged, and B after contact is repelled through an angle of 15°. A is then taken out and discharged, and again introduced. After contact, B parts with half its charge to A, and is repelled. Assuming that no charge is lost by leakage from either B or A, calculate the approximate value of the angle through which B is repelled in the second operation. (Assume distance proportional to angle.)

11. How would you show experimentally that when a charged conductor is introduced into the interior of another conductor without touching it, the charge induced on the external conductor is equal to the inducing charge? (1892)

SUMMARY.

(42) Definition of Electric Field. Electric field associated with a charged conductor. Induction one of its fundamental properties.

(43) Potential. The importance of the conception. Analogous to temperature and pressure. Difference of potential between conductors reduced to nil when the conductors are connected. The final potential intermediate to the two initial potentials. Potential of the earth taken as zero. Relation between positive and negative potential, and positive and negative charge.

(44) Equipotential Surfaces.

(45) Induction within a closed hollow Conductor.

(46) Electric Screens.

(47) Electric Force.

(48) Coulomb's Torsion Balance Experiment.

(49) Quantity of Electricity or Charge. The electrostatic unit.

CHAPTER VIII.

ELECTRICAL ENERGY.

§ 50. For a thorough understanding of electrical phenomena, we must consider them as illustrations of the transformation of energy. Thus in charging an electrophorus, we have in the first place to exert ourselves in electrifying the resinous surface. In its excited state the rubbed substance becomes a source of energy, and is capable of influencing other bodies. It can attract and cause to move light bodies in the vicinity. The energy of motion so acquired by these bodies must, in accordance with the law of the conservation of energy, be derived in some way from energy existing in the system as electrical energy.

When a charged body is placed in an electric field, it tends to move in a certain direction. If we compel it to move in the contrary direction, we are doing work against the electrical forces, and the work so done has its equivalent in the production of so much electrical energy. If, on the other hand, the charged conductor is allowed to move as the electric force urges it, electrical energy will be transformed into energy of motion.

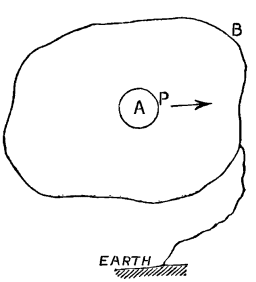


Fig. 85.

For greater definiteness of discussion, let A (fig. 85) be a positively charged conductor inside a conducting shell, B, at zero potential. Suppose that a small part of the surface at P becomes disconnected from A. It will be repelled from A outwards towards the surrounding shell, and it will carry with it a certain charge. This small charge taken from A

will be given up to B. In fact, A will lose charge, and will not therefore be so effective a source of energy as it was before this small charge was allowed to leave it. But the small fragment of conductor has now become a part of the negatively charged inner surface of B. It will therefore be attracted by A, and will start back towards A with a small negative charge. On coming in contact with A, the fragment will lose its negative charge, which will neutralise an equal amount of positive charge, so that the charge on A will be still further diminished. The fragment, now positively charged, will again be repelled from A outwards to B. Thus A will gradually lose its positive charge, and B its negative charge. If no other forces act on the fragment, it will cross and recross the electric field between A and B, carrying positive charge in one direction and negative charge in the other, until the charge left on A becomes too small to be appreciable.*

In this way, without any work being done by any external agency upon the electrified system, we can imagine the system discharging itself by doing work at the expense of its own electrical energy. Positive charge is always passing from A to B, because A is at the higher potential, and will tend so to pass until the potential of A is reduced to that of B.

But this equalisation of potential may be at once effected by joining A and B by a conducting channel. They are then practically instantaneously reduced to the same potential, and the electric field between A and B is destroyed.

In the first case, the equalisation of potential is imagined as the result of the action of the *electric force* in driving a small conductor from side to side. In the second case, the potential is equalised by what is called the *electromotive force* driving the charge along a conducting channel. The result

* This is the principle of the action of an electric toy known as Franklin's Electric Chimes. An insulated ball, which acts as hammer, oscillates to and fro between two bells, one of which is charged and the other is put to earth.

in both cases is the same—namely, the loss of a definite amount of electrical energy. In the one case we see this energy being partially transformed into the energy of motion of a small body. In the other case we have it transformed into the light, sound, and heat, which sensibly accompany the electric spark.

§ 51. **Potential accurately defined.**—If two conductors are at the same potential, there is no tendency for electrical charge to pass from one to the other. Hence, if we carry a small charge from the one to the other, no work on the whole is done against the electrical forces. But if the potentials differ, then, in carrying positive charge from the one at lower potential to the one at higher potential, we do work. It is in terms of work so done that we get the accurate definition of the potential of a conductor. Exactly the same consideration applies to the potential at any point. If we carry a positive charge through an electric field in the direction in which the potential is increasing, we do work against the electric force. From the fact that all points of a conductor are at the same potential, we see at once that the potential of a conductor is simply the potential of any point in it.

Definition: *The potential at any point in an electric field is the work that must be done to bring unit quantity of electricity to that point from a region of zero potential.*

If the potential at the point contemplated is positive—that is, greater than zero—work is done against the electrical forces, which, as we know, tend to drive the positive charge towards regions of lower potential.

If, on the other hand, the potential at the point contemplated is negative—that is, less than zero—the electrical forces will tend to drive the positive charge in the direction in which it is being carried. No work will be done *against* the forces; but work will be done *by* the forces. We may, however, still speak of work being done against the forces, provided we put a *negative* sign before it, and call it *negative work.*

The definition given will then suit every case, negative potential being associated with negative work done, and positive potential with positive work done.

Consider, for simplicity, a charged sphere, A, in the heart of a concentric spherical conductor, B, at zero potential (fig. 86). The system is a symmetrical one, the lines of force are radial, and the equipotential surfaces concentric spheres. The electric force at

any point—that is, the force acting on unit quantity placed there—is the resultant of all the electric forces between this unit quantity and the numerous elements of charge which build up the positive and negative charges on the two spherical surfaces. If we double the charge on a particular element, we double the force between this element and the unit of charge. But any increase of positive charge on A means a corresponding increase of negative charge on B, and all elements of charge increase correspondingly. Hence the resultant electric force at every point increases in the same ratio.

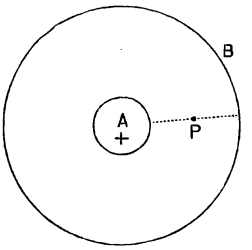


Fig. 86.

But the potential at the point P is the work done against the electric forces in bringing unit charge from B (at zero potential) to that point. And if the electric forces are all increased in the same ratio, the work done must also be increased in this very ratio. But we have just seen that the electric forces are all increased proportionately to the charge on A. Hence the potential at any point must be proportional to the charge on A. It should be noted that we are supposing only the charges to alter. The geometrical or space relations of the system remain unchanged.

This relation between charge and potential is a general one, and may be thus enunciated:

If the geometrical relations of an electrical system remain unchanged, and if the charges on the various charged surfaces are all changed in the same ratio, the potential at any point in the electric field is altered also in the same ratio.

§ 52. **Energy of Charge.**—Return again to the simple case of the spherical conductor A enclosed in the spherical conducting shell B at zero potential. Let the charge be Q, and the potential of the sphere V. Suppose now that we wish to increase the charge on the sphere by a small quantity, q. When this is effected, an equal negative charge $-q$ will have been added to the charge on the inner surface of the shell. That is to say, in *adding* $+q$ to the sphere, we have *subtracted* $+q$ from the shell. We have in effect carried q units of electricity from the shell to the sphere against the electric forces in the electric field between the conductors. The work done in carrying the small charge q through the electric field from B to A is represented by the product Vq, since by definition V is the work done in carrying one unit of charge from B to A.

We might compare the process of charging a conductor to the process of building a tower. Suppose the bricks to weigh each 10 lb. Then in carrying a brick to a height of, say, 20 feet, we do 200 foot-pounds of work against gravity. But by adding bricks we increase the height of the tower, so that when the tower is 50 feet high, we must do 500 foot-pounds of work in adding another brick. The whole amount of work done will be measured by the weight of bricks raised, multiplied by the *average* height to which they are raised. If the tower were of uniform section, this average height would be half the height of the tower.

Now, in the electrical problem, for height substitute potential, and for weight of brick substitute additional charge. Then, as the geometrical relations of the system are supposed to be kept constant, the charge and potential grow proportionately. Hence, the average potential during any process of charging is simply half the final potential. And thus we easily conclude that the work done in charging the conductor with charge Q to potential V is measured by half the product of the final charge into the final potential, namely,

$$E = \tfrac{1}{2}QV.$$

This measures the energy of the charge.

§ 53. **Capacity.**—When two insulated conductors are placed in contact, charged, and then separated, the charges on them will in general be different.

Let there be two conducting spheres, one twice the diameter of the other. Bring them together as in fig. 87*a*, charge them, and then test their charges separately. This is done very effectively by introducing each in succession into a cylindrical conductor placed on the electroscope. The gold leaves will diverge in both cases, but will diverge farther when the larger sphere is used (fig. 87, *b* and *c*).

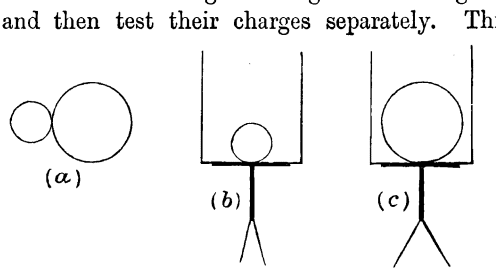


Fig. 87.

This simple experiment proves that the two spheres were

differently charged after being made to touch one another. Their *potentials* were the same, but their *charges* were different. The larger sphere can, so to speak, contain more electricity than the smaller sphere at the same potential. In other words, the larger sphere has the greater **capacity.**

We may define electrical capacity as the charge required to be added so as to raise the potential by unity. From this definition the whole subject may be developed mathematically; but we will illustrate it experimentally.

Let there be three spheres, A, A′, B, of which A and A′ are equal, while B has half the diameter of either of the others. *First*, set B and A at a considerable distance from each other, connect them by a long thin insulated wire, and then charge them. Remove the connecting wire without discharging the conductors, and then use B to charge inductively the electroscope with an equal and opposite charge (§ 49). *Secondly*, bring A (charged) into contact* with A′ (not charged), so that the charge on A is divided into two equal parts, half only being left on A. Disconnect A and A′, and bring either into contact with the electroscope. The electroscope will be completely discharged. Hence *half* the original charge on A was equal to the original charge on B, or the capacity of A is double that of B.

This experiment illustrates the general law, that *the capacities of spheres in free space*† *are as their radii.* The law follows dynamically from the law of the inverse square, as also does the following definition of unit capacity.

Definition: A conducting sphere of unit radius, situated in free space, has unit capacity.

* The two equal spheres may be made to touch directly, or they may be connected by a long thin wire, as in the case of A and B. But a quite different distribution of charge would be obtained if the *unequal* spheres A and B were made to touch directly, instead of at a distance by means of a long wire. The reason is that the two equal spheres are still in similar situations even when they do touch directly.

† 'In free space' is a convenient expression for 'far removed from other conductors.'

Hence the capacity of any sphere in free space is measured by its radius. If the radius of the sphere be a, its charge Q, and its potential V, we have :

$$\text{Charge} = \text{capacity} \times \text{potential}$$

or $$Q = Va.$$

In all the cases mentioned, the conductors are supposed to be in free space, or quite removed from the inductive influence of other conductors. When other conductors are near enough, the capacity of a given conductor becomes altered, in many cases to a very large extent.

§ 54. **Experiments showing Change of Capacity.**—If two equal-sized conducting spheres are charged in contact with one another, and otherwise in similar environments, their charges are equal (§ 49).

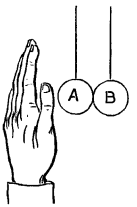


Fig. 88.

But if, while they are hanging together, we bring any conductor (for example, the hand) pretty close to one of the balls, but without touching it (fig. 88), then the charges on the spheres will be found to be different.

The following experiments establish the same truth. (1) Charge any two conductors, A, B, together to the same potential, say positively (fig. 89a). Charge the electroscope inductively with a charge equal and opposite to that on B (fig. 89 b, c).

(2) Bring up an uncharged conductor, C, on the one side of A, but so as not to cause any charge to pass from one to the other. And then again place B in contact with A (fig. 89d). With C permanently in this position, remove B, and introduce it into the nearly closed conductor on the electroscope. The gold leaves will not now collapse completely. Thus B must have lost some of its positive charge to A. In other words, when C was brought into the neighbourhood of A, the potential of A *must have been lowered* below the value it had just before C was brought near.

Had we charged A and B to a negative potential, exactly

similar effects would have been produced. Only, in this case, the presence of the uncharged conductor C would *raise* the potential of A so as to diminish the difference of potential between A and C.

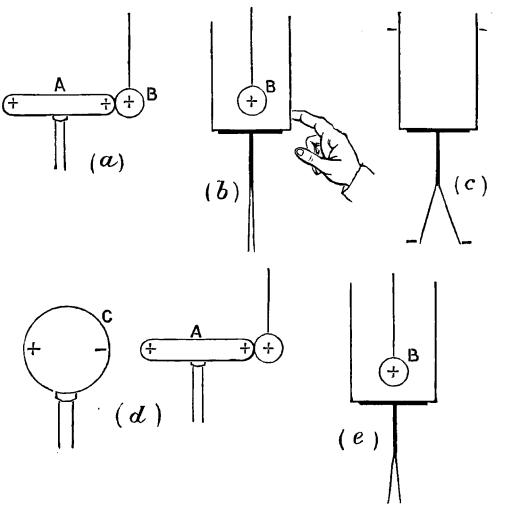


Fig. 89.

Thus the *capacity* of a given conductor is *increased* when any other conductor is brought near it. This is equivalent to the statement that the mere approach of one conductor towards another *diminishes* the *difference of potential* between the conductors.

If the charges are sufficiently great, and if the capacity of A is sufficiently altered when B is brought for the second time into contact with it, the transference of charge from the one to the other may be at once shown by a spark. We could not, however, tell in which direction the positive charge had passed without comparing in some way the initial and final charges on B.

It matters not, so far as the final state of affairs is concerned, whether, in the arrangement of fig. 89*d*, we bring C up before or after B and A are again brought into contact. In either case a transference of charge takes place, and B is left with a different charge from what it had at first.

These experiments all show how the capacity of a conductor is affected by the presence of other conductors. This is far the most important case. But it should be mentioned that the capacity of a conductor may be altered by a change in its own shape or size. A single example of this is shown in fig. 90. A roll of tinfoil, insulated on a glass rod, and connected with the electroscope, is charged. The divergence of the gold leaves will depend upon the charge and the relative capacities of the tinfoil and electroscope. As the tinfoil is unrolled, the gold leaves collapse, showing that the capacity of the foil is increasing. Or we may say that since the surface of the foil is increased by the unrolling, the charge distributes itself with diminished density.

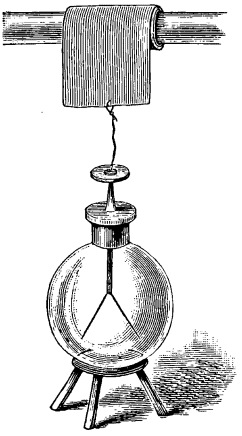


Fig. 90.

A similar experiment can be made by drawing up a chain by means of a silk thread from the plate of the charged electroscope on which it lies coiled.

§ 55. **Relation between Charge, Potential, and Capacity.**—When the geometrical relations of an electrical system remain unchanged, the potential at any point is a function of the charges. Under these conditions the potential of any conductor is proportional to the charge. But capacity is the charge at unit potential; so that the ratio of the charge to the potential is the measure of capacity. Thus if C is the capacity, Q the charge, and V the potential, we have the simple relation, $Q = CV$.

The capacity, we have seen, is in general altered when the geometrical relations of the system are altered; and to keep the capacity constant, we must preserve unchanged these geometrical relations.

The above equation shows that the potential varies in-

versely as the capacity when the charge is constant. This is the complete statement of the experimental result of § 54, that, with constant charge, an *increase of capacity* is necessarily accompanied by a *decrease of potential*, and *vice versâ*.

When two insulated conductors of the same material at different potentials are connected by a conducting channel, they come to the same potential. A redistribution of charge takes place, but the *total charge* on the system of conductors does not alter. Let us assume the capacities to remain unchanged. Then both potentials must undergo change; for the conductors are brought to the same potential. Can this final potential be expressed in terms of the original potentials, capacities, and charges?

Take, first, a simple numerical case. Let the conductors have equal capacities, each 5; and let the original potentials be 2 and 10. Then the original charges are 2×5 and 10×5, or 10 and 50 respectively; and the total charge is 60. But the capacity of the two conductors together is 10, being twice that of either. Hence with charge 60 and capacity 10 we get for the final potential the value $60/10 = 6$. Each conductor will have a charge 30.

Then as to the energy before and after connection. In every case the energy is half the product of charge into potential. Before the conductors were joined,

$$\text{Energy} = \tfrac{1}{2} \times 10 \times 2 + \tfrac{1}{2} \times 50 \times 10 = 260.$$

After the conductors were joined,

$$\text{Energy} = \tfrac{1}{2} \times 60 \times 6 = 180.$$

Thus there is a loss of electrical energy when two conductors at different potentials are joined together. Usually we see and hear the discharge; so that the partial transformation of energy out of the electrical form into other forms is evident.

The general investigation is a simple piece of algebra. Let $C_1V_1Q_1$ and $C_2V_2Q_2$ be the capacities, potentials, and charges of two conductors. By definition,

$$C_1V_1 = Q_1, \quad C_2V_2 = Q_2.$$

Let V be the common potential after connection. Then, since $Q_1 + Q_2$ is still the total charge, and $C_1 + C_2$ is the total capacity, we have

$$(C_1 + C_2)V = Q_1 + Q_2,$$

and

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}.$$

To find the loss of electrical energy when the two conductors are

allowed to come to the same potential, note that the equalisation of potential is accompanied by the transfer of a certain charge from the one conductor to the other. Let q be this charge. Then, if V is the final potential, the initial charges are

$$C_1V + q \text{ and } C_2V - q.$$

But the energy is equal to $\frac{1}{2}QV$ or $\frac{1}{2}\frac{Q^2}{C}$ or $\frac{1}{2}CV^2$. Hence the initial energy is

$$\frac{1}{2}\frac{(C_1V + q)^2}{C_1} + \frac{1}{2}\frac{(C_2V - q)^2}{C_2}$$
$$= \tfrac{1}{2}(C_1 + C_2)V^2 + \tfrac{1}{2}q^2\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

since the products in Vq evidently cancel.

But the expression $\frac{1}{2}(C_1 + C_2)V^2$ is the final energy of the system, and the term $\frac{1}{2}q^2\,(1/C_1 + 1/C_2)$ is always positive, whatever be the sign of q. Hence there is always a definite loss of energy, measured by half the square of the transferred charge into the sum of the reciprocals of the capacities.

§ 56. **Electrical Distribution explained in Terms of Capacity.**—When two charged conductors are connected so as to be at the same potential, the charge divides itself proportionately to their capacities *in the positions in which they are.* If we are dealing with two equal spheres (as in § 84), it is of no consequence whether they are made to touch directly, or are connected by a long insulated wire of comparatively insignificant capacity. But if the conductors are of different size or form, or both, then the capacities, when the conductors are placed in direct contact, are not equal to the capacities when they are far apart and connected by a wire.

For example, when two spheres, whose radii are as 10 to 1, are far removed from one another and from other conductors, their capacities are as 10 to 1. But place them in contact, charge them, then separate them and test the charges on each. The charge on the small sphere will be found to be very much smaller than *one-tenth* of the charge on the large sphere. It will be less than *one-sixtieth.*

Again, take an elongated hollow and nearly closed con-

ductor, blunter at one end than the other (fig. 91), and find what percentage of the total charge collects on a small sphere made to touch the conductor at different points. This is simply the experiment with the proof-plane (§ 40); but it is now to be regarded from a different point of view. Let 1, 2, 3, 4 be four positions of contact; the last being internal. In this 4th position, we know (§ 40) that no charge is communicated to the sphere. *Its capacity in this environment is zero.* Now it is clear that in position 3 the environment has greater resemblance to the environment in position 4 than has the environment in position 1. In short, the positions 1, 2, 3, 4 are in order of decreasing convex curvature of the conducting surface near the sphere; there is an increasingly greater overshadowing as we pass from 1 to 4. The capacity of the sphere diminishes as we take it in succession through these positions, becoming zero when it is wholly enclosed in the conductor.

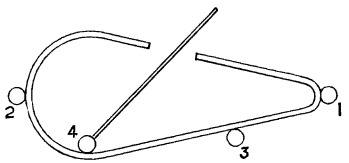


Fig. 91.

Now, what is true of the sphere is true of a disc-shaped body like the proof-plane, and of the immediately contiguous parts of the surface of the conductor itself. Suppose the whole surface of any conductor to be divided into small equal elements. Each has, relatively to the whole, a certain capacity. The element which has, so to speak, the more open aspect has the greater capacity. The area at the end of a pointed region has a much greater capacity than an equal area where it is flat or concave. The charges on equal areas at different parts of the surface are proportional to their capacities. In other words, the electric density is greater where the surface has the greater convexity. In particular, the charge accumulates in greatest quantity at exposed points, where the most pronounced electrical effects are obtained.

§ 57. **The Faraday-Maxwell Theory of Electric Stress in**

the Dielectric.—Faraday was the first to draw special attention to the rôle played by the dielectric or insulating medium in electrical phenomena. He imagined lines of electric force traversing the dielectric; while the charged conductors were simply the surfaces on which these lines of force terminated. Maxwell has greatly developed this conception, the root idea of which is that the dielectric filling a given electric field is in a state of strain under electric stress.

The electric force at any point has a definite direction. If we move a small charge in any direction perpendicular to the electric force, we do no work against the electric force. Consequently, we are confining ourselves to an equipotential surface. Suppose we have the whole field mapped out by a succession of equipotential surfaces, then if we move from point to point always in the direction of the electric force at the point momentarily occupied, we must meet all the successive equipotential surfaces at right angles. A path so described, which is at every point of it tangential to the electric force there, is a **Line of Electric Force.**

Take any small area on an equipotential surface, and draw lines of force through its perimeter. These lines of force will form a so-called **Tube of Force,** whose section will in general vary as we pass along it. Following this tube of force backward to its source, we finally come to a positively charged conductor; and following it forwards, we ultimately come to a negatively charged conductor. Every tube of force has in fact two ends. It springs perpendicularly from a positively charged area, and ends perpendicularly on a negatively charged area. It is along these tubes of force that electric induction takes place; and the negative charge on the terminal area is exactly equal to the positive charge on the area from which the tube springs.

The statement that every tube of force begins and ends somewhere, satisfies the experimental truth that to every positive charge there corresponds an equal negative charge. In the particular case of a charged conductor enclosed in a conducting shell, *all* the tubes of force springing from the conductor end on the shell—a re-statement of the truth that the inner surface of the shell bears a charge equal and opposite to that on the conductor.

The *unit tube of force* is a tube of force which springs from and ends on an area on which unit charge is distributed. We may imagine the whole surface of a conductor to be divided into such areas, each bearing unit charge. The areas will be large or small according as the electric density is small or large. Some areas may

bear negative charges; these will be *ends* of unit tubes of force. Others, again, may bear positive charges; these will be *beginnings* of unit tubes of force. The total charge on any conductor will be the difference between the number of unit tubes which spring from, and the number of those which end on, the surface.

For simplicity, take the sphere A inside a concentric spherical shell, B, at zero potential (fig. 92). Let the charge on A be + Q, so that Q unit tubes of force will spring from A and radiate outwards till they meet B. The equipotential surfaces will be concentric spheres. Let them be drawn so that each differs in the value of its potential by unity from the one next it. That is to say, in carrying unit charge from any one surface to the next within it, we have to do unit work. The field is stronger as we approach the sphere A, so that the equipotential surfaces are closer. If V is the potential of A, the equipotential surfaces between A and B will divide the electric field into V bomb-shell shaped regions.

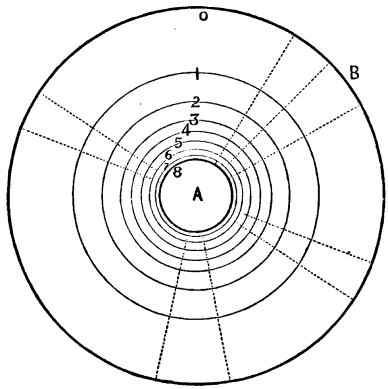


Fig. 92.

The tubes of force radiating out across the equipotential surfaces will divide the region between any two into Q parts. Hence by the intersections of the equipotential surfaces and tubes of force the whole field will be divided into QV spaces or *cells*, which become smaller and smaller as we approach A. If, following Maxwell, we imagine each cell to contain half a unit of energy, we form a distinct conception of the way in which electrical energy may be stored up in the dielectric. The medium is in a state of electric strain. Where the strain is greatest, there the cells are smallest, and the energy contained in unit volume greatest.

The electric stress is of the nature of a tension acting along the lines of force. Each tube tends to contract so as to bring its positively charged and negatively charged ends together. And this is a very accurate description of what takes place when the dielectric yields to the stress, and allows a spark to pass through it.

EXERCISES.

1. An insulated conductor, A, is charged with electricity. Another conductor, B, earth connected, is placed near to A. Is the induced charge on B greater than, equal to, or less than the charge on A? Give reasons for your answer. (1890)

2. The extremity, B, of a wire, AB, is attached to the plate of a gold-leaf electroscope. By means of an insulating handle, the other end, A, is placed in contact, first with the blunt, and then with the more pointed end of a pear-shaped insulated and electrified conductor. Describe and explain the movements of the leaves of the electroscope. (1889)

3. Contrast the result of this last experiment with the effect produced when the proof-plane is used in the customary manner as an intermediary between the conductor and the electroscope. Discriminate carefully between the conditions of the two experiments.

4. Four precisely similar insulated balls, A, B, C, D, are placed in a row. The two inner balls, B and C, are in contact, and the distances AB and CD are equal. If A and D are electrified, what will be the electrical states of B and C after first one and then the other have been removed from the neighbourhood of A and D: (1) When the charges on A and D are equal and opposite; (2) when the charges are equal and similar? (1892)

5. In Exercise 8 of the preceding chapter (p. 109), consider how the potential of the electrified body will be affected by the operations there described.

6. Explain the source of electrical energy when a gold-leaf electroscope is charged inductively by the presence of a charged body, or when the movable plate of the electrophorus is charged in the usual way. How do the electric forces during separation of the bodies compare with the electric forces during their approach?

7. Three insulated spheres of equal size are set in line so as just to touch one another. They are then charged. Consider (roughly) the relations of the charges on the balls after they are separated: (1) When one of the end spheres is first removed; (2) when the middle sphere is first removed.

8. How should four equal-sized spheres be arranged in close contact so that a charge communicated to them will be equally divided amongst them?

9. Given a sphere and a circular disc of equal radius, how would you show that the capacity of the former was greater than that of the latter?

SUMMARY.

(50) Transformation of Energy in producing or destroying Electrification. Distinction between electric force and electromotive force.

(51) Potential accurately defined. Simple case of sphere inside spherical shell.

(52) Energy of Charge. Gravitation analogy. $\frac{1}{2}QV$.

(53) Capacity. Experimental comparison of capacities of spheres. Isolated sphere of unit radius has unit capacity.

(54) Experiments showing Change of Capacity. Effect of neighbouring conductors. With constant charge, increase of capacity means decrease of potential.

(55) Relation between Charge, Potential, and Capacity. $Q=CV$. Loss of energy at discharge.

(56) Electrical Distribution explained in Terms of Capacity.

(57) The Faraday-Maxwell Theory of Electric Stress in the Dielectric.

CHAPTER IX.

ELECTRIC MACHINES OR GENERATORS.

§ 58. **Unlimited Generation of Electricity.**—We have seen that, when a charged conductor is brought into contact with another conductor, the total charge is divided in the ratio of the capacities which the conductors have at the time of contact. If the one conductor be wholly within the other, the former will have zero capacity, and the whole charge will accumulate on the latter. If the one conductor be to a great extent surrounded by the other at the time of contact, its capacity will be very small, and nearly all the charge will be received by the overlapping conductor. This truth should be carefully borne in mind when the action of machines for the generation or accumulation of large quantities of electricity is being studied.

Let B be a hollow conductor (as in fig. 93), set on an insulating stand; and let A be a smaller conductor, which may be introduced within B. Charge A while outside B by

means of an electrophorus. Introduce it into B, and let them touch. Practically, the whole charge on A will be given up to B. Remove A, re-charge it, and again introduce it as before. On contact, this second charge will be practically given up to B. This operation may be repeated almost indefinitely. At each contact, A parts with nearly its whole charge to B. Thus if A carries one unit of charge each time it is introduced into B, after 100 operations B will be charged with very nearly 100 units. The charge would be exactly 100 if B quite surrounded A.

Again, take two hollow conductors, B, C (fig. 93), set apart on insulating stands. As in the last experiment, charge B positively by introducing the charged body A within it and discharging it by contact with B. When A, now discharged, is being removed, let it be put to earth when it is just outside B. Before being so reduced to zero potential, A was at a potential intermediate to the potentials of B and the earth. Hence, on being connected to earth, A loses positive electricity, and is left negatively electrified. Insulate A again, and introduce it within C, and transfer the negative charge to C by letting A and C come into contact. C is now at a slightly lower potential than the earth. Lift A away, and again put it to earth when it is just outside C. A *positive* charge will be left on A. Insulate A, and carry it with its positive charge to B, there again to give it wholly up to B. Repeat this series of operations indefinitely, reducing A to zero potential for a moment just when it is outside each hollow conductor. It is evident that A will always carry a negative charge to C, and a positive charge to B. And thus, without the use of any other apparatus

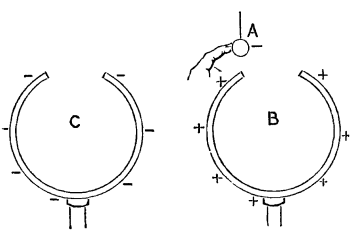


Fig. 93.

than two conductors and one carrier, we can, once a difference of potential between the conductors is established, increase that difference indefinitely. The electrical energy is derived from the work done in carrying the small conductor with its induced charge always in the direction contrary to that in which the electric forces of the field would naturally urge it. If the potential differences are great, contact between A and either conductor is established by means of a spark. One of the hollow conductors may be put to earth; but in this case two carriers must be used, connected and disconnected at suitable intervals.

We now proceed to study the action of some typical machines for generating electricity in considerable quantities.

§ 59. **The Frictional Machine.**—The Friction Electrical Machine is a mechanical improvement on the earliest method of electrifying by friction. A glass or other non-conducting surface is made to revolve so that successive portions are electrified by friction against the same rubber. After being so electrified, each portion of glass passes close to an insulated conductor, and virtually gives up to it the greater part of its charge. From this conductor, known as the **Prime Conductor,** the charge may be nearly all removed and collected on other conductors.

There are two types of frictional machine, distinguished as the **Cylinder** and **Plate Machine.***

They are represented in figs. 94, 95, the distinguishing feature of the one being a cylinder, and of the other a plate, generally of glass, sometimes of vulcanite. The particular figures shown are, of course, simply types; for different machines differ greatly in detail. The important parts of these machines are:

(1) The cylinder or plate, G, supported on wooden stems,

* Historically, the Plate Machine is a modification of the Cylinder Machine, which, in its turn, is a natural evolution from the glass globe used by Newton, and the still earlier sulphur globe which Von Guericke invented in 1671, and electrified by revolving it against the dry hand.

W, and capable of being turned briskly by means of the handle, H.

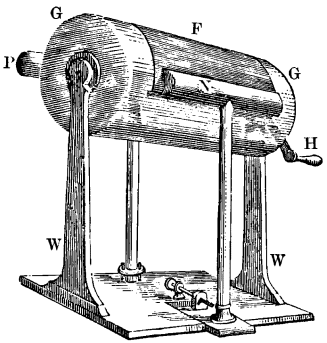


Fig. 94.

(2) A rubber of leather, padded with horse-hair, and fixed to the conductor, N. A flap of silk, F, is attached to the rubber, and passes well over the cylinder or round the plate. The action is greatly improved by covering (with the aid of a little grease) the leather surface with an amalgam of mercury, zinc, and tin.

(3) The prime conductor, P, which is armed with a comb

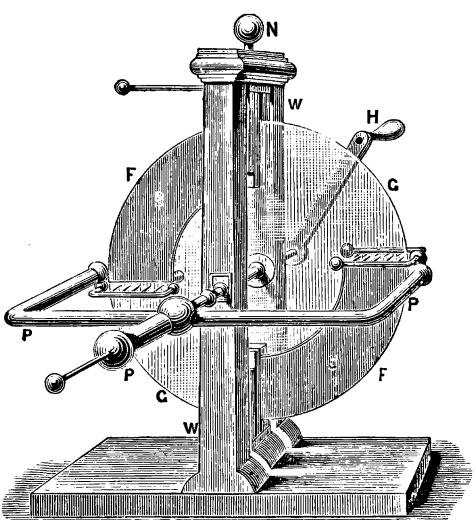


Fig. 95.

of metal points projecting towards the glass. Under the

inductive influence of the charged glass surface, these points become highly charged. In accordance with the principle explained in § 40, the electric density on the metal points is so great that discharge between them and the glass is much facilitated (see also below, § 62). This neutralisation of the induced negative charge by combination with the inducing positive charge leaves the prime conductor positively charged. In the case of the plate machine, the comb of points is bent so as to embrace the glass plate. Thus the charged glass surface passes to some extent *within* the conductor, and the conditions of the experiment described in last paragraph are partially realised.

For the efficient working of the machine, it is necessary to have one of the conductors earth-connected—either N or P. Generally, it is the conductor in connection with the rubber (N, namely) which is so kept at zero potential. But if we desire to get negative electricity from the machine, we may insulate N, and connect P to earth.

In taking the charge from the prime conductor, we have simply to bring an insulated conductor near enough, when, by the usual process of induction and neutralisation, it becomes charged with an amount depending on its capacity, and on the potential and capacity of the prime conductor.

§ 60. **The Electrophorus.**—The general action of the electrophorus has been described in § 38. Let us here look at it in the light of the principles established in last chapter. In its most improved modern form, it consists of two plates, the one of metal, and the other of vulcanite or ebonite backed with metal. Insulating handles can be screwed on to the backs of the plates; and one at least must be so insulated.

Now let the surface of the ebonite be electrified in the usual way by friction. This electrification being negative, the ebonite surface will have a negative potential. The lower plate will be either at zero potential (if it is uninsulated), or

at some potential less than zero, but higher than the average potential over the ebonite surface (if it is insulated). In either case, its potential is somewhat higher than that of the ebonite surface. When the insulated metal plate is brought very close to the ebonite surface, it will, because of its greater proximity to the negatively charged surface, be at a lower potential than the metal attached to the ebonite. If these are then brought into contact—conveniently effected by means of the metal pin passing through the ebonite—a transference of charge will take place from the plate of higher to that of lower potential—that is, from the metal back to the movable plate. This plate, when lifted away, will therefore be found positively charged.

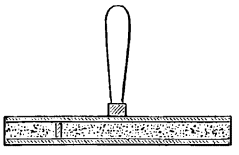


Fig. 96.

§ 61. **Continuous Electrophori or Influence Machines.***—Just as the frictional machine is a mechanical improvement of the rubbed glass or sealing-wax, so the electrophorus has been improved into a machine for the continuous development of electrical charge. Such a machine is called an influence machine. If in good condition, it requires only to be once fairly started, when it will continue to produce electricity as long as it is kept working.

These machines are generally known by the names of the inventors, such as Töpler, Holtz, Bertsch, Voss, and Wimshurst. Of these the Wimshurst is the latest, and apparently the most satisfactory.

The Wimshurst machine (fig. 97) consists of two circular glass plates, mounted on a common spindle, and capable of rotation in opposite directions with equal angular speeds. Each plate carries 12 or 16 strips of sheet-metal, fixed

* The word 'influence' here used is borrowed from the French, and means simply 'induction.' Since, however, 'induction' is used in two other senses in electrical science, the French word has been adopted in this connection to prevent confusion or circumlocution.

radially at regular intervals apart. These strips are attached to the outer sides of the closely opposed glass plates. At the extremities of the horizontal diameter of the plates the conductors are placed, insulated on glass or vulcanite pillars. Horizontal arms with the usual combs project inwards, embracing both plates as far as the inner ends of the metal strips. In front, as it appears in the figure, is fixed a diagonal conductor, called a 'neutralising rod;' and a similar rod is fixed behind approximately at right angles to the one in front. These neutralising rods terminate at each end in a small metal brush, which touches the metal strips or *carriers* as they pass. By this contact of brushes and strips, every strip on either plate is, very soon after it has passed under the collecting combs, brought into metallic connection for a moment with the strip diametrically opposite it on the same plate. The importance of this will be seen immediately. The main conductors are provided with adjustable arms, which reach out towards one another, and between whose terminal knobs discharge takes place. In the diagram the two sets of strips are distinguished by the one set being shaded. This set belongs to the plate of glass to the back. When the machine is working, the front plate revolves clockwise, and the back plate counter-clockwise. The direction in which the plate must revolve depends on the

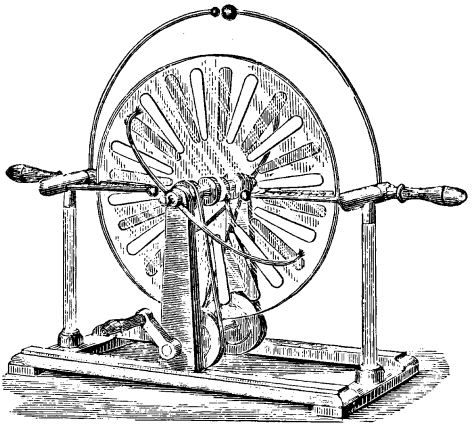


Fig. 97.

position of the neutralising rod, which each strip must meet soon after it has left the main conductor.

To understand the action of the machine, suppose the conductors to be at different potentials, and compare the conditions of any two diametrically opposite strips just before they touch the neutralising rod. Evidently they also will be at different potentials, because of their positions relatively to the conductors. Hence there will be a transference of charge from one to the other. On passing beyond the touch of the neutralising rod, the strips become disconnected. The one, which at first was at the higher potential, carries a small negative charge on towards the main conductor of lower potential; while the other strip, with its positive charge, moves on towards the main conductor of higher potential. Now exactly the same thing is going on on the other glass plate, only in a somewhat reversed manner. For example, as a strip on the upper half of the front plate is travelling, with its positive charge, from the negative to the positive conductor, the strips on the upper half of the back plate are going in the opposite direction, with their negative charges, from the positive to the negative conductor. The pair of strips on the back plate, which are for the moment connected by the neutralising rod, are opposite to strips on the front plate, whose opposite charges accentuate the inductive action of the main conductors.

Now each strip, with its proper charge, passes close to the combs of the main conductor. It virtually passes within the conductor, and delivers up its charge to it, just as in the experiment of § 58. Hence strips pass out from the neighbourhood of the conductors in a comparatively uncharged condition. Their potentials, however, have the average value of the region they happen to occupy. Thus the neutralising rod is always reducing to the same potential two strips originally at different potentials. When the machine is working briskly, there is a constant stream of positive electricity along the neutralising rods on to the travelling strips, and thence across the air-spaces to the one main conductor, and of course a constant stream of negative electricity in the other direction. The principle of the machine is identical with the principle illustrated in § 58, fig. 93. That simple experiment mastered, there should be no difficulty in understanding the general action of the Wimshurst, or of any other influence machine.

We began by assuming the main conductors to be at different potentials. Since it is found necessary at the start to join the conductors by bringing their arms into contact, this assumption may appear to be unwarranted. But it must be remembered that in such a machine we are dealing with electricity *in motion*. If any one

strip is at a different potential from the others, or, what is the same thing, if the glass plate has any portion electrified, conditions are favourable for a transference of charge along the neutralising rods, and therefore of necessity from one conductor to the other. Hence a *tendency* to difference of potential between the conductors is at once established. Once the electric flux becomes sufficiently great, we may separate the knobs of the conductors, which then charge to a certain difference of potential, and discharge across the air-space between. The limit to the difference of potential attainable depends upon the distance of the conductors apart, and the general condition of the machine as regards insulation.

§ 62. **Dissipation of Charge.**—The great density of electrical charge at exposed points of conductors has already been discussed (§ 40). With large charges and high potentials, such as are derivable from the prime conductor of an electrical machine, the electrical density becomes so great at any exposed point that the charge rapidly dissipates into the air. The action is easy to explain. The air immediately in contact with the point becomes powerfully charged, and is repelled violently from the point, carrying with it some of its charge. This discharge causes what is called the electric wind. It may be powerful enough to bend over the flame of a candle held near, as shown in fig. 98.

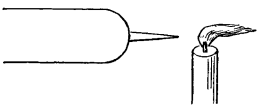


Fig. 98.

Another good illustration of the same effect is given by the electric whirl. This consists of a number of wires fixed radially like the spokes of a wheel to a brass cap, which rests on a pointed support. The wires are bent towards the ends, so that each end points at right angles to the radial direction (fig. 99). When this little apparatus is set on a charged conductor, the charge accumulates at the sharp points, and electrifies the air there. Repulsion takes place between the points and the contiguous air. There is a rush of air outwards at every point; and, in virtue of the reaction upon each point, the little wheel-like apparatus is pushed round in the direction opposite to that in which the air is driven off.

All the points, being bent the same way, act together, and the wheel whirls rapidly in the direction shown by the arrow-heads.

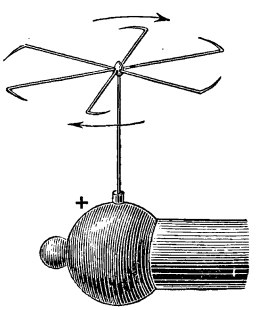


Fig. 99.

The following experiments are similarly explained. If one of two conductors, otherwise identical, has a sharp point fixed to it, and if both are put into the same electrical field, the one provided with the sharp point will gradually become charged. Or if both are charged, and then left to themselves, the one with the sharp point will gradually lose its charge.

A sharp-pointed conductor put to earth is very efficacious in gently removing the charge from any conductor towards which it is pointed. For the highly-condensed induced charge on the point escapes through the air towards the conductor, neutralising some of the charge there. A loud hissing or crackling noise accompanies the action.

This is the manner in which lightning conductors act so as to protect buildings from what would otherwise be lightning flashes. The thunder-cloud, highly charged, passes over the pointed conductor, which gently robs the cloud of so much of its charge. It is necessary to have such lightning conductors in good metallic connection with the earth, so that they cannot become permanently charged.

EXERCISES.

1. A glass rod is electrified by being rubbed with silk. A small sewing needle, held in the hand, is then passed along the rod from end to end, the point of the needle not quite touching the glass. What effect is produced, and how do you explain it? (1890)

2. One [bare] end of a gutta-percha covered copper wire is connected with [the prime conductor of] an electrical machine, and the other [bare end] dips into a non-conducting liquid. When

the machine is worked, the liquid is agitated. Explain this. (1891)

3. Under what circumstances is it possible to transfer the whole of the charge on a conductor to another insulated conductor? (1891)

4. An insulated hollow metal vessel has a charge of positive electricity, and is at some distance from other conductors. An uncharged metal ball supported by a silk thread is (1) introduced into the vessel without touching it, (2) connected momentarily with the earth, and (3) removed to a distance. State how the potential changes during these operations. (1891)

5. Two gold-leaf electroscopes, similar in all respects except that a needle projects from the cap of one of them, are placed at equal distances from an electrical machine. When the machine is worked, both pairs of leaves diverge. When it ceases to work, one pair of leaves collapses rather quickly and the other pair very slowly. Explain this difference in their behaviour. (1892)

6. The prime conductors of one influence machine are joined by insulated wires to the prime conductors of another, in which the merely mechanical resistances to the rotation of the plates are diminished as much as possible. When the first is worked in the usual way, the plates of the second begin to revolve. Explain this effect.

7. When the handle of a frictional machine is turned, sparks can be drawn from the prime conductor. Explain carefully how the prime conductor becomes charged with electricity. (1888)

8. A sharp point attached to a conductor, A, is held near an insulated charged conductor, B. What will be the effect on B if A is (1) insulated, (2) uninsulated? (1888)

SUMMARY.

(58) Indefinite Generation of Electricity.

(59) The Frictional Machine.

(60) The Electrophorus.

(61) Continuous Electrophori. Influence Machines. Wimshurst's described and explained.

(62) Dissipation of Charge at Points. The Electric Whirl. Lightning Conductors.

CHAPTER X.

CONDENSERS AND DIELECTRICS.

§ 63. **The Principle of the Condenser.**—The capacity of a conductor depends, as we have seen (§ 54), upon its environment as well as upon its shape and size. Thus, given any insulated conductor, it is always possible to increase its capacity by a rearrangement of other conductors near it. If we suppose the conductor to be charged with a definite and unvarying charge, whatever influence increases the capacity diminishes the potential (§ 54).

Now the energy of charge is measured by half the product of the potential into the charge ($\frac{1}{2}QV$). Hence a loss of energy, with constant charge, means a diminution of potential. It was proved in § 55 that, when two charged conductors, originally at different potentials, are connected so as to come to the same potential, energy is always lost. But an essentially similar equalisation of potential takes place throughout a single conductor suddenly introduced into an electric field. Thus the mere separation of electricities (to use the language of the old two-fluid theory) involves a loss of electrical energy in the system. And this loss of energy is proportional to the square of the quantity of electricity transferred as the equalisation of potential is taking place throughout any region (see end of § 55).

To fix our ideas, let A (fig. 100) be a conductor, with charge Q and potential V, situated in free space. Let the curve drawn round it represent an equipotential surface, and let v be the value of the potential at every point of this surface.

Now, suppose this surface to become a thin conducting sheet whose potential is the same as that of every point on the original surface. This change will affect neither the character of the distribution on A nor the values of the potential at different points.* But the conducting sheet now separates two independent electrical fields. It is an electric screen (§ 46). Whatever may be done in

* It is only because the conducting sheet is infinitely thin that this conclusion is a warrantable one.

the outer field has no influence upon the electrical distributions and forces in the inner field.

Thus the energy associated with the outer field is the energy due to the distribution + Q on the outer surface of the conducting sheet—that is, $\frac{1}{2}Qv$. The total energy of the system is $\frac{1}{2}QV$.

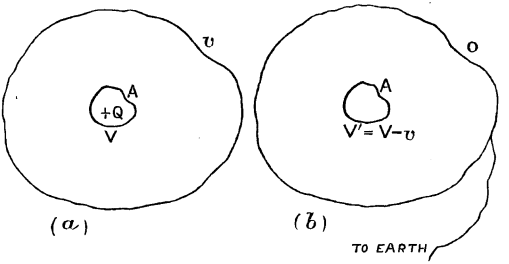


Fig. 100.

Hence the energy associated with the inner field is the difference of these—namely, $\frac{1}{2}Q(V-v)$. Or we may look at it in this way. Here is an electric field, the inner field namely, bounded by two conductors at potentials V and v, and bearing respectively charges + Q and − Q. The energies of the charges are respectively $\frac{1}{2}QV$ and $-\frac{1}{2}Qv$; hence the total energy in this inner field is $\frac{1}{2}Q(V-v)$.

Now, because of the screening effect of the conducting sheet, the energy of the inner field will remain unaffected, whatever change takes place outside. Let us connect the conducting sheet to earth—that is, reduce it to zero potential. This will annihilate the outer field altogether, and an amount of electrical energy $\frac{1}{2}Qv$ will disappear as such, and will be transformed into the heat, light, and sound of the spark. What happens to the potential of the inner conductor? Its value V′ (fig. 100*b*) must be such as to keep the electric forces of the inner field just as they were. The energy of this field will be unchanged, or

$$\frac{1}{2}QV' = \frac{1}{2}Q(V - v).$$

Hence the potential of A is diminished by the same amount as is the potential of the conducting sheet.

If C, C′ are the capacities of A in the two states represented by fig. 100 (*a* and *b*), then

$$Q = CV = C'V' = C'(V-v),$$

or C′ is greater than C in the ratio of V to $(V-v)$.

In other words, by merely surrounding a charged body by a conducting shell at zero potential, we greatly increase its capacity. This is the principle of all condensers.

In this the discussion is simplified by taking a special case in which the conductor at zero potential does not alter the distribution of the charge on the conductor, originally taken as situated in free space. The conducting sheet completely encloses the conductor in such a way that not only is the capacity of the conductor as a whole increased, but the capacities of its different surface portions are increased all in the same ratio. Practically such a case rarely exists.

§ 64. **Experiments with the Aepinus Condenser.**—As a general rule, when, as described in § 54, the capacity of a conductor is increased by the approach of a second conductor at zero potential, this influence is exerted mainly upon a particular part of the conductor, on which part, accordingly, the greater portion of the charge is *accumulated* or *condensed*. The apparatus known as the **Aepinus Condenser** is specially constructed to show this effect.

It consists of two parallel conducting plates, C, E (fig. 101), supported on insulating rods, and capable of being

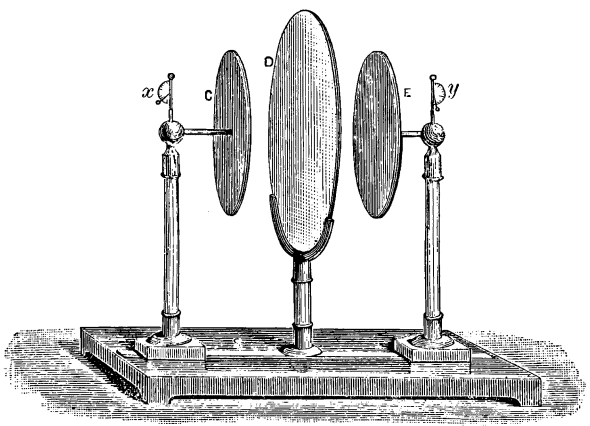


Fig. 101.

adjusted at different distances from one another. The large disc D is made of some insulating material, usually glass. (Very instructive experiments on the screening effects of

conductors can be made by substituting a large metal plate for this disc.) To the conductors are attached electroscopes x and y, whose mode of action is very simple. An upright metallic rod (see fig. 102) bears, suspended from its upper end, a light rod (also metal), with a pith-ball at its free end. A divided arc (usually a quadrant) of non-conducting material is fixed to the upright rod, so as to measure the deflection of the light rod when the apparatus is charged. For when (fig. 102) the instrument is fixed to a charged conductor, the pith-ball, being similarly electrified, is repelled by the rod, and rises until gravity and the electrical forces produce equilibrium. Such electroscopes (called after Henley, who first constructed them) are very serviceable in indicating changes of electrical condition. For effective working they need stronger charges than are needed with the gold-leaf electroscope. We may dispense with the Henley electroscope, and use the gold-leaf electroscope instead.

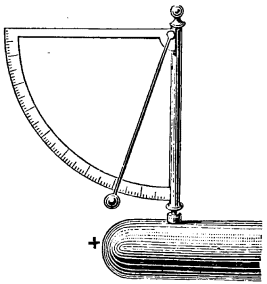


Fig. 102.

The glass disc D is chiefly useful inasmuch as it enables us to bring the plates highly charged much nearer each other than would be possible if only air intervened. As we shall see later, glass as a dielectric has a most important influence on the inductive action between the two plates. But in discussing the experiments now to be described, we may disregard the glass disc altogether.

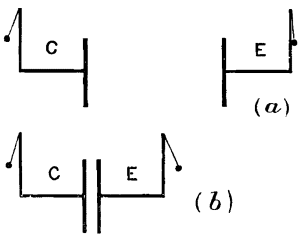


Fig. 103.

(1) With both plates far apart and insulated, charge one, C, positively. The one electroscope will show C strongly

charged, and the other (if delicate enough) will show E slightly charged by induction (fig. 103*a*). E will be at a potential intermediate to that of C and zero.

(2) Slide E nearer. The potential of E will rise, and that of C will fall (fig. 103*b*).

Work has been done in effecting a certain transference of charge along E. Electrical energy ($\frac{1}{2}QV$) has therefore been lost. But the charge on C has remained constant. Hence the potential must have diminished, and the capacity increased. The *chief increase of capacity* has taken place *on the plate* because of the proximity of the other plate. Consequently the charge, distributing itself over the plate and connected electroscope, in accordance with the various capacities of the different parts, accumulates *more on the plate* and *less on the electroscope.*

(3) Touch E to earth—that is, bring it to zero potential. A spark will pass showing a further and very considerable loss of energy. The potential of C will fall off the more, and the capacity as a whole will increase proportionately. But, again, this increase of capacity will take place chiefly on the plate, and a still smaller fraction of the charge will be left on the electroscope. By far the greater number of lines of force (§ 57) will spring from the plate of C, and the majority of these will fall on the opposing plate of E. A negative charge will accumulate on E opposite to the condensed positive charge on C.

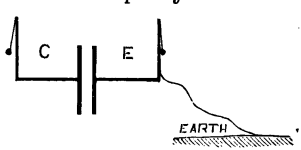


Fig. 104.

(4) Insulate E once more, and then slide it back to its first position. Just before removal, it was at zero potential in a region which, because of contiguity to C, would naturally be at a considerably higher potential. The potential of E must therefore diminish as it is removed from the neighbourhood of C. After removal, E will consequently have a negative potential and a negative charge. Simultaneously the

potential of C will rise; the capacity will diminish *chiefly* on the plate. The electroscope will receive more of the charge, and a greater deflection will result (fig. 105).

(5) Bring the condenser plates close, but not in contact, as in fig. 104, and let C be still positively charged. Touch E to earth. Then insulate E. Now touch C to earth. A slight shock will be felt, as C is by this means reduced to zero potential. At the same time E suffers also a reduction in potential, which now becomes less than zero. The electroscope on E will show a slight deflection. Next, insulate C once more, and again touch E to earth. A slight positive charge will flow into E, or a slight negative charge will flow out of E, so as to raise it up to zero potential again. Simultaneously C will be slightly raised in potential. Finally, insulate E, and yet again touch C. Exactly similar effects will be obtained.



Fig. 105.

In this way, by touching each conductor alternately, the other being at the same time insulated, we gradually diminish the charge by taking away successive small portions.

But connect both to earth, or join the one to the other, and a comparatively powerful discharge will occur.

Or, remove the one far from the other, and touch either; a strong discharge will result.

(6) Set the condenser plates close together without touching, and let both be insulated and uncharged. Begin to charge one, C, with an electrophorus. After a few sparks have passed from the electrophorus plate to the condenser plate, no more will pass. The condenser has been raised to practically the same potential as that possessed by the electrophorus plate when it is brought freshly charged from the electrophorus. But connect E to earth, and begin again to charge C with the electrophorus. It will be found that the charged electrophorus plate may be discharged a great

number of times into the condenser plate. By connecting E to earth we so increase the capacity of C, that practically the whole charge on the electrophorus plate passes into it at each successive discharge.

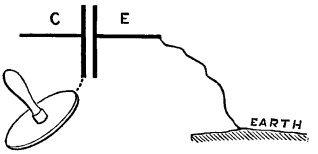


Fig. 106.

This experiment is a particularly instructive one, and may be performed with any form of condenser, such as Franklin's Pane or the Leyden Jar.

Franklin's Pane consists simply of a plate of glass coated on both sides with tinfoil. The pieces of foil separated by the glass play the same rôle as the plates in the Aepinus condenser. With both foils insulated, a comparatively small charge can be accumulated on either. But with one foil, U, to earth, as in fig. 107 (*t*), a large charge may be accumulated on the other foil, I.

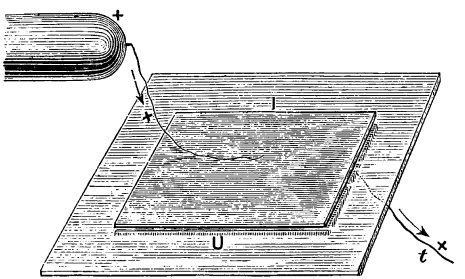


Fig. 107.

A simple form of condenser may be made with sheets of tinfoil separated by sheets of paraffin paper—that is, paper soaked in melted paraffin, lifted out and allowed to cool.

Of all condensers the Leyden jar is one of the most important. It is described below (§ 67).

§ 65. **Calculation of the Capacity of an Ideal Condenser.**—Let the conductor be a sphere, whose capacity in free space is measured by its radius *a*. The equipotential surfaces are also spherical. Choose one of radius *b*, and let it, as in the previous

paragraph, become a conducting sheet. Regarded as a conductor in free space, the capacity of this shell is b. The charge on A being + Q, its potential is Q/a. Similarly, the potential of the conducting sheet is Q/b. But now reduce this outer shell to zero potential. Then at once the potential of A falls off to the value

$$V = \frac{Q}{a} - \frac{Q}{b} = Q\frac{b-a}{ab},$$

or

$$Q = \frac{ab}{b-a}V,$$

where the quantity $\frac{ab}{b-a}$ is the capacity of the sphere in its new environment. If the sphere and shell are separated by a very small distance, t, of air, we may write

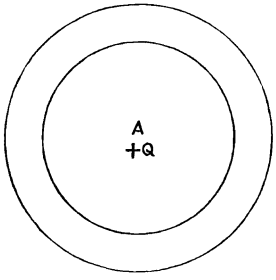


Fig. 108.

$$\frac{ab}{b-a} = \frac{a(a+t)}{t} = \frac{a^2}{t}$$

approximately as the capacity of this condenser.

For example, compare the capacities of a sphere of radius 10 in free space and an equal-sized sphere enclosed in a shell of internal radius 10·1.

The capacity of the sphere = 10

" " " condenser $= \frac{101}{\cdot 1} = 1010.$

The capacity is increased fully 100-fold by simply surrounding the sphere with the shell at zero potential.

Since the area of a spherical shell is $4\pi a^2$, we find at once for the capacity of unit area of the spherical surface the quantity

$$\frac{a^2}{t}\Big/4\pi a^2 = \frac{1}{4\pi t}.$$

This quantity is independent of the radius, and may be applied to the case of an ideal condenser formed of two infinite parallel planes (which are simply spheres of infinite radius) separated by the distance t. It may also be applied approximately to plane discs of finite size, provided their distance apart is very small compared to their smallest diameter. For such a pair of discs, the capacity will vary inversely as the (small) distance between them.

§ 66. **Specific Inductive Capacity of the Dielectric.**—It was by the use of spherical condensers that Faraday discovered

the rôle played by the dielectric in inductive processes. He found that the capacity of such a condenser depends not only on the radius of the sphere, and the thickness of the dielectric layer, but also on the nature of this dielectric. We now know that Cavendish had made this important discovery last century. One example will suffice to show its significance.

Suppose we have a series of condensers, made of the same conducting material, and all exactly alike as regards their geometrical and space relations; but all differing as regards the dielectric which separates their plates. For example, let one have air for its dielectric, another plate-glass, another paraffin, and another mica. Connect the one set of condenser plates to earth, and charge the others all to the same potential. Test each charge separately. The charges will be found to be all different, being in the four cases mentioned approximately proportional to the numbers 1, 6, 2, 6·6. Since the condensers differ only in the character of the dielectric, these numbers measure what Faraday called the **specific inductive capacities** of air, plate-glass, paraffin, and mica.

Thus, by merely inserting a plate of mica between two parallel plates of an air condenser, we increase the capacity by 6·6 times. Now (§ 65) the capacity of an air condenser composed of two parallel plates very close together, is inversely as the (small) distance between the plates. Hence the insertion of the mica has the same effect on the capacity as if we had approached the plates through a distance equal to ·85 (= 5·6/6·6) of the thickness of the mica.

In using glass as the dielectric in a condenser, we secure two advantages. First, we are able to have the charged conducting surfaces much closer than if only air were used; and second, we get a greater capacity, other things being equal.

§ 67. **The Leyden Jar.**—This is one of the most familiar and useful forms of condenser. As generally constructed, a Leyden jar is a cylindrical glass bottle lined inside and

outside with metal foil up to within a short distance from the top (fig. 109). A brass rod connected below with the inside coating passes upward through the cork or stopper, and terminates generally in a ball or knob. A Leyden jar, in short, consists of two conductors, the one almost completely enclosed in the other, and separated from it by the thickness of the dielectric. If either conductor is earth-connected, and the other insulated and charged, an opposite and nearly equal charge is induced on the former. If we could completely surround the one conductor by the other, the induced charge would be exactly equal but opposite to the inducing charge.

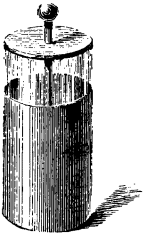


Fig. 109.

Let the outside coating be connected to earth. Then the inside coating is the part of the other conductor which has the great capacity. The brass rod and knob which form the rest of the conductor will have a comparatively small capacity, and a very small fraction of the whole charge will be found on the knob. This is called the Free Charge. We may remove it by first insulating the outside coating, and then touching the knob with the finger. A very slight shock will be felt. If, however, the outside coating be to earth, and the knob of the charged jar be brought near any conductor also to earth, a violent discharge will take place.

A Leyden jar is charged very simply by holding the outside coating in the hand and presenting the knob to the prime conductor of an electrical machine. It is not necessary to have the knob in contact with the conductor, but it is better to be so. We are then sure that the jar has the potential of the conductor at the instant of separation.

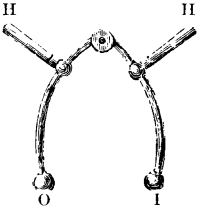


Fig. 110.

Because of the powerful charges that can be accumulated in the Leyden jar, it is dangerous to

discharge it by touching with the finger. It is necessary to use 'discharging tongs,' which are simply adjustable tongs with insulating handles. One form is shown in fig. 110. H, H are the insulating handles, and by them must the instrument be held. In using it for discharging a Leyden jar, we place the one leg, O, in contact with the outside coating, and bring the other end, I, near the knob. At a certain distance, depending upon the potential of the charged conductor, the discharge occurs.

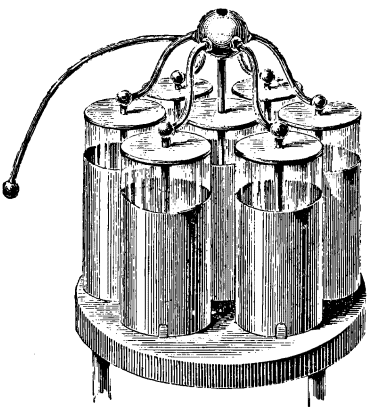


Fig. 111.

For large effects, Leyden jars may be used in combination. Such a combination is represented in fig. 111, and is called a Leyden Battery. Here all the inside coatings are joined together so as to form one conductor; and the outside coatings are all connected to earth. The capacity of these six equal jars will be six times the capacity of one. If the capacity of each jar is C, and if V is the potential of the prime conductor from which the jar is charged, the total charge accumulated in the battery will be 6CV.

A very neat experiment is the following. Construct a Leyden jar with removable coatings (fig. 112). Charge it in the usual way, with the outside coating to earth. Insulate the outside coating, and lift out the inside conductor. Only a slight shock, due to the 'free charge,' will be felt. Now lift the glass portion, D, out of the outside conductor, which may then be itself lifted up with impunity. The two conductors will be devoid of charge; but the glass surfaces will be found to be strongly charged. Restore the glass to its

position in the outside conductor, and place the inside conductor back again, care being taken to have one conductor at least always insulated. On testing, we find a large charge still in the jar. This experiment emphasises the importance of the dielectric in all electrical phenomena. The conductors simply act as terminal surfaces, from which the lines of force spring, and on which they end. The surfaces of the dielectric may perform the same function.

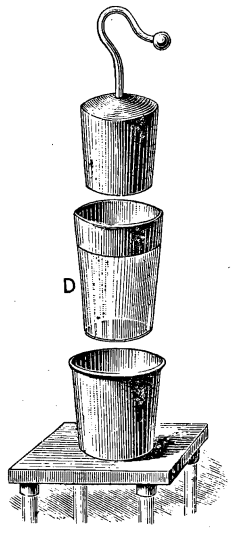


Fig. 112.

Charge a Leyden jar, and leave it alone for some little time. Now discharge it, and again leave it to itself for a few minutes. Bring the outside and inside conductors again into connection by means of the discharging tongs. A small but evident spark will be seen. This is due to what is known as the **Residual Charge.**

Closely connected with this residual charge is the gradual loss of potential which occurs for a short time after a Leyden jar has been charged. The effect is as if some of the charge penetrated into the dielectric. After the first discharge, the residual charge begins slowly to collect, and appears as if it were returning out of the dielectric. The real explanation of the phenomenon, however, is to be found in the heterogeneousness of the dielectric and the effect of the electric stress upon it. The fall in potential immediately after charging is due to the dielectric yielding in places to the electric stress acting on it. The rise in potential immediately after discharge is due to the gradual recovery of the dielectric from its strained state. The phenomenon occurs in most condensers with solid dielectrics, and depends upon the nature of the dielectric.

EXERCISES.

1. The inner coating of a Leyden jar is connected by a wire with the prime conductor of an electrical machine and also with a gold-leaf electroscope. If the jar rests upon a sheet of glass, a quarter of a turn of the machine produces a large divergence of the leaves of the electroscope. If the glass be removed, ten turns of the handle are required to produce the same deflection. Explain this. (1888)

2. A Leyden jar was twice charged and discharged. The first time the knob was held a quarter of an inch from the prime conductor of the electric machine; the second time the knob was in contact with the prime conductor. Both times the machine was worked for the same interval. In which case will the discharge spark be the brighter? Give reasons. (1890)

3. In the Aepinus condenser, suppose one side of the glass disc that is usually placed between the metal discs to be coated with tinfoil. What effect upon the operations described in § 64 will this tinfoil have (1) when it is insulated, (2) when it is connected to the earth?

4. Given one Leyden jar charged in the usual way with the outside coating to earth. A second Leyden jar of equal capacity is seized by the knob, and its insulated outside coating is brought into contact with the knob of the first jar. Both jars are then set on insulating stands, and the outside coatings are connected through the discharging tongs. What are the electrical conditions of the two jars before and after this last operation? With the outside coatings still connected, join the knobs. What will be the final electrical conditions of the jars?

5. The prime conductors of a Wimshurst machine are connected to the knobs of two Leyden jars (each to one), whose outside coatings are to earth. What effect is produced upon the rate and character of discharge, as the machine is turned steadily with the discharging terminals at a given distance apart?

6. The charge in a condenser is shared with two other condensers of equal capacity which were originally uncharged and at zero potential. Compare the final common potential of the three with the original potential of the one; and express the loss of energy in terms of the original charge and potential.

SUMMARY.

(63) Principle of the Condenser. Simple case of sphere inside spherical conductor.

(64) Experiments with the Aepinus Condenser. Franklin's Pane.
(65) Calculation of the Capacity of an Ideal Condenser.
(66) Specific Inductive Capacity of Dielectrics.
(67) The Leyden Jar. Residual Charge.

CHAPTER XI.

THE ELECTRIC DISCHARGE.

§ 68. **Dielectric Strength.**—The spark, so familiar in all electrical experiments, is an electric discharge in air. Similar discharge may take place in any dielectric, gaseous like air, liquid like paraffin-oil or turpentine, or solid like glass or mica. When the discharge takes place through a solid dielectric, the dielectric remains pierced or shattered. Thus Leyden jars have been cracked under the influence of a powerful charge.

In all such cases, the phenomenon is essentially the same. Two conducting surfaces, separated by the dielectric, are at different potentials. At a particular difference of potential, depending on the form and size of the conductors and their distance apart, the electric stress which constitutes the electric field in the insulating medium becomes so intense that the medium yields, and the electrical energy becomes transformed into light, sound, heat, and mechanical energy. The strength of field depends upon the closeness of the lines of force, and this closeness is directly as the electrical density on the conducting surface on which the lines of force begin and terminate. Thus there is greater chance of a discharge taking place when these conducting surfaces are much curved, than when they are plane.

In comparing the strengths of dielectrics in resisting discharge through them, we must have the conducting terminals, between which the discharge takes place, under the same conditions with all. Experiment then shows that the difference of potential needed to effect the discharge is

different for different dielectrics, which accordingly differ in what is known as dielectric strength.

§ 69. **The Character of Discharge.**—The character of the discharge depends very greatly upon the form and size of the terminals between which the discharge takes place.

Take, for example, two rounded conductors, and place them

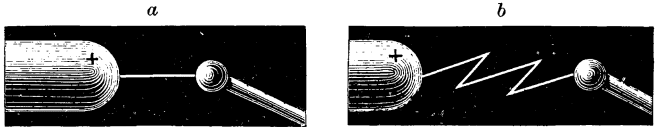


Fig. 113.

at different distances. When they are close, the discharge is an approximately straight spark (fig. 113*a*). Draw them a little further apart, and the spark becomes zigzag (fig. 113*b*). Draw them still further apart, and the spark becomes branched (fig. 114). These are all cases of the *spark discharge.*



Fig. 114.

The *Brush Discharge,* so called from its appearance as seen in the dark, is best obtained when a very large conductor is held at a little distance from a small knob or ball in connection with a highly charged conductor. It is shown in fig. 115. This discharge is accompanied by a hissing or crackling noise. It is often to be seen, and always to be heard, escaping from the points or knobs of the prime conductors of a well-conditioned electrical machine.

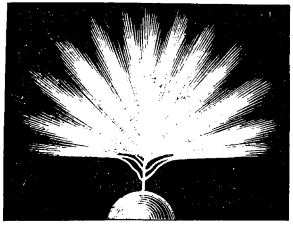


Fig. 115.

The *Glow Discharge* occurs round any sharp point of a charged conductor. The electric convection, by which the charge at a sharp point is dissipated,

has already been described (§ 62). To see the glow, we must make the experiment in a darkened room. This glow is sometimes seen at the tops of lightning conductors and masts of ships. Sailors call it St Elmo's Fire.

The colour of the electric discharge depends upon the nature of the metal terminals between which the discharge takes place, and the nature of the dielectric through which it passes. With powerful discharges, the influence of the metal predominates; with feeble discharges, that of the dielectric. In rarefied gases, the colour of the electrical discharge is very characteristic. Thus, in air or oxygen, the colour is white, with a touch of blue; it is blue in nitrogen, red in hydrogen, and green in carbonic acid gas.

The most familiar of all electrical discharges is lightning. See below, § 72.

§ 70. **Effects of Electrical Discharge.**—The electric spark is evidently a source of light and of heat. It may, accordingly, be used for producing effects similar to, or even identical with, those produced by light and heat. Such effects may be distinguished as indirect effects.

As an example, we may refer to the explosion of a mixture of oxygen and hydrogen to form water. If the explosion is made in a strong vessel, with a cork fitting moderately tightly, the explosion will drive the cork out. This is the construction of an electrical toy known as *Volta's Pistol.*

Similarly, we may use the electric spark to fire gunpowder or light a gas-jet; but all such experiments, striking though they are, are hardly, in a strict sense, electrical.

The shattering and piercing effects of the electric discharge have already been referred to. A useful piece of apparatus for studying these effects is the **Universal Discharger** (fig. 116). It consists of two movable brass arms insulated on glass rods. The lower ends are adjusted at a suitable distance from the object to be experimented on, which rests on the small table S. One of the upper ends (O, say) is joined to the outside coating of a Leyden Battery. The

other end, I, is then brought into connection with the inside coating by means of the discharging tongs, when a violent discharge passes between the knobs and the object. Experiments of this kind impress upon the mind the extraordinary concentration of energy associated with the electric discharge.

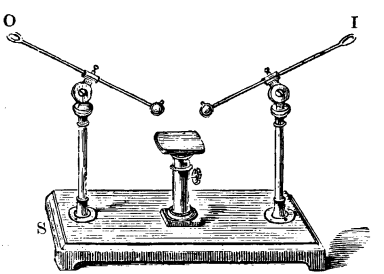


Fig. 116.

The **Electric Shock** is the most familiar of the physiological effects of the electric discharge. It is hardly possible to work to any extent with electrical apparatus without experiencing such shocks. They are not unpleasant when the charges are small. But with large Leyden jars and condensers too great care cannot be taken, as a shock from a battery may prove fatal.

The **Chemical Effects** of the electric discharge were distinctly recognised towards the close of last century, when Van Troostwijk and Deiman effected the decomposition of water by its means. Similar decomposition will occur if the discharge be taken through a saline solution. For example, dip two wires leading from the prime conductors of a Wimshurst machine into a solution of sulphate of copper, and then set the machine in operation. A flow of electricity will pass through the solution, and the wire in connection with the *negative* conductor will become coated with copper. The significance of this result will be discussed in Chapter XV.

The very peculiar odour which is developed by the working of an electrical machine is best classified as a chemical effect. It is due to the presence of *ozone*, into which the electrical discharge transforms the oxygen of the air.

The **Magnetic Effects** of the electric discharge early attracted attention. Magnets were made or altered by dis-

charging electricity through them; but the results were too irregular and haphazard to lead to any certain conclusion. Not till after Oersted's discovery in 1820, which laid the foundations of electro-magnetism, was it recognised that magnetism was not a property peculiar to iron or lodestone. The magnetic effect of an electric discharge may be shown thus. Coil a piece of well-insulated wire round an astatic magnetic needle adjusted so as to point in the plane of the coils. Discharge a Leyden jar through the coil. At the instant of discharge the needle will be slightly deflected. The direction of deflection will depend on the direction in which the positive electricity passes round the coil.

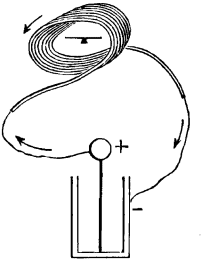


Fig. 117.

The **Duration** of the electric spark is extremely short. Luminous impressions on the retina of the eye persist for about $\frac{1}{10}$th of a second; consequently it is impossible for the eye to judge as to the duration of any short-lived luminous phenomenon. Wheatstone proved by means of a rotating mirror that the electric spark did not last longer than $\frac{1}{24000}$th of a second. That it lasts a very short time may be demonstrated to the eye in the following manner. Set a wheel or radially striped disc in rapid rotation. Looked at in ordinary light, the individual spokes or stripes are indistinguishable. But place the wheel in a dark room, and illuminate it by means of a spark. Then at each flash the rapidly rotating wheel will appear to be perfectly steady. Bullets moving swiftly through the air have been sharply photographed by simply using the electric spark as an instantaneous source of light.

§ 71. **Discharging Power of Flames.**—Pass a rod of sealing-wax, which has been electrified by friction, quickly through a flame. It will completely lose its charge. This method of removing the charge from an electrified dielectric is most

efficient. A conductor may be discharged by mere connection with the earth; but it is not possible so to discharge a non-conductor.

A flame placed above a charged conductor, but insulated from it, also acts as a rapid discharger. The simplest way to make this experiment is to set a spirit-lamp on a charged gold-leaf electroscope. As soon as the lamp is lighted, the gold leaves collapse.

A white-hot iron ball, at zero potential, acts similarly if held over an electroscope. As it cools to dark heat, it loses the power of so discharging a positively charged electroscope, but it will still discharge one negatively electrified. In these experiments it appears that the hot-air current must be the really efficient factor; but the precise nature of the phenomenon is not well understood.

§ 72. **Atmospheric Electricity.**—The atmosphere is generally electrified—that is, it is at a different potential from the earth. In fine weather the potential is usually positive. Under certain conditions this potential may vary quickly from positive to negative, or from negative to positive; and contiguous regions in the air have frequently quite different potentials. The great development of electricity in thunder-clouds shows to what an extent the electrical conditions in the atmosphere may vary.

Exactly how the charges begin to develop on the rain-drops which constitute the cloud is not well understood. But if we suppose that the very minute drops first formed are all charged, then it can be shown that the coalescence of these to form larger drops must be accompanied by a rise in potential. For example, let eight equal-sized drops at the same potential coalesce into one. This single drop will have twice the diameter, and therefore twice the capacity of each of the original drops. Now the potential is equal to the charge divided by the capacity; hence we find

$$\frac{\text{Potential of final drop}}{\text{Potential of each original drop}} = \frac{8/2}{1/1} = 4.$$

Now, although we cannot apply this simple calculation to the case of a cloud of drops, since the potential of any one drop, say, in the heart of the cloud, depends to a great extent upon the presence of the surrounding drops, nevertheless this coalescence into larger drops is a process which is always going on all through the cloud, and its effect must be in the direction indicated. In this way the potential of the region occupied by the cloud becomes to a greater and greater degree different from the potential of the earth, or from the potential of a contiguous atmospheric region; and when the difference of potential is sufficiently great, a discharge will take place in the form of a lightning flash, either from cloud to earth, or from cloud to cloud.

Franklin first demonstrated the electrical condition of the thunder-cloud by means of a kite. The kite and wet string formed a conductor, on which the inductive effect of the charged cloud was made manifest.

Forked Lightning is evidently the electric discharge itself. It has all the characteristics of the electric spark upon a large scale. It may take place between cloud and cloud, or between cloud and earth. Its zigzag form is probably due to the varying condition of the air through which it passes. If the flash take place behind a cloud or at a great distance, so that it is itself invisible, its luminous effect is evident as *sheet-lightning.*

Thunder is the audible accompaniment of the lightning flash, as it passes through the air. According to the character or intensity of an electric discharge, the accompanying noise may be a hiss, a crackle, a snap, or a thunder-clap. The rolling and reverberation so familiar in thunder-storms are due to echoes of the original thunder-clap from clouds or masses of air of different densities. Also with long flashes, the sounds from the various points of the path, having different distances to travel, may reach the ear at different times, and give a succession of peals. Since light travels practically instantaneously between terrestrial points, whereas

sound takes about five seconds to the mile, it is a simple piece of arithmetic to calculate the distance of the flash from us, if we know the interval of time between the lightning and the thunder.

When a highly charged thunder-cloud is poised over the earth, the region between it and the earth is an electric field; and corresponding to (say) the positive charge in the cloud there is an induced negative charge on the surface of the earth immediately beneath. When a lightning flash passes, the cloud and earth are momentarily reduced to the same potential, the electric field is destroyed, and the opposing charges disappear. This sudden change of electrical condition may be felt as a shock by a person at a considerable distance from where the flash took place. This is called the *return shock*, so called from the fact that the negatively charged condition of the surface of the earth just before the flash is neutralised by an inflow of positive electricity.

The function of lightning conductors has been already discussed in § 62.

The **Aurora Borealis** is of the nature of an electric discharge through the upper regions of the atmosphere, and is closely connected with the abrupt changes in the magnetic elements known as magnetic storms.

EXERCISES.

1. Sparks pass between the prime conductor of an electrical machine and a metal knob, connected with the earth, held near it. Describe the changes, if any, in the phenomena observed, as the knob is gradually moved away from the prime conductor. (1889)

2. The ebonite portion of an electrophorus is charged with electricity. What means would you take to completely discharge it? (1891)

3. A frictional machine is being worked steadily so that sparks pass at regular intervals between the prime conductor and a neighbouring conductor put to earth. If the prime conductor be connected to the knob of a Leyden jar, whose outside coating is also to earth, what changes are produced in the character of the sparks, and the rate at which they come, other conditions being the same?

4. Discuss the experiment described in § 58, when one of the hollow conductors is kept connected to the earth, the other being initially at a small negative potential.

SUMMARY.

(68) Dielectric Strength, or Resistance of Dielectrics to Discharge taking place through them.

(69) Character of Discharge. Spark. Brush. Glow. Colour.

(70) Effects of Electrical Discharge. Indirect effects. Direct effects. Electric Shock. Decomposition of certain Liquids. Development of Ozone. Magnetic Effects. Duration of Spark.

(71) Discharging Power of Flames.

(72) Atmospheric Electricity. Lightning. Thunder. The return shock. Aurora Borealis.

CHAPTER XII.

VOLTAIC AND CURRENT ELECTRICITY.

§ 73. **Difference of Potential due to Contact.**—In § 43 (footnote), we have referred to the discovery made by Volta,* that, when any two different metals are placed in contact, insulated and separated, the one will be found positively and the other negatively electrified, although the region in which they are is not an electric field. The charges produced in this way are very small compared to the charges usually obtained with frictional electricity; but they may be shown with a sufficiently delicate gold-leaf electroscope of ordinary construction. The gold leaves must be as light as possible, and the plate as large as possible.

* The remarkable discoveries of Volta were historically connected with a chance observation made by Galvani, a physician of Bologna. In 1786 Galvani noticed that, by mere contact with two dissimilar metals, the leg of a recently killed frog could be set into convulsive motions exactly similar to those that could be excited by electrical means. Galvani gave a physiological explanation of the phenomenon; but Volta showed that electrical effects could be produced from the contact of the dissimilar metals. Thence sprung the whole science of voltaic or current electricity, also known as galvanism, or galvanic electricity.

Place over the plate of such a gold-leaf electroscope a thin sheet of non-conducting material, as thin as possible. A sheet of mica acts admirably. Over this place a plate of zinc, with an insulating handle fixed to it (fig. 118). Now bring the two metal plates, the zinc and the brass, into contact by means of a wire (fig. 118*a*). Remove the wire, and lift away the zinc plate. The gold leaves will diverge slightly (fig. 118*b*). Discharge the zinc plate by touching it, bring it back again insulated, connect the plates again by means of an insulated wire, disconnect, and lift away the zinc plate again. The gold leaves will diverge still farther. With a copper plate instead of the brass, somewhat larger effects will be obtained. By testing with a rubbed piece of sealing-wax, we find that the brass or copper plate is left *negatively* electrified. If we interchange the plates, and manipulate with the copper plate, the electroscope will become *positively* charged.

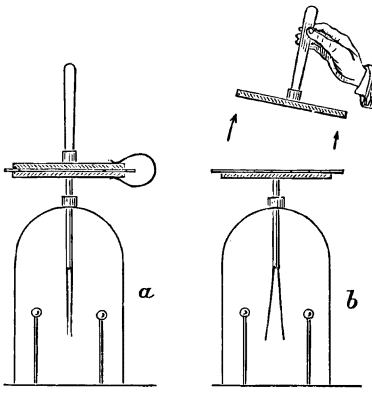


Fig. 118.

If, again, we discharge the plates separately to earth, lay them in position, but do not connect them by a wire or other conductor, then, on raising the upper plate, we shall get no effect on the electroscope.

From this experiment we learn that two different metals in contact are at different potentials. In the first case described, the zinc is at a higher potential than the brass. By lifting the zinc away, we diminish the capacity of each plate. Hence a portion of the charge passes into the gold leaves.

If we use two plates of the same kind, say two brass plates, and go through exactly the same series of operations, no charge will be obtained on the electroscope. In this form the electroscope is known as the *condensing electroscope.*

§ 74. **The Voltaic Cell.**—Instead of connecting the two like plates of the condensing electroscope by means of a single wire, let them be connected through a chain of different metals in contact with one another. Then, on disconnecting and lifting away the upper plate, we shall find no effect on the electroscope. But let the connecting chain of materials be a copper plate, a zinc plate, a piece of cloth or paper moistened with dilute acid, and a copper plate as shown in fig. 119. Then, on disconnecting and lifting away the upper plate, we shall find the gold leaves diverging slightly with *positive* electricity.

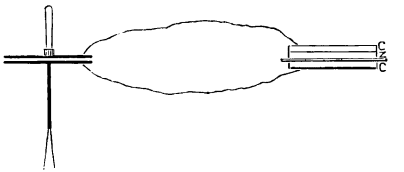


Fig. 119.

Or, let two strips, one of copper and one of zinc, dip into a glass vessel filled with dilute sulphuric acid. Join the strips, by means of (say) copper wires, to the brass plates of the condensing electroscope. Suppose the zinc strip to be connected to the lower plate. Insulate the plates, and lift away the upper one. The gold leaves will diverge with a *negative* charge. Thus we learn that, if we form a chain of the materials brass, zinc, dilute acid, copper, brass, the initial and final brasses are at different potentials. If we substitute for the dilute acid any metallic conductor, the two brasses will be at the same potential. Or, if we have the dilute acid in contact with the *same metals on both sides*—as, for example,

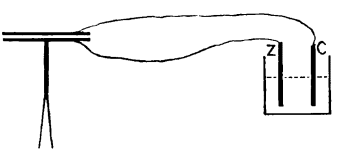


Fig. 120.

copper, acid, copper—no difference of potential will be obtained.

The important substances are clearly the *zinc*, *dilute acid*, and *copper*, in *combination*. They form what is called a **voltaic cell.** Since they give to the terminal brasses different potentials, we should expect to get a transference of electricity from the one to the other, when they are joined by a conducting wire. This is found to be the case. The transference is not a mere short-lived discharge as from a Leyden jar. It is a continuous discharge or *current*, whose properties are now to be studied.

The important *electrostatic* feature of the voltaic cell is the difference of potential it establishes between its terminals. By using a great many cells suitably joined together, we can increase the difference of potential. Volta's Pile is such an apparatus. It is shown in fig. 121. It consists simply of a pile of single cells—zinc, moistened cloth, copper being repeated in order throughout.

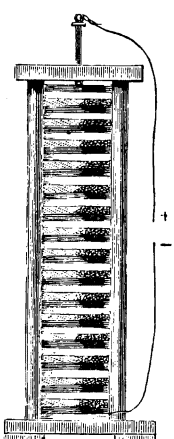


Fig. 121.

Or, we may join together in similar succession a series of simple cells of the kind just described. Four such cells are shown in fig. 122. The copper of the first is joined to the zinc of the second; the copper of the second to the zinc of the third; and so on. Each succeeding zinc is higher in potential than the preceding zinc by an amount equal to the difference of potential produced by one cell. Hence the terminal wires differ in potential by four times this difference of potential. And, generally, the difference of potential produced by a number of like cells in series is equal to the number of cells multiplied by the difference of potential produced by one. Such a combination of cells is called a Voltaic Battery. With a battery of ten or a dozen cells there is no difficulty in making the experiments already described in § 73.

§ 75. **The Chief Effects of the Voltaic Current.**—The

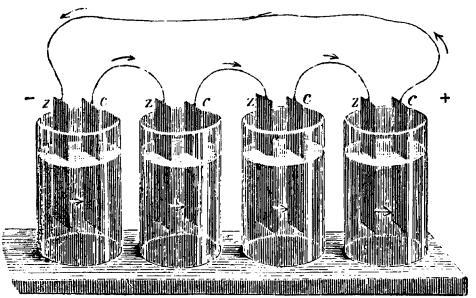


Fig. 122.

chief effects of the voltaic current are three—namely, *magnetic*, *thermal*, *chemical*. These are all effects likewise of the electrical discharge. Thus the electrical discharge and voltaic current both require for their existence a difference of potential; both produce the same effects. Hence we conclude that they are really identical.

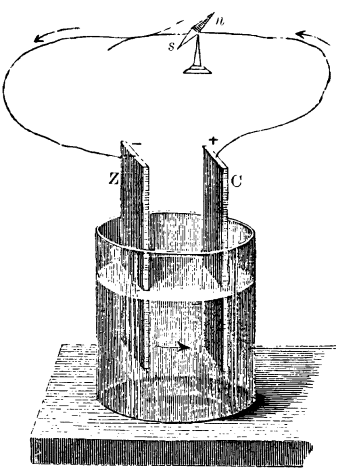


Fig. 123.

The *magnetic effect* is easily observed. We have simply to join the terminals of a cell by a wire, part of which is placed close to a magnetic needle. The needle will be deflected (fig. 123). This experiment was first made by Oersted in 1820, and forms the fundamental experiment of electro-magnetism.

To show the *chemical effect*, join three or four simple cells

in series. Dip the terminal wires (best of platinum) into a vessel of slightly acidulated water (fig. 124). Bubbles of gas

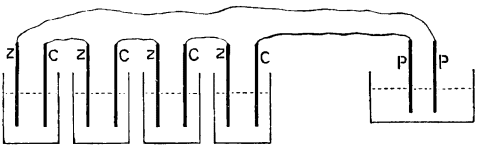


Fig. 124.

will rise from the wires. Or dip them into sulphate of copper, and a copper film will form on the wire connected to the zinc—that is, to the *negative* terminal (compare § 70).

To show the *thermal effect*, join the stout terminals of a voltaic battery by a short and very thin piece of iron wire. The wire will become red hot, or even white hot.

These effects mean that when the terminals, or *poles*, as they are called, of a voltaic cell are joined, work is being done, energy is being transformed. There must be in the cell somewhere a source of energy. On inspection, it will be found that changes are going on in the cell itself. Bubbles of gas will be seen forming on the zinc strip, and the zinc will gradually be consumed. Without this combustion of the zinc, no current could be produced; and all experiments go to show that the production of the voltaic current depends on chemical action.

§ 76. **Types of Voltaic Cells.**—There are two important types of voltaic cell, distinguished as the *One-fluid* and the *Two-fluid* cell.

The simple cell described at the beginning is evidently a one-fluid cell, the fluid being dilute sulphuric acid. When such a cell is working, bubbles of hydrogen gas are given off from the positive pole. Some of it clings to the positive metal, and greatly diminishes the effective electromotive force of the cell. The reason of this will appear later. For our present purpose it is sufficient to note that accumu-

lation of gases on the poles renders such cells very inconstant in their action. One of the most serviceable of one-fluid cells is the Bichromate of Potash cell. The poles are zinc and carbon; and the chromic acid derived from the decomposition of the bichromate of potash is utilised as an oxidising agent to get rid of the film of hydrogen which would otherwise collect. This cell is very useful when strong currents are required frequently, but for short periods. For a steady constant supply it is not efficient.

The two-fluid cells are, as a general rule, much more constant in their action. In these cells the two poles dip in *different* liquids, and the liquids are in contact usually through the pores of a porous wall.

The most familiar of the two-fluid cells are the Daniell, Grove, and Bunsen cells. The series of substances which build these cells up are shown in the following table:

Name of Cell.	+ Pole.	Liquids.		− Pole.
Daniell.....	Copper.....	Copper Sulphate (saturated).	Sulphuric Acid..... (dilute).	Zinc.
Grove.....	Platinum...	Nitric Acid (strong).	"	"
Bunsen.....	Carbon.....	"	"	"

When great constancy of action is aimed at, semi-saturated sulphate of zinc is substituted for the sulphuric acid in the Daniell cell. In the Grove and Bunsen cells, the hydrogen is prevented from accumulating on the positive pole by the chemical action of the nitric acid. In the Daniell cell, the hydrogen, which is set free by the formation of sulphate of zinc, replaces in its turn the copper of the copper sulphate, and this copper is deposited, in accordance with principles to be discussed later (Chap. XV.), in the metallic form on the copper pole.

There is one defect common to all these cells, known as *local action*. This is due to lack of homogeneity in the zinc plates. Particles of iron may be present, or some parts may be harder than others and differently acted upon by the acid.

The result seems to be that some parts of the zinc act as positive poles with regard to other parts, generating currents which pass from the zinc to the liquid and back to the zinc again. These exist whether or not the outside circuit be closed, and lead to a considerable dissipation of energy. Cells, in which this action is considerable, become inconveniently overheated. The defect may be largely remedied by amalgamating the zinc surface with mercury, an operation which makes it more homogeneous.

One simple form of Daniell cell is shown, both in general appearance and in section, in fig. 125 (*a* and *b*). The con-

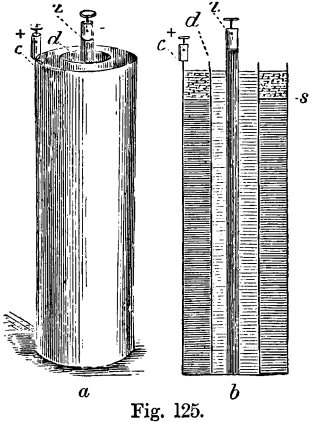


Fig. 125.

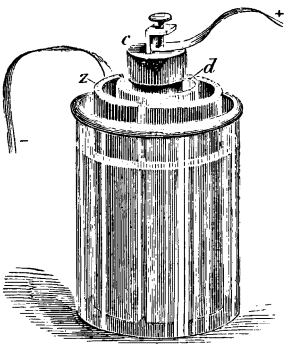


Fig. 126.

taining vessel, *c*, is of copper, and serves as the positive pole. Inside this is a vessel, *d*, of porous earthenware, containing a rod of zinc, *z*. The space between the vessels is filled with the sulphate of copper, which is kept concentrated by crystals of the salt lying on a shelf, *s*. The sulphuric acid fills the space between the zinc rod and the porous wall.

In fig. 126 is shown a very usual form of Bunsen cell. Here the zinc, *z*, is in the form of a cylindrical plate, surrounding the porous vessel, *d*, in which the nitric acid and carbon, *c*, rest. The whole is contained in a glass (or glazed

porcelain) vessel, and sulphuric acid is filled in between the vessels. The substitution of platinum for carbon changes the Bunsen into the Grove cell.

We postpone the consideration of the kind of cells known as Secondary cells, Storage cells, or Accumulators, until the principles of electrolysis have been established (Chap. XV.).

§ 77. **Conductors and Non-conductors.**—A current can flow in a circuit only when the circuit is composed of a continuous sequence or chain of conducting substances. As in electrostatics, so here, all ordinary metals are good conductors. But if we separate any metallic parts of the circuit by such materials as wood, dry cloth, dry paper, and generally any solid non-metallic substance (with the important exception of charcoal or carbon), we virtually break the circuit, and no appreciable current flows. Such substances are bad conductors, and are practically non-conductors when the differences of potential are small. But when the differences of potential are great, many of these substances conduct measurable currents. The differences of potential associated with voltaic currents are generally very much smaller than those associated with electric discharges. Consequently many substances which are practically non-conductors for current electricity cannot be used as insulators in frictional electricity. On the other hand, all dielectrics or insulators, such as glass, ebonite, gutta-percha, mineral and vegetable oils, benzol, &c., are perfect non-conductors for electric currents. Metals remain conductors in the liquid or molten state; and mercury is a most indispensable conductor in all electrical experiments.

Solutions, in general, belong to a peculiar class of conductors, known as electrolytes. They conduct after a fashion of their own, which will be discussed hereafter (Chap. XV.). The mixture of sulphuric acid and water, which entered into the composition of the simple cell described above, is an electrolyte; and, indeed, one of the constituents of a voltaic cell must belong to this class of conductors.

It is necessary, in making experiments, to have the wires

covered with some non-conducting material. The wires are then said to be insulated, and may be placed with their covers in contact, and yet be as electrically disconnected as if they were separated by a large air-space. The necessity for this insulation will be seen as we proceed. For ordinary purposes, a cotton thread covering serves sufficiently well. But if high potentials are being used so that really good insulation is needed, silk thread or gutta-percha covering is indispensable.

EXERCISES.

1. A strip of platinum and a strip of zinc dip into a vessel of acidulated water. How would you show that two copper wires fastened one to the zinc and the other to the platinum are in different electrical states? (1890)

2. How would the action of a Daniell's cell be modified if the solution of copper sulphate in the porous vessel were replaced by dilute sulphuric acid? (1890)

3. An insulated positively electrified gold leaf hangs half-way between two vertical insulated and unelectrified copper plates, A and B, to each of which a copper wire is attached. If plates of zinc and copper, which are partly immersed in dilute acid, are touched by the wires attached to A and B, respectively, and if afterwards the wires are both dipped into the acid, describe the movements of the gold leaf in each case. (1888)

4. Plates of zinc and copper are soldered together, and dipped into dilute sulphuric acid. What action will go on at the surfaces in contact with the liquid?

SUMMARY.

(73) Difference of Potential due to Contact. Condensing electroscope.

(74) Voltaic Cell. Permanent difference of potential between its poles.

(75) Magnetic, Thermal, and Chemical Effects of the Voltaic Current.

(76) Types of Voltaic Cells. One-fluid and two-fluid cells. Bichromate of potash cell. Daniell, Grove, and Bunsen cells.

(77) Conductors and Non-conductors. Electrolytes. Insulation.

CHAPTER XIII.

ELECTRO-MAGNETISM.

§ 78. **The Magnetic Field associated with a Current.**—Oersted's great discovery of the magnetic effect of a current has been referred to. It is this effect which provides us with the readiest means both of detecting and measuring a current.

In all ordinary cells in which zinc is one of the elements, the zinc forms what is called the *negative* pole. Towards it the current flows in the external wire which joins the poles. The direction of a current is determined by the same convention which originally distinguished positive and negative electricity. We think of a current as being a flow of positive electricity in a given direction; we might as well regard it as the flow of negative electricity in the opposite direction. The convention once made, it is well to adhere strictly to it. Brevity of expression is gained, ambiguity of meaning is avoided.

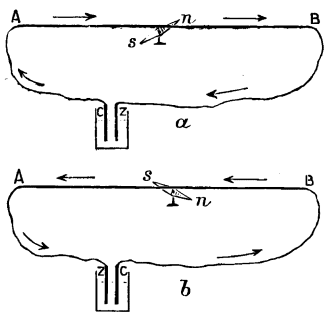


Fig. 127.

Let AB be a straight wire placed directly over the magnetic needle, and parallel to its axis. Join A by a wire to the copper of the cell, and B to the zinc. The magnetic needle will be deflected, with its north pole to the *west*, as shown in fig. 127*a*. But now join A to the zinc, and B to the copper. The magnetic needle will be deflected towards the other side, with its north pole to the *east* (fig. 127*b*).

From such experiments we conclude that the direction of the magnetic field at any point is reversed when the current is reversed, and has a definite relation to the direction of the current.

Now let the straight wire, AB, through which the current is passing, say, from A to B, be moved about in the vicinity of the magnetic needle. We learn at once that the deflection increases as the current is brought nearer. In other words, the *magnetic field associated with a given current is greater at the smaller distance.* Also the following facts are easily established.

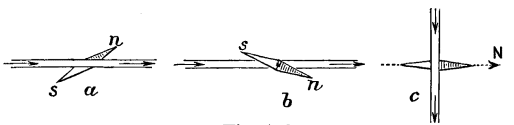


Fig. 128.

When the current flows from south to north *above* the magnetic needle, the needle turns towards the *west* (fig. 128*a*); when it flows from south to north *below* the needle, the needle turns towards the *east* (fig. 128*b*); when it flows from east to west, or west to east, either above or below the needle, the needle suffers no deflection (fig. 128*c*).

If we make the magnetic needle *astatic* by means of another magnet (see § 12), we find at once that the needle always sets itself, if possible, at right angles to the direction of the current. Such experiments show that the magnetic lines of force associated with a straight-line current are circles, with the current for their common axis. The direction of any line of force at any point is determined by a very simple rule. Let AB (fig. 129) be the conductor carrying the current from A to B. Round this conductor as axis draw a series of circles as indicated in the figure—each circle being

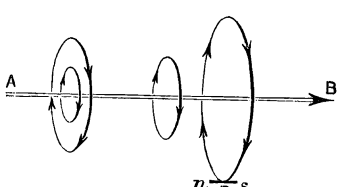


Fig. 129.

in a plane perpendicular to AB, and having its centre in AB. These circles are lines of magnetic force, and the direction of the lines of force is that direction which is right handed with reference to the direction of the current. That is to say, the direction AB is to the direction in the circles as the forward motion of an ordinary (right-handed) screw is to its rotation round its axis. Or otherwise, look along the current in the direction in which it is flowing (that is, from A towards B), then a clockwise motion round the current will give the direction of the lines of force. A magnet at any point, P for example, will tend to set itself along the line of force through P, and will so set itself, if no other force, magnetic or mechanical, prevent it.

§ 79. **Coils.**—In fig. 127 we have considered the field in the neighbourhood of only one part of the wire that helps to make the circuit. Suppose that part fixed in position ; and then bring up some other part of the wire also. When this second part is near enough, the needle will be distinctly affected, and will shift into a new position of equilibrium. The greatest deflection will be produced when the two strands are, the one close above the needle, and the other close below (fig. 130*a*) ; and the smallest deflection will be produced when the two strands are close together on the same side of the needle (fig. 130*b*).

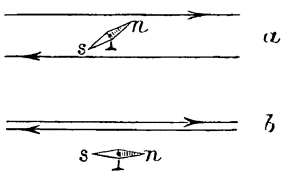


Fig. 130.

Twist an insulated wire back upon itself, so that everywhere its two halves lie as close to one another as possible. Pass a current through it. No appreciable effect will be produced on a magnetic needle placed near it. This is because the magnetic field due to the current going along the one part of the wire is balanced by the magnetic field due to the same current coming back along the other part. Now pull out the wire (fig. 131) so as to form a fair-sized loop, A. Inside this loop A there will be a considerable

magnetic field, while at B there will be little or none. The lines of force pass through A in that direction, which is forward in relation to right-handed circulation of the current in the bounding wires. If the current appear to circulate clockwise, the lines of force pass through the loop from the nearer to the farther side.

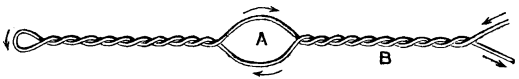
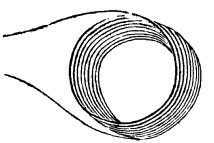


Fig. 131.

To increase the magnetic field near any part of a circuit, we make a close *coil*, or close combination of loops of the wire, as indicated in fig. 132. The advantage of such a coil over a single loop is obvious. The current is the same and similarly directed in all, and the total magnetic field at the centre is the sum of the magnetic fields due to each loop. In other words, by coiling a wire which is carrying a current, we concentrate the field, and get correspondingly increased effects. Usually coils are wound on bobbins, and often consist of many successive layers wound one over the other. Having connected the ends of such a coil to a voltaic cell, so that an electric current flows through it, substitute it for the bar magnet in the experiments of the early chapters. Precisely similar deflections of the magnetic needle will be obtained. In short, the **coil with current flowing in it is a veritable magnet.** For any given direction of current, the one aspect or face of the coil corresponds to a north or positive magnetic end, the other to a south or negative magnetic end. Reverse the current, and the coil magnet has its aspects or ends at once reversed.


Fig. 132.

Set the coil so that an astatic needle lies within it. Suppose the needle to lie as shown in fig. 133—namely, in the plane of the coil. Now pass a current along the coil from + to −. The needle will be turned round so as to lie parallel to

the axis of the coil, as indicated by the dotted lines. It will set itself tangential to a line of force. If the coil be fairly symmetrical about an axis perpendicular to the planes in which the component loops lie, we can imagine drawn a medial plane dividing the coil into two equal ring-shaped portions. Then all lines of force pass through this medial plane in a direction parallel to the axis. The direction of the field across this plane is related to the direction of the current in the coil, according to the same rule as was found to hold for the straight current and the lines of magnetic force associated with it. The relation is shown in fig. 134, where the circular ring represents the coil, the curved arrows the direction of the current in it, and the straight arrows the direction of the field across the medial plane.

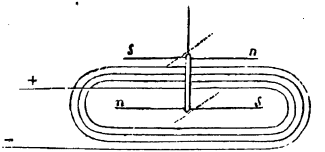
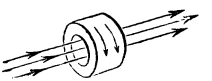


Fig. 133.


Fig. 134.

§ 80. **Action between Coils and Wires, with Currents flowing in them.**—Since a coil with current flowing in it has properties similar to those of a magnet, then two such coils should behave towards one another as magnets. The following simple experiment demonstrates this. Let A be a coil of wire tied to the end of a wooden rod (fig. 135), and let the ends be led to a voltaic battery. Suspend the rod as shown in the figure, and place a weight on the other arm of the rod, till the whole rests horizontally, like an ordinary balance. Now bring a second coil, B, close

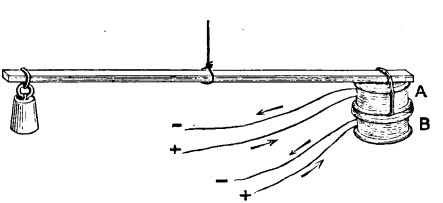


Fig. 135.

under the first, or above it, if it be more convenient. When currents are made to flow in both coils, the rod will be at once tilted out of its horizontal position. The coil A will move up or down—that is, will be repelled or attracted—according to the relative directions of the currents. Reversal of either current will produce reversal of effect; reversal of both at the same time will produce no change in the direction of motion. If the currents are similarly directed in both coils, so that both have, say, their northern sides below, there will be attraction between the coils; and similarly if the northern sides are above.

The following experiment is due to Ampère, who, shortly after Oersted's discovery, established the laws of the mutual action of two circuits with currents flowing in them. A rectangular coil, *cdef*, is supported by its point terminals *ab*, in cups containing mercury, to which the poles of a battery are joined. The character of the suspension gives the coil freedom of rotational motion round a vertical axis. We will suppose the current to be circulating in the direction of the arrows.

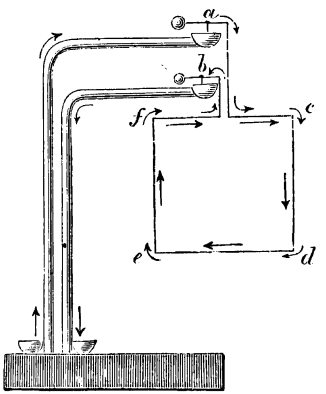


Fig. 136.

Bring a wire (it may conveniently be part of the same circuit) conveying a downward current near *cd*, so as to be parallel to it on either side. Attraction will take place. On the other hand, let the same downward current be presented to *df*. Repulsion will ensue. Thus we prove that *conductors conveying parallel and co-directional currents attract one another; conductors conveying parallel and anti-directional currents repel one another.*

This law agrees with the *magnetic* characteristics of the

two circuits, as does the more general law regarding the theoretical action of portions of circuits inclined to one another. Experimentally it is with the *whole closed circuit* that we are dealing; and it is an incomplete view to regard the mutual action of circuits as *demonstrating* the mutual action of the contiguous parts of these circuits. Thus the wires *ab*, *cd* (fig. 137*a*) can convey steady currents when they are portions of complete circuits. Let the dotted lines be the completed circuits. Then, as they are shown, the upper circuit corresponds to a magnet with its north pole up; and the lower circuit, to a magnet with its south pole up. The magnetic field associated with either acts upon the other so as to bring these poles together. Hence *ab* and *cd* seem to attract one another. A similar discussion of the case when the currents are anti-directional shows that there will be repulsion. Again, take the parts *ab*, *cd*, inclined as in fig. 137*b*. Complete the circuits as shown. Then each circuit will tend to set itself, so that its magnetic axis becomes co-directional with the magnetic axis of the other circuit. Hence the tendency will be for *ab*, *cd* to become parallel to each other.

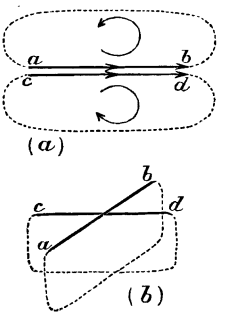


Fig. 137.

If we suspend a helix of wire, such as is shown in fig. 138, from the mercury cups of fig. 136, the helix will, when conveying a current, behave like a bar magnet in the earth's field. It will set its axis, SN, pointing in the direction of magnetic north. Such a helix, when conveying a current, is called a **Solenoid.** If a

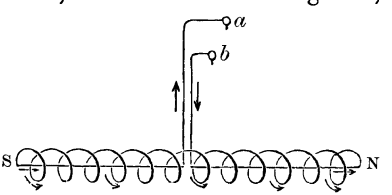


Fig. 138.

second solenoid be brought near, exactly the same phenomena of attraction and repulsion between the ends will be obtained as in the case of two bar magnets.

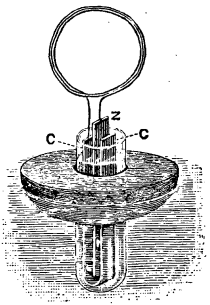
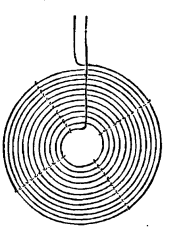


Fig. 139.

A neat form of apparatus, due to De la Rive, is shown in fig. 139. The terminals of a small coil are attached, the one to a zinc and the other to a copper strip. These dip into acidulated water contained in a test tube, and the whole floats, by means of a cork, on water. A current begins to flow through the coil, which becomes an electro-magnet, and will be found to be very sensitive to the approach of a bar magnet. The test tube may be dispensed with, and the coil floated on dilute acid.

A flat spiral formed by coiling a wire on a piece of cardboard, and stitching it down to it, is a very convenient form for illustrative purposes. Such a coil (fig. 140), with a current flowing through it, is essentially a flat magnet. The one aspect has north polarity, and the other south. With two flat spirals hung facing one another, the mutual actions of currents can be well seen.


Fig. 140.

§ 81. **Galvanoscopes and Galvanometers.**—The most important use to which a simple coil magnet is put is the measurement or detection of currents.

Set a coil with its medial plane vertical and parallel to the direction of magnetic north. Place anywhere in its axis—for example, at its centre—a magnetic needle. Under the influence of the earth's field this needle lies in the plane of the coil. When a current is made to flow in the circuit, the needle will be acted upon by the field associated with the current. This field will be in the direction of the axis of the coil—that is,

perpendicular to the earth's field. The magnet will accordingly be deflected. The stronger the field, the greater will be the deflection. The tangent of the angle of deflection will be proportional to the ratio of this field and the earth's horizontal field (§ 21).

Now if we use, first, one cell in the circuit, then two cells, then three cells, and so on, we shall get larger and larger deflections as we use more and more cells. Two cells will give approximately twice the field given by one cell. Thus we see that a stronger current is associated with a stronger field. By assuming the field to be proportional to the current, we get at once a magnetic method for comparing strengths of currents. The comparison of currents is reduced to the comparison of associated fields.

An instrument consisting of a coil with a magnetic needle in its axis is called a Galvanoscope or **Galvanometer,** according as it is used for the detection or measurement of *galvanic,* that is, voltaic currents. Every galvanometer is also a galvanoscope, and is often merely used as such. There are many forms of galvanometer, of which the following may be taken as types.

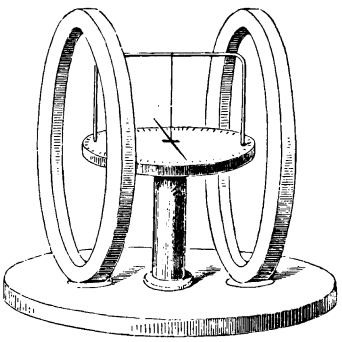


Fig. 141.

The *Tangent Galvanometer* should be a carefully constructed instrument of fairly large dimensions. The diameters and positions of its coils can thus be determined with great accuracy, so that it is possible to calculate with corresponding exactitude the value of the current which produces a particular deflection in a known magnetic field. In its usual form it consists of one large coil with a magnetic needle at its centre. This needle should be as small as possible, so that the different parts of it may be under the influence of practically the same field. A much

superior form is the double coil arrangement, known as the Helmholtz Galvanometer. Two equal-sized and co-axial coils are set vertical at a distance apart equal to their mean radius (see fig. 141). When the same current flows through both coils in similar directions, the field of force between the two coils is remarkably constant throughout a considerable region around the point midway between them. Any slight error in adjusting the magnetic needle to this midway point causes practically no difference in the deflection produced.

The *Mirror Galvanometer* is ordinarily used for the detection of minute currents. It does not differ *electro-magnetically* from other galvanometers, and its essential feature is the arrangement for observing the very small deflections produced on the magnetic needle by the small currents. To this end it is usual to fix to the needle, or to a part of the vertical axis with which it is rigidly connected, a small mirror, from which a beam of light is reflected upon a scale. A lens properly placed focuses sharply upon the scale the image of a thin wire placed in the original path of the ray. Through whatever angle the magnet is deflected, the attached mirror moves through the same angle, and the reflected ray (in accordance with optical principles) through double the angle. The reflected image of the wire is thus made to move along the scale; and its motions give us all the needed information regarding the motions of the magnetic needle in the galvanometer. One

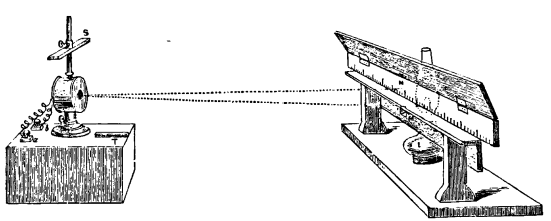


Fig. 142.

arrangement of galvanometer and scale is shown in fig. 142. In this form a small light mirror hangs by a silk fibre in the heart of the coil, and small magnets are attached to the back of the mirror. A bar magnet is represented clamped to the vertical rod above the coil. By moving this bar magnet up and down, we may vary the field in the heart of the coil within considerable limits.

Astatic combinations (see p. 27) are also frequently em-

ployed, in which either only one magnet is placed in the heart of a coil, or each magnet is placed in the heart of its own coil. In the latter case the coils are arranged one above the other, and so joined together that the same current may flow round the coils in different directions, and produce like deflections on the oppositely directed magnets. A diagram of the simplest form of astatic galvanometer with one coil is shown in fig. 133.

§ 82. **The Electro-magnetic Measure of Current.**—The magnetic field at any point due to a current flowing in a given coil is proportional to the strength of the current. Let the coil be a single loop, small compared to the distance of the point at which the magnetic field is being measured. Then it can be shown, as was done first by Ampère, that the magnetic action of such a small plane circuit at distances great compared with the dimensions of the circuit is the same as that of a magnet whose axis is normal to the circuit, and whose magnetic moment is proportional to the area of the circuit and the strength of the current conjointly. By taking the magnetic moment to be equal to the product of the area and current, we virtually define the unit current in terms of magnetic moment. This is called the electro-magnetic unit of current. If the coil consist of several loops, the effective area is the average area of all the loops multiplied by the number of loops. Thus a coil of twenty-five loops having a mean area of one square centimetre will, if traversed by unit current, produce at a distance of 5 centimetres in the axial line a field equal to $\frac{2 \times 1 \times 25}{5^3} = 0{\cdot}4$.

In accordance with the definition just given, it can be shown that a current, i, flowing in a circular circuit of radius a, produces at the centre of the circuit a field equal to $2\pi i/a$. By setting this circuit in the magnetic meridian, and placing at its centre a magnetic needle, we are able to compare the field due to the current with the earth's field, H; for, if δ be the deflection, we have $2\pi i/a = \mathrm{H} \tan. \delta$. Thus the measurement of a current in electro-magnetic units is reduced to the comparison of magnetic fields. This is the principle of the tangent galvanometer.

The unit of current now in universal use is the ampere, which is defined as being one-tenth of the electro-magnetic unit. One ampere flowing round a coil of 25 turns of average radius 10 centimetres, produces at the centre a field approximately equal to

$$2\pi \times \tfrac{1}{10} \times 25 \div 10 = \frac{\pi}{2} = 1{\cdot}57,$$

that is, about nine times the horizontal force of the earth's field in Great Britain.

§ 83. **Powerful Electro-magnets.**—Insert a piece of iron into the heart of a coil with a current flowing round it. A very great increase takes place in the strength of the magnetic field associated with the electro-magnet. The effect is due to the inductive action of the original field upon the iron. So long as the current flows, the iron remains powerfully magnetised. When the current ceases to flow, the iron loses more or less of its magnetic moment, according to its character (§ 18).

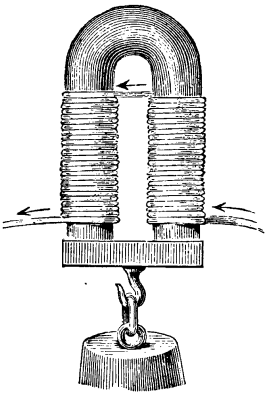


Fig. 143.

A usual form of powerful electro-magnet is shown in fig. 143. Here a horseshoe of soft iron, powerfully magnetised by a current in the coil surrounding it, is able to support a heavy weight attached to the armature. When the current is broken, the weight and armature immediately fall.

The iron is magnetised in the direction of the field associated with the current; and the north pole of the iron

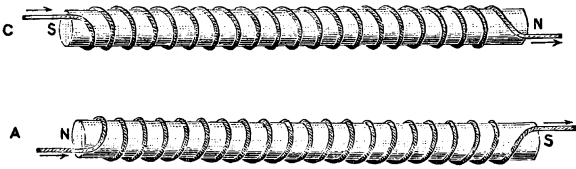


Fig. 144.

core is at the same end as the north end or aspect of the coreless coil.

In fig. 144 we have two iron rods magnetised by currents

circulating in a right-handed (CD) and left-handed (AB) helix respectively. The magnetic axis of the rod and the direction of circulation of the current are related to one another as the translatory motion of a right-handed screw is to the corresponding rotation.

If the core is of hard iron or steel, it becomes permanently magnetised under the influence of the solenoid surrounding it. This method of preparing magnets is superior to the older methods described in § 19.

The ordinary electric bell is a very good illustration of the rapid gain and loss of magnetic moment in a core of soft iron. The essential parts of this instrument are: (1) The electro-magnet, AB (fig. 145), with cores of soft iron; (2) the armature, E, attached to a spring, F, and prolonged into the hammer, H, of the bell; (3) the bell; (4) the 'push,' G, or mechanism for making and breaking the current automatically. The point of this presses upon the end of the armature to which the clapper is fixed. The armature is itself a part of the circuit. When the terminals of the circuit are joined to a battery, the current passes round the coils of the electro-magnet, into the armature at D, and back through the 'push.' The electro-magnet at once attracts the armature, and draws it away from the point pressing on it, so that the circuit is broken and the current ceases to flow. The spring then brings the armature back into contact with the point, the circuit is again closed, and the current made to flow. Immediately the armature is drawn towards the electro-magnet. This again breaks the circuit, and the armature springs back upon the point. And so on indefinitely, the armature oscillating

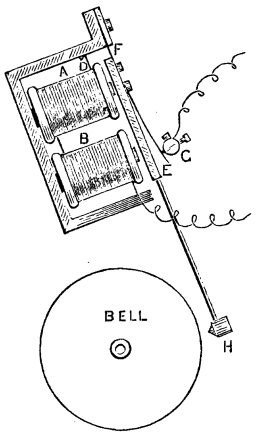


Fig. 145.

to and fro as the current is made and broken in rapid alternation. The clapper, of course, moves with the armature, and thus the bell continues to ring as long as the terminals are joined to the poles of the battery.

§ 84. **Magnetic Qualities of Substances: Diamagnetism.** —Electro-magnetism affords us the means of making a careful study of the magnetic qualities of substances. By means of currents suitably applied we are able to form magnetic fields of remarkable uniformity and power, and these fields we can create and destroy at will.

The magnetic field associated with a coreless electro-magnet is proportional to the current. But with cores of the so-called magnetic metals—iron, nickel, and cobalt—there is no longer simple proportionality of field and current. When the current is powerful, a considerable increase in its value will produce hardly any increase in the magnetisation of the iron, nickel, or cobalt core. The metal is then said to be saturated. The point at which saturation is practically reached depends partly on the nature of the metal, and partly on the form of the core. When once this point is reached, very little magnetic advantage is gained by using stronger currents.

The three metals, iron, nickel, and cobalt, are peculiar among substances, because of their great susceptibility to being magnetised. When placed in any magnetic field, they virtually bring into existence a great many more magnetic lines of force than previously existed there. These lines of force converge towards one end of the piece of metal, and diverge from the other.

Many other substances show also magnetic properties, but their susceptibility to being magnetised is so feeble that strong fields are necessary for studying their magnetic qualities. One of the most peculiar of these is bismuth, which we will take as the type of what is called a *diamagnetic substance.*

Let A, B (fig. 146) be pieces of iron laid on the iron poles of a powerful horseshoe electro-magnet. They are pointed so as to produce as powerful a field as possible

in the region between them. When a strong current is made to flow in the stout wire wound round the soft iron core, a very strong field is produced between the opposing points of A and B. A rod of iron or nickel or cobalt, suspended between the poles, will set itself along the lines of force passing from A to B.

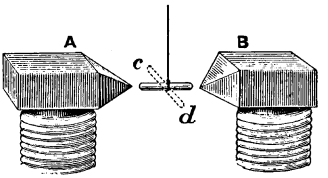


Fig. 146.

But suppose we suspend a rod of bismuth. First, let no current be flowing, and adjust the rod so that it lies pointing from A to B. Now make the current flow; then, under the influence of the powerful field between A and B, the bismuth rod will turn round, and will come to rest in the position *cd*, at right angles to the lines of force. For this reason bismuth is said to be *diamagnetic*, the ordinary magnetic substances being for distinction called *paramagnetic*.

Also a piece of bismuth will be driven from the stronger to the weaker parts of the field—that is, outwards from the poles; whereas a piece of iron will be drawn from the weaker to the stronger parts—that is, towards the poles.

Bismuth is the most powerfully diamagnetic substance known. Compared, however, to iron, or even nickel and cobalt, it is very feebly susceptible to magnetic influences.

Metals, crystals, powders, liquids and gases, almost all substances indeed, show feeble magnetic susceptibilities in powerful fields. Recently it has been demonstrated that *liquid oxygen* is strongly paramagnetic.

Magnetic changes in iron, nickel, and cobalt are accompanied by mechanical strains. Thus an iron rod lengthens when moderately magnetised, and contracts when strongly magnetised. Nickel rods always contract; while cobalt rods contract in moderate fields, but lengthen in very high fields. Recently it has been found that the internal capacity of iron and nickel tubes is altered by magnetisation. Again, when

a current is passed along a magnetised wire, the wire twists —the twist being, in moderate fields, in one direction for iron, and in the opposite direction for nickel and cobalt. Sounds sometimes accompany these changes. In the case of nickel the magnetic click is particularly distinct, both when the metal is magnetised in a solenoid, and when it is magnetised by a current passing along it.

Conversely, mechanical stress (such as tension, torsion, bending) affects the magnetic condition of magnetised rods and wires; but the relations are too complicated to be discussed in a short paragraph.

We conclude with a brief sketch of **Ampère's Theory of Magnetism.** Led by consideration of the mutual action between currents, Ampère formed the theory that the magnetism of the ultimate molecules is due to electric currents circulating round them. Each molecule is regarded as a tiny circuit with a current circulating in it. De la Rive's floating coil (fig. 139) may be taken as a rough model of Ampère's current molecule. In the molecule, however, there can be no dissipation of electrical energy; and the molecular currents must be essentially different from the ordinary current, with which dissipation of energy is always associated. The ordinary current draws upon the chemical energy of the voltaic cell; but in the molecular current there is no cell to draw upon. The molecular theory of magnetism (§§ 6, 16) requires us to assume magnetic molecules of permanent magnetic moment. Whether or not this permanent magnetic moment is due to circulation of some kind round the molecules, we cannot say. Nevertheless, Ampère's theory of magnetism is an ingenious attempt to explain magnetism as a property of the electric current.

EXERCISES.

1. Describe and sketch an electric bell which continues to ring as long as the circuit is closed. (1892)

2. A strong electric current flows through a copper wire, which passes through the centre of an iron ring, and is at right angles to the plane of the ring. Describe the magnetic state of the ring. (1892)

3. A current is flowing through a rigid copper rod. How would you place a small piece of iron wire with respect to it, so that the iron may be magnetised in the direction of its length? Assuming

the direction of the current, state which end of the iron will be a north pole. (1891)

4. Two long wires are placed parallel to each other in the same horizontal plane, and in the magnetic meridian. A magnetic needle, capable of turning in any direction about its point of suspension, is placed exactly half-way between them. How will it behave, if the same electric current flows through the easterly wire from south to north, and through the westerly wire from north to south? (The action of the earth on the magnetic needle may be neglected.) (1890)

5. A gutta-percha covered copper wire is wound round a wooden cylinder, AB, from A to B. How would you wind it back from B to A, (1) so as to increase, (2) so as to diminish the magnetic effects which it produces when a current is passed through it? Illustrate your answer by a diagram drawn on the assumption that you are looking at the end B. (1889)

6. Why is an astatic galvanometer better adapted for the measurement of weak currents than a galvanometer with a single needle? (1889)

7. An insulated copper wire is wound round a glass tube, AB, from end to end, and a current is sent through it, which to an observer looking at the end A, appears to go round in the same direction as the hands of a watch. A rod of soft iron is held (1) inside the tube; (2) outside, but parallel to the tube. What will be the magnetic pole at that end of the bar which is nearest to the observer in each case? (1888)

8. A strong current is flowing along a vertical rod, which passes through a hole in a horizontal card. Iron-filings are sprinkled on the card in the neighbourhood of the rod. How will they arrange themselves?

9. Show that the direction of the magnetic field in the heart of a coil, as given in § 79, may be deduced from the direction of the lines of magnetic force associated with a rectilinear current, as given in § 78.

10. A coil of wire is suspended in front of the one pole of a bar magnet. A current is made to flow in the coil. How will the coil move (1) when its axis points in the line of the magnet, (2) when it points at right angles to that line?

11. A small coil is suspended between the poles of the powerful electro-magnet of § 84, and is movable about a vertical axis. How will it move when a current flows in it? If set in a certain position it will not move. What is this position?

SUMMARY.

(78) The Magnetic Field associated with a Current. Lines of force round straight currents.

(79) Coils. Principle of multiplication of effect. A coil with current is a magnet. Lines of force through the heart of the coil.

(80) Action between Coils and Wires conveying Currents. Attraction and repulsion between parallel currents according as they are co-directional or anti-directional. Solenoids. Floating coil. Flat spirals.

(81) Galvanometers. Tangent galvanometer. Helmholtz galvanometer. Mirror galvanometer. Astatic galvanometer.

(82) Electro-magnetic Measure of Current. The ampere.

(83) Powerful Electro-magnets. Effect of iron core. Magnetisation of iron in solenoid. Electric bell.

(84) Magnetic Qualities of Substances. Iron, nickel, and cobalt, the magnetic metals *par excellence*. Saturation. Bismuth. Diamagnetism and paramagnetism. Magnetic strains. Ampère's theory of magnetism.

CHAPTER XIV.

OHM'S AND JOULE'S LAWS.

§ 85. **Electromotive Force.**—As mentioned in § 74, the important feature of the voltaic cell is the difference of potential it establishes between its terminals or poles. When these poles are joined by a wire, a current passes along the wire. Now electricity passes in the direction of diminishing potential. We should expect, therefore, to find that the potential falls off as we pass along the wire in the direction in which the current is flowing. And this may be proved by connecting any two points in the proper manner to an electrometer; or it may be still more simply shown by connecting them to the terminals of a delicate galvanometer. A current will flow round the galvanometer, showing that these points taken anywhere in a wire that is conveying a current are at different potentials. The difference of potential between the extremities of any portion of a conductor that is conveying a

current is called the *electromotive force* acting along that conductor (compare § 50). Along every portion of a circuit such an electromotive force will act, and its value in any particular case will depend upon how much of the circuit we take into consideration. The total electromotive force of the circuit will be the sum of the electromotive forces acting along the successive parts of it. Now it is evident that the seat or source of the electromotive force acting round a circuit is in the voltaic cell, which, in a way that is not well understood, keeps up the difference of potential necessary for the electrical flow. If we construct a series of cells exactly similar, we shall find that each, introduced into a circuit in all other respects the same, will give the same current. Hence we conclude that each such cell has got the same electromotive force. Because of the steadiness of their action, we speak of cells of the Daniell, Grove, or Bunsen type as being cells of constant electromotive force.

§ 86. **Relation between Current and Electromotive Force.** —Join up in circuit a voltaic cell (B) of constant electromotive force, a tangent galvanometer (G), and connecting wires as short and stout as possible. Note the deflection. Now substitute for one of the stout connecting wires a long thin wire, x (fig. 147), and again note the deflection. The current, measured by the tangent of the angle of deflection, will be smaller in the second case than in the first case. The circuit with the stout wire conducts more current than does the circuit with the thin long wire, other things being the same. We may easily vary the strength of the current in the circuit by simply altering the length of the wire x. We find that, the shorter the length of a given wire included in a circuit otherwise invariable, the greater is the current circulating

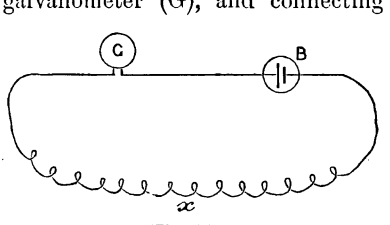


Fig. 147.

in it. So long, however, as we keep the circuit unchanged in all respects, the current circulating in it remains constant.

Thus the conducting powers of circuits or of conductors vary through wide limits; but the conducting power of a given conductor, as shown by the constancy of the current sustained in it by a constant cell, is remarkably constant—is indeed accurately constant so long as the conductor does not change in other physical properties, such as length, cross section, temperature, state of strain, and so on.

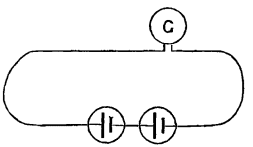


Fig. 148.

Keeping the galvanometer and connecting wires the same, introduce in succession a number of different voltaic cells. In general, different currents will be obtained in the different cases, because of the differences in the electromotive forces. Thus if a Daniell cell give a current 10, a Grove cell will give a current 18, and a Bunsen a current 16. We conclude that the electromotive forces of these cells are in the ratios of these numbers.

Again, still keeping the circuit otherwise unchanged, introduce into the circuit a second cell similar to the first, and in such a way that it drives the current in the same direction as the first (fig. 148). The electromotive force is now doubled, and it will be found that the current also is almost exactly doubled. Three cells will give nearly three times the current, and so on. The introduction of each additional cell changes the circuit as regards its conducting power. If, however, we keep the conducting power of the circuit the same throughout, then we shall find that the current is proportional to the electromotive force. This may be done as follows. Join up any number of similar cells, say five, in series (§ 74), and include this battery in circuit with the galvanometer. The electromotive force is five times the electromotive force of one cell. Now invert one of the cells so that its electromotive force will act against the electro-

motive force of the other four. The electromotive force acting round the circuit will now be three (= 4 − 1) times that of one cell. The current in this case will be $\frac{3}{5}$ of the current obtained at first. Invert a second cell, so that the total electromotive force (3 − 2) will be the same as that of one cell. Then the current will be $\frac{1}{5}$ of the current first obtained, and $\frac{1}{3}$ of the current in the second arrangement. In this way we may prove rigorously the simple law connecting electromotive force and current, namely:

The Current in a given circuit is proportional to the Electromotive Force acting round it.

Now we know that if we *add to the length* of a circuit, we diminish the current derived from a given cell, we *decrease the conducting power* of the circuit. This conducting power or **conductance** we define as being measured by the ratio of the current to the electromotive force. It is therefore a quantity which does not alter with the electromotive force, for electromotive force and current increase or decrease proportionately.

The whole subject is contained in the very simple relation known as **Ohm's Law,** which asserts that if E is the total electromotive force acting round a circuit, I the current in the circuit, and K the conductance of the circuit, then

$$I = KE.$$

§ 87. **Resistance and Conductance.**—Ohm's Law is more usefully expressed in the form

$$RI = E,$$

in which the quantity R, measured by the reciprocal of the conductance, is known as the *Resistance.* The resistance increases when the conductance diminishes. Thus by adding to the length of a circuit, we add to its resistance. The operation of putting in extra wires into a circuit is usually referred to as *putting in resistance.* In so putting in resistance, we cut down the current.

Take a circuit including cell, galvanometer, and any fairly long piece of wire. After the deflection has been noted,

substitute for the first piece of wire a portion of another piece of the same or of any other kind. If the new deflection is less than the former, include in the circuit a shorter and shorter piece of the substituted wire until the deflection is brought back to its old value; or if the second deflection is greater than the former, then lengthen the included portion of the wire until the same original deflection is obtained. When in this way we get again exactly the same current, we conclude that the original piece of wire and the substituted piece of wire have equal resistances.

By such experiments we can prove that two equal lengths of the same uniform wire have equal resistances.

Let AB (fig. 149) be a wire of uniform section joining the poles of a cell. The electromotive force acting along the wire is measured by the difference of potential between its ends (§ 76). Similarly, if we take any part of AB, say PQ, the electromotive force acting along it is measured by the difference of potential between P and Q. Experiment shows that if PQ is a particular fraction of the length AB, the difference of potential between P and Q is the same fraction of the difference of potential between A and B.

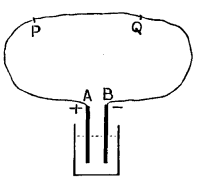


Fig. 149.

Suppose, for example, that AB is 4 feet long, and that PQ is any length of 1 foot taken anywhere in AB. Then, if we take P, Q, A, B to represent also the potentials of these points, we have

$$P - Q = \tfrac{1}{4}(A - B).$$

By experiments of this kind we may convince ourselves of the truth that Ohm's Law applies to portions of the circuits as well as to whole circuits.

In fact, the difference of potential between the ends of a conductor along which a current is flowing is such that the ratio of this difference of potential (or electromotive force) to the current is the measure of the resistance of the conductor.

Since, as may be proved by inserting galvanometers at different points, the current is the same at every point of a circuit, and since (§ 85) the sum of the electromotive forces along successive parts of a conductor is the electromotive force along the whole of the conductor, we easily see that Ohm's Law (RI = E) implies the following law:

The resistance of any continuous conductor is the sum of the resistances of any parts into which it may be divided.

Or, *the sum of the resistances of a number of conductors joined consecutively end to end—that is, joined* **in series**—*is the resistance of the single conductor so formed.*

In particular, the resistance of a wire of uniform section varies as its length. It may also be shown by experiment that the resistance of a wire of a given material varies inversely as the cross section. For example, suppose the thicknesses of two silver wires to be as 1 to 2, so that their cross sections are as 1 to 4. Then equal lengths of these wires will have resistances in the ratio of 4 to 1—that is, 4 yards of the thicker wire will have the same resistance as 1 yard of the thinner. Thus *the resistance of a wire* (or conductor of uniform section) *made of a given substance is directly as the length of the wire, and inversely as its cross section.*

The practical unit of resistance is called the **Ohm**. It is the resistance of a column of mercury 106·3 cm. long and 14·452 grammes in mass (1 sq. mm. section) at a temperature of 0° C. Having once formed such an ohm, we may make copies of it in the more permanent form of wires. Suppose a yard length of a particular wire to have this resistance of 1 ohm. Then two yards length of the wire will have 2 ohms resistance, three yards length 3 ohms, and so on. This particular length and section of mercury column is chosen so as to bring the ohm into simple relation with the centimetre-gramme-second units of current and electromotive force. The ohm being so defined, and the ampere being taken as one-tenth the electro-magnetic unit of current (§ 82), we get the definition of the corresponding unit of electromotive force from the equation IR = E. This unit is called the **Volt**. If we multiply the current in amperes by the resistance in ohms, we get the electromotive force

acting along that resistance in volts. Thus an electromotive force of 1 volt will sustain a current of 1 ampere in a circuit of 1 ohm resistance. The electromotive forces of the Daniell, Bunsen, and Grove cells are respectively 1·08, 1·73, and 1·96 volts.

§ 88. **Shunts and Conductors in Multiple Arc.**—Take any circuit with a steady current circulating round it, and note the deflection on the galvanometer. Choose any two points P and Q, one on each side of the galvanometer, and join them with a wire A. At once the current through the galvanometer will be diminished. By *shortening* the connecting wire, we may make the current through the galvanometer as *small* as we please; and by *lengthening* the connecting wire, we make the current through the galvanometer approximate more and more closely to its first value before P and Q were joined. The wire A, in fact, serves as a *shunt* along which a greater or smaller part of the current supplied by the cell passes. The shorter A is made, the smaller becomes its resistance and the greater its conductance in proportion to the resistance and conductance of the branch containing the galvanometer.

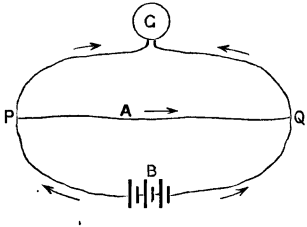


Fig. 150.

Let P, Q (fig. 151) be two points connected with the poles of a battery, and let *a*, *b*, *c* be three wires joining P to Q. The current I enters at P, and there breaks up into three parts, i_1, i_2, i_3, respectively (fig. 151), which reunite at Q to form I once more. If we take P, Q to represent the potentials at these points, then we have, by Ohm's Law:

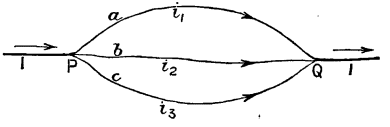


Fig. 151.

$$\frac{i_1}{P - Q} = \text{conductance of } a,$$

$$\frac{i_2}{P - Q} = \text{" } b,$$

$$\frac{i_3}{P - Q} = \text{" } c.$$

Hence $\frac{I}{P - Q} = \frac{i_1 + i_2 + i_3}{P - Q}$ = sum of the conductances of a, b, and c. But $\frac{I}{P - Q}$ measures the conductance of the three-branched system taken as a whole, which conductance accordingly is equal to the sum of the conductances of the branches.

Conductors arranged as in fig. 151 are said to be arranged in **multiple arc.**

The conductance of any number of conductors arranged in multiple arc is the sum of the conductances of the separate conductors.

For example, let wires of resistances 1, 2, 3 be arranged in multiple arc. The conductances are respectively 1, $\frac{1}{2}$, $\frac{1}{3}$. Adding, we find $\frac{11}{6}$ for the conductance of the compound conductor, and therefore $\frac{6}{11}$ for its resistance.

Take two equal lengths of a wire of uniform section, and let r be the resistance of each. Arrange them in multiple arc. The conductance of the conductor so found will be

$$\frac{1}{r} + \frac{1}{r} = \frac{2}{r},$$

and hence the resistance is $\frac{1}{2}r$. This doubling of the conductance has been effected by virtually doubling the area of the section across which the current flows. This agrees with the truth already stated, that the resistance of a wire of given length and material varies inversely, and that therefore the conductance varies directly, as the cross section.

When conductors are arranged in multiple arc, as in fig. 151, the current in each conductor is directly proportional

to its conductance. This is evident from the equations given above.

Returning to the experiment of fig. 150, we see at once what effect the introduction of the *shunt* A has upon the current in the galvanometer branch. If s is the resistance of A, and g the resistance of the galvanometer branch, then the currents in the branches are inversely as the corresponding resistances, or

$$\frac{\text{Current in A}}{\text{Current in G}} = \frac{\text{resistance of G}}{\text{resistance of A}}.$$

Add 1 to both sides and rearrange in fractional form, then

$$\frac{\text{Current in A + current in G}}{\text{Current in G}} = \frac{\text{resistance of G + resistance of A}}{\text{resistance of A}}$$

or, inverting, $$\frac{\text{current in G}}{\text{total current entering at P}} = \frac{s}{s+g}.$$

Thus the fraction of the current supplied which is allowed to pass through the galvanometer is equal to the ratio of the resistance of the shunt to the sum of the resistances of shunt and galvanometer branch. The smaller the resistance of the shunt, the less is the current which gets through the galvanometer.

For example, if the resistance of the shunt is one-ninth that of the galvanometer branch, only one-tenth of the whole current goes through the galvanometer, nine-tenths being shunted. Or, if $s = \frac{1}{9}g$, then only one-hundredth of the whole current passes through the galvanometer, $\frac{99}{100}$ths being shunted.

§ 89. **Internal and External Resistances.**—The voltaic battery which supplies the current is itself a combination of conductors, which must accordingly have a certain resistance. This is called the *Internal Resistance*, and is practically the resistance of the fluid part of the cell. The resistance of the rest of the circuit is called the *External Resistance*.

The whole current depends upon the total resistance, which is the sum of the external and internal resistances. The internal resistance of a battery of cells depends upon the arrangement of the cells. For example, let there be 36 cells, each having a resistance of $\frac{1}{4}$ ohm. Arranged in series, the resistance of the whole 36 will be $\frac{1}{4} \times 36 = 9$ ohms.

Arrange them in two parallel series of 18 in each, and then join the terminal positive poles together, and likewise

the terminal negative poles. We have thus joined the two series in multiple arc. Each series has a resistance of $\frac{1}{4} \times 18 = \frac{9}{2}$ ohms, and therefore a conductance of $\frac{2}{9}$. The conductance of the two in multiple arc is consequently $\frac{4}{9}$, and the resistance is $\frac{9}{4}$ or $2\frac{1}{4}$ ohms.

Again, join them six in line and six abreast, as indicated in fig. 152. Each row has resistance $\frac{1}{4} \times 6 = \frac{3}{2}$, and therefore conductance $\frac{2}{3}$. The conductance of the whole is $6 \times \frac{2}{3} = 4$, and consequently the resistance of the whole is $\frac{1}{4}$, the same as for one cell. Similarly, we may arrange the cells in 9 rows of 4 each, or 4 rows of 9 each. Each arrangement has its own resistance.

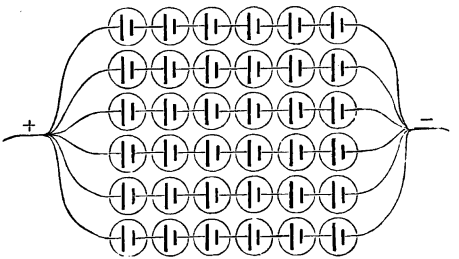


Fig. 152.

Suppose now that this battery is to be used for driving a current through a given external resistance. What arrangement will give the strongest current? Here we must take into account the electromotive force of the battery, and this depends upon the number of cells in series. Let us take the electromotive force of one cell as unit. Then the electromotive forces and resistances of the various possible arrangements of the 36 cells will be as in the following table:

	Number of Cells in Series.	Number of Cells abreast.	Electromotive Force.	Resistance.
(1)	36	1	36	9
(2)	18	2	18	$2\frac{1}{4}$
(3)	9	4	9	$\frac{9}{16}$
(4)	6	6	6	$\frac{1}{4}$
(5)	4	9	4	$\frac{1}{9}$
(6)	2	18	2	$\frac{1}{36}$
(7)	1	36	1	$\frac{1}{144}$

If R is the external resistance, B the internal resistance, and E the electromotive force; then by Ohm's Law the current I is

$$I = \frac{E}{B + R}.$$

If R = 9, arrangement (1) gives a current 2, while arrangement (2) gives only $18/11\frac{1}{4}$ or 1·6, and other arrangements still smaller.

If R = 3, arrangement (1) gives a current 3; arrangement (2) gives $18/5\frac{1}{4} = 3\frac{3}{7}$; arrangement (3) gives $9/3\frac{9}{16} = 2\frac{10}{19}$, &c. Hence the best arrangement is 2 rows of 18 cells in each.

If R = 1 or $= \frac{1}{2}$, similar calculations will show that No. 3 is the best arrangement.

The general rule is very simple. It is that the best effect is got with a given number of cells, when the internal resistance is as nearly as possible equal to the external resistance.

Let there be N cells altogether, and let them be arranged x cells in a row, so that there will be N/x rows in multiple arc. The resistance of the battery is then $\frac{x}{N/x}$ or $\frac{x^2}{N}$ times the resistance of one cell, and the electromotive force is x times the electromotive force of one cell. Let r be the resistance of one cell, and R the external resistance. Then the current will be

$$I = \frac{E}{B + R} = \frac{x}{\frac{x^2}{N}r + R} = \frac{1}{\frac{x}{N}r + \frac{1}{x}R}.$$

Hence I will have its greatest value when the denominator $\frac{x}{N}r + \frac{1}{x}R$ has its smallest value. Now here are two quantities whose geometric mean $\frac{x}{N}r \times \frac{1}{x}R$ $\left(= \frac{rR}{N}\right)$ is constant, and they are to be adjusted by varying x, so as to have their arithmetic mean as small as possible. But the arithmetic mean of two numbers is *never less* than the geometric mean, and is equal to it only in the case in which the numbers are equal to one another. Hence the quantity $\frac{x}{N}r + \frac{1}{x}R$ has its smallest value when

$$\frac{x}{N}r = \frac{1}{x}R, \text{ or } \frac{x^2}{N}r = R, \text{ or } B = R.$$

This proves the general rule.

§ 90. **The Heating of Conductors conveying Currents.**—In Chap. VIII., § 55, we saw that the transference of charge from one conductor to another involved of necessity a loss of electrical energy proportional to the square of the charge transferred. Now a current is simply a continuous transference of charge, to sustain which there is a constant drain upon the energy of the battery. As the current passes, we may utilise some of its energy in moving magnets, driving an electro-magnetic machine (an electric bell, for example), or producing chemical effects. But a great part of its energy is lost, and takes the form of heat, which raises the temperature of the conductors through which it passes. With strong currents the heating may be so great as to raise the temperature of the conductors to a white heat.

The familiar incandescent electric light (fig. 153) is a good illustration of the heating effect of a current. The incandescent lamp consists simply of a very thin carbon filament of considerable resistance. Through it a powerful current is passed. The effect is one of pure incandescence. There is no burning or combustion. To prevent combustion, which would rapidly deteriorate the carbon, the filament is enclosed in a vessel which contains no oxygen. Even with this precaution a certain deterioration occurs, so that an incandescent lamp in constant use lasts only a few months. The arc light, in which the current overcomes the resistance of an air-space between carbon points, illustrates the same principle.

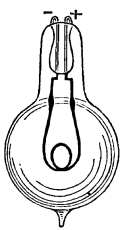


Fig. 153.

We owe to Joule the complete investigation of the laws governing the heating effects of currents.

Joule's Law: The heat produced in a given time in any part of a circuit is proportional to the square of the current, and to the resistance of that particular part.

This may be illustrated by the following simple experiments. Cut off from the same piece of thin wire two lengths,

of which one is twice as long as the other. Coil each round a glass tube, and connect their ends to stout wires. Fill two similar jars with *exactly equal quantities of pure water*, sufficient to cover completely the coils of thin wire when they are immersed. Insert two thermometers close to the coils. Then join the coils up in series with a few cells, and let the same current pass through the two coils. The arrangement is shown in fig. 154. The water surrounding the coils will gradually rise in temperature; and the thermometer in the vessel with the longer coil of wire will rise at *twice the rate* at which the thermometer in the other vessel rises. That is, the amounts of heat produced by the same current are directly as the resistances of the coils.

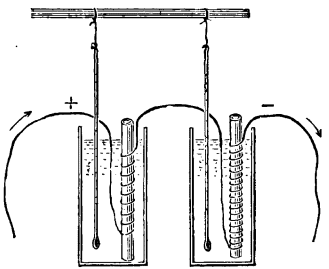


Fig. 154.

Now connect the coils together in multiple arc, and join up in circuit with the battery as before. The same current will not now flow through both coils. The longer wire, having twice the resistance of the other, will convey only half the current. In this arrangement the heating effect will be greater around the *shorter* coil of wire. The thermometer in the vessel containing the *smaller* resistance will rise at twice the rate of the other. The reason of this will be readily seen if we apply Joule's Law. For the heat evolved is proportional to the resistance and to the *square* of the current. But in this case the double current flows through the coil of half resistance. Hence the amount of heat evolved in the shorter coil is to that evolved in the longer coil in the ratio of 1×2^2 to 2×1^2, or as 2 to 1.

In general, as may be easily shown, the amounts of heat generated in conductors set in series are *directly* as the resistances of these conductors, while the amounts of heat gener-

ated in conductors set in multiple arc are inversely as their resistances.

Let us compare the increments of temperature produced by the same current in two wires of the same material, but different cross section. To fix our ideas, let the diameters be as 2 to 1, and therefore the cross sections as 4 to 1, and the resistances of equal lengths as 1 to 4. By Joule's Law the amounts of heat evolved in equal lengths will also be as 1 to 4. But a given length of the thinner wire has only one-fourth the mass of an equal length of the thicker wire; so that if the same amount of heat were applied to both, the thinner wire would experience a rise of temperature *four times* as great as the rise of temperature experienced by the other. But in the case imagined, there is four times as much heat generated in the thinner wire as in the thicker wire. Hence, loss by radiation being neglected, the temperature of the thinner wire will rise sixteen times as fast as the temperature of the thicker wire.

The general rule is, that the rise of temperature in a conductor of given form and material is directly as the square of the current, and inversely as the fourth power of the diameter. This consideration at once shows how, by diminishing the diameter, we greatly increase the incandescence in a conducting wire or filament.

Heating effects take place at all parts of the circuit, within the cell or battery itself, as well as in the external conductors. With strong currents the heating of the cells is very perceptible.

§ 91. **Comparison of Resistances.**—To obtain a rough comparison of the resistances of two conductors, A and B, we may proceed thus. First, note the deflection on a tangent galvanometer which is joined up in circuit with a constant cell. Second, introduce A into the circuit, and again note the deflection. Third, remove A and introduce B into the circuit, and note the deflection. Suppose, now, that A and B are the resistances of the conductors, and that C is the resistance of the rest of the circuit; and let i_1, i_2, i_3 be the currents as measured in the three different experiments. Then, since the electromotive force is assumed to be constant, we have by Ohm's Law:

$$Ci_1 = (C + A)i_2 = (C + B)i_3,$$

whence $$\frac{A}{C} = \frac{i_1 - i_2}{i_2}, \quad \frac{B}{C} = \frac{i_1 - i_3}{i_3}, \quad \frac{A}{B} = \frac{i_1 - i_2}{i_1 - i_3}.$$

Thus we get the ratios of the resistances in terms of the observed currents. The method is not capable of great accuracy.

We may also use Joule's Law in the comparison of resistances. The two conductors are joined in series, and the quantities of heat developed are measured by means of thermometers. This method

is one which requires great precision in the measurement of comparatively small changes of temperature, and in the proper estimation of specific heats.

But there is a third method, known as the method of the Wheatstone Bridge, which is simple and yet accurate. It requires only a delicate galvanoscope. Let two conductors be joined in multiple arc, and suppose the current from the battery B to enter at P and leave at Q. Let P, Q also represent the potentials of these points. As we pass from P to Q along either wire, the potential falls off steadily. Hence to any point A_1 on the one conductor, there corresponds some point A_2 on the other which has the same potential. If these points are joined by wires to the terminals of a galvanometer, G, no current will pass through the galvanometer. If we shift the point of contact, say from A_2 to a (that is, nearer to P), then a will be at a higher potential than A_1 and a current will pass through the galvanometer from a to A_1. If, on the other hand, we shift the point of contact to a', that is, nearer Q, then the current will pass through the galvanometer in the other direction. Thus, by feeling about with one of the galvanometer wires, we find the point A_2 which is at the same potential as the point A_1.

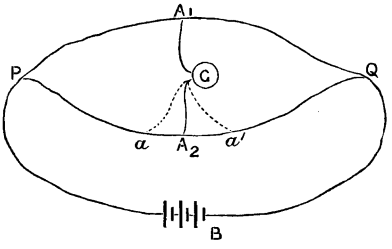


Fig. 155.

Calling the common potential of these points A, we get, by applying Ohm's Law to the four conductors, PA_1, PA_2, A_1Q, A_2Q, the following relations :

$$\begin{aligned} P - A &= \text{current in } PA_1 \times \text{resistance of } PA_1 \\ &= \text{current in } PA_2 \times \text{resistance of } PA_2 ; \\ A - Q &= \text{current in } A_1Q \times \text{resistance of } A_1Q \\ &= \text{current in } A_2Q \times \text{resistance of } A_2Q. \end{aligned}$$

Hence,
$$\frac{\text{resistance of } PA_1}{\text{resistance of } PA_2} = \frac{\text{current in } PA_2}{\text{current in } PA_1},$$

and
$$\frac{\text{resistance of } A_1Q}{\text{resistance of } A_2Q} = \frac{\text{current in } A_2Q}{\text{current in } A_1Q}.$$

But, since no current flows through the bridge conductor A_1GA_2, the current in PA_1 is the same as the current in A_1Q, and the current in PA_2 is the same as the current in A_2Q. Thus the two ratios just given are equal, namely :

$$\frac{\text{resistance of } PA_1}{\text{resistance of } PA_2} = \frac{\text{resistance of } A_1Q}{\text{resistance of } A_2Q},$$

or

$$\frac{\text{resistance of } PA_1}{\text{resistance of } A_1Q} = \frac{\text{resistance of } PA_2}{\text{resistance of } A_2Q}.$$

To show how the method may be applied, let it be required to copy a standard ohm. Let A_1 be the middle point of the stretched uniform wire, PA_1Q, and let PA_2 be the standard ohm. Join A_1A_2 through the galvanometer. Adjust the length, QA_2, of the wire which is to be used for making the copy, until, when the battery connections are made and broken, no effect is observed in the galvanometer. Then, since $PA_1 = A_1Q$, we find that the resistance of A_2Q is equal to the resistance of PA_2. If we make $PA_1 = \frac{1}{2} A_1Q$, then A_2Q will have a resistance equal to two ohms. Or, if we make PA_1 equal to $2\ A_1Q$, we shall get A_2Q equal to half an ohm.

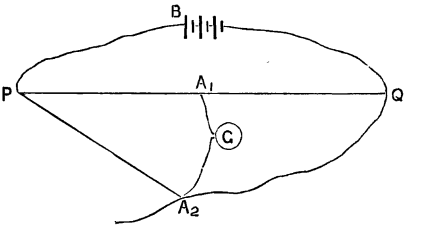


Fig. 156.

The following table shows the approximate resistance in ohms of 1 metre length, of different substances, all 1 millimetre in diameter.

Substance.	Resistance at 0° of material 1 metre long, 1 millimetre diameter.
Silver	·019
Copper	·020
Platinum	·072
Iron	·124
Nickel	·160
German-silver	·269
Mercury	1·211

These numbers must be taken only as indicating the approximate resistances of the different substances. Slight impurities make considerable changes, so that different coppers may differ more from one another than does copper from silver. With increase of temperature, the resistance of metal conductors increases. The resistance is also slightly affected by strains, such as twisting,

pulling, bending, &c., and (in the case of the magnetic metals and bismuth) by magnetisation.

The resistance of electrolytes is much higher than the resistance of metal conductors of the same length and section, and is diminished with rise of temperature.

EXERCISES.

1. It is intended to set up 100 Grove cells in series, but by mistake 10 cells are arranged in opposition to the rest. What is the relation of the potential difference of the terminals on open circuit to that which would have been obtained if the mistake had not been made? (1892)

2. A circuit includes an insulated battery and a galvanometer. Will the indication of the galvanometer be affected if a point in the circuit (for example, the negative pole of the battery) is directly connected with the earth? Give reasons for your answer. (1891)

3. The poles of a battery are connected by a long wire. Of this, two lengths of six inches, one near the positive pole, the other halfway between the poles, are immersed in two similar vessels of water. Will there be any difference in the amounts of heat produced in the two? Give reasons. (1891)

4. A hundred Grove cells are arranged in series, and wires to the terminals are momentarily brought into contact with the inner and outer coatings of a Leyden jar. What result will follow if the operation of discharging the jar is afterwards performed? Give reasons for your answer. (1891)

5. Two Grove cells, alike in all respects except that in one the plates are twice as far apart as in the other, are arranged in series, and the poles of the battery so constituted are united by a copper wire. The liquid in both cells becomes heated. In which is the rise in temperature the greater, and why? (1889)

6. A current flows through a copper wire, which is thicker at one end than the other. If there is any difference either (1) in the strength of the current at, or (2) in the temperature of, the two ends of the wire, state how they differ from each other, and why? (1888)

7. Two points of a conducting sheet (for example, tinfoil pasted on to a glass surface) are connected to the poles of a voltaic cell. Show how, by means of a galvanometer, we may trace out, point by point, a series of equipotential lines.

8. A galvanometer whose resistance is 1000 ohms is joined by its terminals to the extremities of a wire of uniform section, along which a current is flowing from a constant cell. The wire has a resistance of 1 ohm. What fraction of the whole current passes

through the galvanometer? If one terminal is shifted to the middle of the wire, what change is produced in the galvanometer current?

SUMMARY.

(85) Electromotive Force.

(86) Relation between Current and Electromotive Force. The current proportional to the electromotive force. Ohm's Law.

(87) Resistance and Conductance. Resistance the ratio of difference of potential to current. Resistance of conductors in series. The unit of resistance or ohm. The unit of electromotive force or volt. Relation between ohm, ampere, and volt.

(88) Shunts and Conductors in Multiple Arc. The conductance of a system of conductors in multiple arc. Effect of a shunt or branch circuit.

(89) Internal and External Resistances. Arrangement of the cells of a battery so as to get maximum current in the circuit.

(90) The Heating of Conductors conveying Currents. Incandescent lamp. Arc light. Joule's Law. Experimental illustrations.

(91) Comparison of Resistances. The Wheatstone bridge. Resistances of equal-sized wires of different substances. Effect of temperature on resistance.

CHAPTER XV.

ELECTROLYSIS.

§ 92. **Chemical Action of the Current.**—When certain liquids form part of a circuit, they conduct the current after a fashion peculiar to themselves. The passage of the current is accompanied by a change of constitution of the liquid.

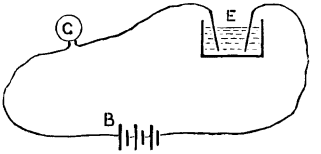


Fig. 157.

Put a little water, slightly acidulated, into a vessel, E (fig. 157), and dip into it two wires—preferably platinum, although copper will show the existence of the phenomenon sufficiently well. Join the wires to the poles of a battery, B, of several

cells (not less than two if Daniell's are used) in series. If a galvanometer be included in the circuit, a current will be observed to be passing. If the wires that are dipping in the liquid be inspected, bubbles of gas will be seen to form upon them and to rise from them through the liquid. The stronger the current, the more rapidly will these bubbles of gas be given off.

Invert over each of the wires a long test tube filled with liquid (as in fig. 158). As the current passes, the bubbles given off from the wires will collect at the top of the test tubes, and by displacing the liquid, gradually fill them. It will be noticed that one tube fills with gas at double the rate of the other; and that this gas is what is given off at the wire which is joined to the negative (zinc) pole of the battery. It may easily be shown that these gases are oxygen and hydrogen, the well-known constituents of water. Two volumes of hydrogen and one of oxygen enter into the composition of water; and in the present case the volume of hydrogen given off is double the volume of oxygen.

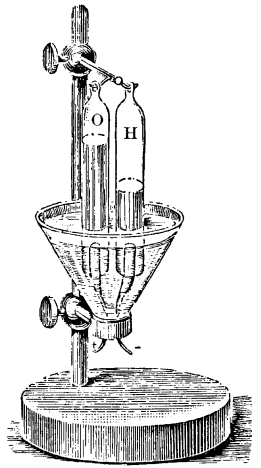


Fig. 158.

If the current from the battery continues steady, as shown on the galvanometer, the rate at which the gases are given off will be constant.

By such experiments we learn that one of the effects of the electric current is chemical decomposition, and that a steady current decomposes the substance at a steady rate.

This action is called *Electrolysis*. The substance decomposed is called the *electrolyte*. The solid wires or plates by which the current enters and leaves the electrolyte are known as the *electrodes*. The substances produced as the immediate

result of the decomposition are the *ions*. The electrolyte and electrodes form what is known as the *electrolytic cell*. This terminology was introduced by Faraday.

In the case described, the acidulated water is the electrolyte, and hydrogen is one of the ions. The hydrogen is given off at the negative electrode. The other ion is probably a complex radical, which decomposes at the positive electrode into sulphuric acid and oxygen.

To show the process in its simplest relations, consider the electrolysis of fused silver chloride. In this case, the ions are silver and chlorine. Silver is deposited on the negative electrode, and chlorine gas is given off at the positive electrode. Thus the ions or products of decomposition appear at the electrodes; but this is not always the case. Generally, indeed, what appears at one, if not both, of the electrodes is not the ion, but a different substance altogether. For example, in the case of the sulphate of copper solution, the one ion is the metallic copper, which is deposited on the negative electrode, while the other ion is a complex molecule which cannot exist by itself. Consequently it breaks up into sulphuric acid and oxygen, the former portion remaining in the solution, and the latter being given off as gas. Again, in the case of the solution of sulphate of soda, the one ion is sodium, which as soon as it is produced at the negative electrode, acts upon the water of the solution, forming soda and hydrogen. The other ion breaks up as in the preceding case. Thus in the electrolysis of sulphate of soda, hydrogen and oxygen are given off at the electrodes, exactly as in the case of acidulated water. A similar result is produced by the electrolysis of caustic soda and caustic potash, which Davy proved to be compound substances by this very process, and so prepared the way for the discovery of the metals sodium and potassium. By suitable precautions these highly oxidisable metals may themselves be obtained by electrolysis.

These cases illustrate the truth that the substances found at the electrodes are not necessarily the actual ions of the electro-

lysis, but are the products of the action of these ions on the electrolyte. When these secondary actions occur, we are guided to a knowledge of what the real ions are by comparison with other cases.

Although the substances given off in many instances are the constituents of water, it is doubtful if pure water is itself an electrolyte. The purer water is, the higher is its resistance, and there is strong evidence that really pure water, if it could be prepared, would be found to be a non-conductor.

It is well to bear in mind that in all cases of electrolysis it is the metal in the electrolyte, or substance corresponding thereto, which is produced at the negative electrode. It is, so to speak, *carried with the current.*

§ 93. **The Laws of Electrolysis.**—These were discovered by Faraday.

Law I.—The amount of any electrolyte decomposed by a current passing through it is proportional to the strength of the current, and to the time during which the current flows.

Join up in circuit a voltaic battery, a tangent galvanometer, and an electrolytic cell of the form shown in fig. 158. Let the current run for a definite time, say five minutes, and measure the quantity of gas given off. Now alter the strength of the current. This may be done either by changing the number of cells in the voltaic battery, or by changing the resistance of the circuit. Allow this second current to run for the same time, and again measure the quantity of gas given off. The quantities of gas given off in these two cases will be found to be directly proportional to the currents as measured on the tangent galvanometer.

An apparatus of the kind figured in fig. 158 is called a Voltameter, since it may be used for measuring voltaic currents. The current to be measured must run for a sufficient time, so that a measurable quantity of gas may collect in the tubes. The voltameter cannot therefore measure a current *at an instant*, as can be done by the galvanometer; it can measure only *the average current* during an interval of not less than several minutes. Instead of acidulated water, we may use copper sulphate solution, and measure the current by the amount of copper lost from the one electrode in a given time. For this experiment it is best to use copper electrodes,

say copper plates or copper wire wound up in a flat spiral. Both electrodes must be weighed dry before and after the electrolysis. When the experiment is well conducted, it is found that the one electrode has gained nearly as much copper as the other has lost.* In this case no oxygen is given off at the positive electrode. The oxygen that is produced there oxidises the copper electrode, which passes into the electrolyte in the form of sulphate of copper.

Law II.—The amounts of different electrolytes decomposed by the same current are proportional to the chemical equivalents of the electrolysed substances.

For example, join up in series, with a battery of six or more cells, two voltameters containing sulphuric and nitric acids, more or less diluted. Both will be electrolysed by the same current, and both will give off at the negative electrode (or cathode) *exactly the same amounts of hydrogen*. This can only be if the amounts decomposed are proportional to the chemical equivalents of the substances.

Or take the following more complex case. Three electrolytic cells are put in series with a strong battery. The first is an ordinary voltameter, with dilute sulphuric acid as electrolyte. The second contains fused silver chloride, with silver wire for negative electrode, and a piece of carbon for positive electrode. The third is a solution of copper sulphate with copper electrodes. Let a current from a battery of a dozen Daniell cells be run through these electrolytes in series; and let the quantities electrolysed be carefully measured. The hydrogen and oxygen given off from the first electrolytic cell are easily measured by volume. The silver deposited in the second electrolytic cell must be measured by weighing on a delicate balance; and similarly with the copper deposited in the third cell. Now it will always be found that the masses of hydrogen, silver, and copper so collected or deposited electrolytically by the same current have a constant ratio one to another. In the same way the masses of oxygen and chlorine collected over the positive electrodes in the first and second cells respectively have a definite relation to each other and to the other masses electrolysed.

To fix our ideas, suppose 2 milligrammes of hydrogen (occupying 22·6 cubic centimetres, or about 1·4 cubic inches) and 16 milligrammes of oxygen to have collected in the voltameter. Then it will be found that 71 milligrammes of chlorine have collected over

* Theoretically, the gain on the one should be equal to the loss on the other; but, practically, this is rarely if ever realised.

and on the carbon, 218 milligrammes of silver on the silver wire, and 63·5 milligrammes of copper on the copper electrode. Now these are exactly the quantities which are chemically equivalent in forming similar combinations. And thus we prove Faraday's law *that the electrolytic equivalents of different substances, those quantities, namely, which are electrolysed by the same current in the same time, are proportional to the chemical equivalents.*

The electro-chemical equivalent of a substance is the quantity electrolysed by a unit of current passing through the substance for unit of time. The practical unit of current known as the ampere will reduce out of a solution of nitrate of silver 1·118 milligrammes of silver per second. This quantity is therefore the electro-chemical equivalent of silver. A knowledge of the chemical constitution of other substances enables us to calculate their electro-chemical equivalents. For example, ·0932 milligrammes is the amount of water decomposed by an ampere in one second.

As long ago as 1840, Joule used this principle to define a thoroughly scientific unit of current. He defined as his unit the current which would decompose 9 grains of water (1 grain of hydrogen and 8 grains of oxygen) in one hour. Now 9 grains is equivalent to 583 milligrammes, and 1 hour is equivalent to 3600 seconds. Hence Joule's unit of current would decompose 0·162 milligrammes of water in one second, and was therefore very nearly equal to 1·74 amperes.

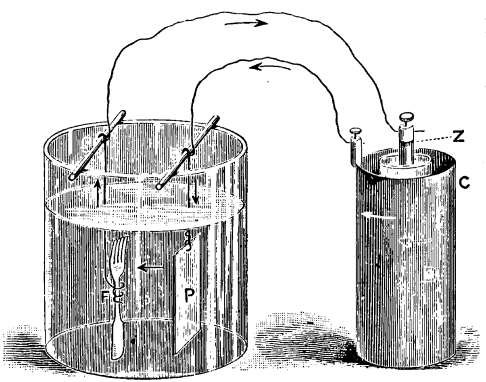


Fig. 159.

§ 94. **Electroplating and Electrotyping.**—When one of the ions is a metal, the negative electrode may become coated with it. For example, we may cover any conductor with a deposit of copper, by simply making it the negative electrode in a solution of copper sulphate.

There are many useful applications of this process. Thus in fig. 159 is shown an arrangement for silver-plating a fork. The fork, F, and a silver plate, P, are the electrodes dipping in a solution of a silver salt. The fork is connected with the negative pole of a voltaic cell, and the plate with the positive pole. The current passes through the electrolyte from the silver plate to the fork. Silver is gradually deposited on the fork, and removed from the plate, so that the strength of the solution remains constant.

Nickel-plating is conducted after the same fashion. As an industry, it has greatly developed in recent years. Copper-plating and electro-gilding are also important industries. In all these cases the current must be feeble but steady, so as to ensure uniformity and compactness in the deposit.

In **Electrotyping,** the object aimed at is to obtain an exact reproduction of coins or other engraved surfaces. To this end, a mould of the object is obtained, say, in plaster of Paris. But since plaster of Paris is a non-conductor, it cannot be used as an electrode. Its surface must be covered with a thin film of graphite, so as to become conducting. On this graphite surface the copper or other metal is deposited electrolytically. The plaster of Paris and the layer of graphite are then removed, and a metallic surface is obtained, which is an exact reproduction of the original object.

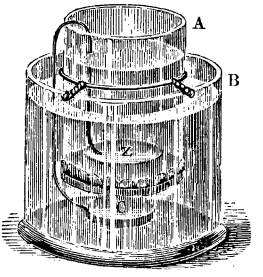


Fig. 160.

In electrotyping, an electrolytic cell other than the battery cell itself need not be used. Thus, in fig. 160, B is a vessel containing sulphate of copper, half immersed in which is the vessel A containing sulphate of zinc. There is no glass bottom to A; it is closed below by a bladder. A wire connects the piece of zinc Z in A with the impression C, on which, accordingly, copper begins to be deposited. The

apparatus is, in fact, a Daniell cell, having for its positive pole the object that is to be electrotyped. It is evident that any impression may in this way be reproduced in copper.

§ 95. **The Action of the Voltaic Cell.**—Electrolysis is an essential process in the action of the voltaic cell. In the case of a Daniell cell producing a current, it is easy to show that the zinc pole is being gradually consumed, while at the same time the copper pole is growing. As already mentioned, the Daniell cell in one of its forms consists of zinc dipping in semi-saturated sulphate of zinc, and copper dipping in saturated sulphate of copper, the liquids being in contact with each other. The current, which flows from copper to zinc outside the cell, flows from zinc to copper through the solutions. Here electrolysis takes place. Zinc passes into the liquid as zinc sulphate, and simultaneously the copper sulphate is decomposed, and copper is deposited on the copper pole. At the boundary of the liquids, zinc is replacing copper in the sulphate. Thus the zinc sulphate solution is always growing stronger and stronger. The copper sulphate solution would at the same time grow weaker if it were not for the crystals of sulphate of copper, which are always kept lying in the liquid, ready to be dissolved in it. On the whole, the action of this form of Daniell cell depends upon the substitution of zinc for copper in the sulphate. Now, in virtue of what is called the greater chemical affinity of zinc for sulphuric acid, this substitution liberates so much energy, part of which takes the electrical form when the current is flowing.

Consider also the case of a simple one-fluid cell formed, let us say, of zinc and platinum dipping in a solution of sulphuric acid. When the current is flowing, hydrogen is liberated at the platinum surface. Its place in the sulphuric acid is taken by the zinc, which gradually passes from the metallic form into the solution. Here, again, because zinc has greater affinity for sulphuric acid than hydrogen has, there is a liberation of energy, part of which sustains the current. When zinc is dipped in sulphuric acid without the

presence of platinum or other metal, a similar chemical action takes place. Zinc replaces the hydrogen, which comes off as gas. As it does so, a considerable amount of heat is generated, causing an appreciable increase in temperature of the solution. For every 2 milligrammes of hydrogen liberated, 65 milligrammes of zinc pass into the solution as sulphate of zinc, and a definite amount of heat is generated, that is, a definite amount of energy is transformed from the potential form of chemical affinity into the kinetic form of heat. When this same action takes place in the cell, most of the transformed energy appears as electrical energy. The kinetic energy associated with the electric current so sustained finally takes the form of heat. Part of the electrical energy may, however, be used for magnetising or electrolysing purposes. But whatever be the various transformations through which the electric energy may pass, its total amount can never exceed the energy liberated by the chemical and thermal actions going on in the cell.

§ 96. **Electrolytic Polarisation.**—If two perfectly clean and similar electrodes of the same substance be dipped into any electrolyte, and then connected with the terminals of a delicate galvanometer, no current will be obtained. It is, however, almost impossible to obtain two such electrodes. In general, some slight difference exists, so that with a very delicate galvanometer a feeble current may be detected. For ordinary purposes we may clean platinum electrodes sufficiently by heating them to a bright red heat in a flame.

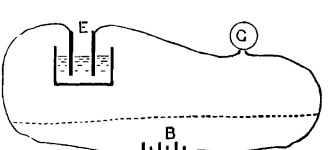


Fig. 161.

Having constructed a fresh electrolytic cell consisting of clean platinum electrodes dipping in dilute sulphuric acid, join it in circuit (fig. 161) with a battery and a tangent galvanometer, and carefully observe the indications of the galvanometer. During the first few

minutes the current will be seen falling off distinctly in strength. It will finally attain a steady value considerably less than the value first shown. This diminution of current means that one (if not both) of two things must have happened. Either the electromotive force acting round the circuit has decreased; or the resistance of the circuit has increased.

There has indeed come into existence through the action of the current an electromotive force opposite to the electromotive force of the battery. To show this, throw the battery out of the circuit, and put in its place a single wire, as indicated by the dotted line in the diagram. A small current will be observed passing round the circuit. This current will, however, steadily fall off in strength, and after a few minutes will be inappreciable. It will be noticed that its direction is opposite to the direction of the original current. Thus we see that after electrolysis has gone on for some time, the electrolytic cell has become capable of giving a current—short-lived, no doubt, but still a current.

The explanation of the phenomenon is simple enough. The substances given off, the oxygen and hydrogen, namely, do not wholly pass up through the liquid. A certain quantity of each clings to its electrode. The electrodes are no longer simply platinum and platinum, but platinum with a film of oxygen, and platinum with a film of hydrogen. We may call them oxygenised platinum and hydrogenised platinum. They are, in fact, different substances, having, for example, different powers of being oxidised, just as any two different metals (e.g. copper and zinc) have. The electrolytic cell has in effect become a voltaic cell, capable of sustaining a current when the circuit is closed. As this current flows, the electrolyte is decomposed, hydrogen being produced at the oxygenised platinum, and oxygen at the hydrogenised platinum. The hydrogen film on the latter electrode is gradually oxidised until the electrode becomes incapable of further oxidation, and the oxygen film is used up in oxidising the hydrogen

that appears there. When this stage is reached the current ceases, and the cell loses its temporarily acquired voltaic characteristic. This phenomenon, which consists in the electrodes of an electrolytic cell assuming temporarily the function of *voltaic poles*, is known as **electrolytic polarisation** or **polarisation of the electrodes.** Its effect is to bring into existence a reversed electromotive force, that is, an electromotive force acting against the electromotive force which sustains the electrolysing current.

Electrolytic polarisation is met with in the voltaic cell, especially in the simple one-fluid cell. For example, in Smee's cell, which consists of zinc and platinum dipping in dilute sulphuric acid, the current flows through the cell from the zinc to the platinum. Zinc is consumed, and hydrogen appears at the platinum surface. Part clings to the platinum, and alters its voltaic character. The result is, that after such a cell has been working some little time, its electromotive force falls off, and the cell deteriorates. It is in this particular that the two-fluid cell is so superior to the one-fluid cell. From the description already given, it is evident that the action of the Daniell cell does not tend to coat either the zinc or the copper with another substance. The zinc is simply consumed, and copper is deposited on the copper.

§ 97. **Secondary Batteries, Storage Batteries, or Accumulators.**—Construct an ordinary voltameter with very long wire electrodes reaching well up the collecting tubes, and let the current flow through it until the ends of the electrodes project above the liquid surfaces. Such a cell will give a constant though small current, so long as the electrodes project above the liquid. It is called a gas battery. The polarisation is kept up because the one electrode is partly in an oxygen atmosphere and the other in a hydrogen atmosphere. The energy of the current is derived from the recombination of the hydrogen and oxygen; and as the current flows, the liquid gradually rises in each tube.

This kind of cell is one of a class known as secondary cells

or accumulators.* In virtue of its action it tends to run down, and can be revived only by being electrolysed.

The 'gas cell' just described is practically of no value because of its high resistance.

The *Storage Batteries* of commerce consist of cells containing large sheets or grids of lead immersed in dilute sulphuric acid. The surface of the lead is first coated with the oxide of lead, and each cell is then simply an electrolytic cell from which no current can be obtained. One form is shown in fig. 162. Here the one electrode consists of a set of parallel plates of lead which fit in between the similarly arranged plates of the other electrode, care being taken that each set is insulated from the other. In this way the resistance of the cell is made as small as possible. When a powerful current is passed through the cell, it polarises it. Peroxide of lead accumulates on the positive plates, and spongy metallic lead on the negative. After some time, the lead cell becomes to all intents a true voltaic cell, and can be used as such. Its electromotive force is about 2 volts. While the cell is yielding current, the peroxide of lead loses oxygen, and the spongy metallic lead oxidises, so that in the end both are reduced to the same condition. The cell then ceases to act, and must be polarised or *charged* again before it can be utilised as a source of current. The polarising or charging is effected by means of a dynamo or machine for generating current by direct transformation of mechanical energy (see Chap. XVIII.).

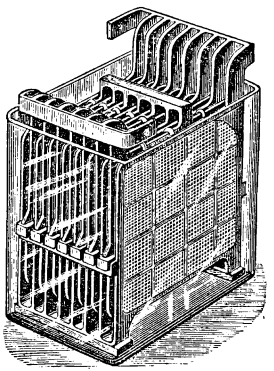


Fig. 162.

* These must not be confounded with electrostatic accumulators or condensers.

EXERCISES.

1. Plates of copper and of platinum are dipped into a solution of copper sulphate, and a current is passed through the cell from the copper to the platinum. Describe the effects produced; also what happens when the current is reversed. (1892)

2. A vertical partition of porous earthenware is fitted into a tumbler, and dilute sulphuric acid is poured into each compartment. Rods of common zinc and copper are placed respectively in the two compartments, and connected by a wire. State what will be observed with regard to the evolution of gas, and how the observed phenomena will be modified when copper sulphate is poured into the compartment containing the copper rod. (1891)

3. Two copper plates of the same weight are connected, one with the positive and the other with the negative pole of a voltaic battery, and immersed side by side in a solution of copper sulphate. If, after some time, the plates are removed, dried, and re-weighed, they are found no longer to weigh alike. Account for this, and explain how, by the continued action of the current, the equality of the plates could be re-established. (1890)

4. What are the materials used in the construction of a Daniell's cell, and what chemical changes occur in the cell when in action? (1888)

5. The experiment described in § 88, fig. 150, is modified by introducing an electrolytic cell in the shunt PQ. When this is done, the current in the galvanometer is at once diminished, but thereafter begins to increase gradually to a final value. Explain this, and point out the direction of the temporary current obtained when the voltaic battery is disconnected altogether.

SUMMARY.

(92) Chemical Action of Current. Decomposition of acidulated water. Electrolysis: electrolyte, electrodes, ions. Decomposition of chloride of silver and of solutions of sulphate of copper and sulphate of soda. Secondary reactions at the electrodes. Discovery of metals by electrolysis.

(93) The Laws of Electrolysis. Amount electrolysed proportional to current. The voltameter. Electro-chemical equivalents proportional to chemical equivalents.

(94) Electroplating and Electrotyping.

(95) The Action of the Voltaic Cell. Electrolysis of one or more elements in the cell. Chemical affinity the source of the energy.

(96) Electrolytic Polarisation. Accumulation of the products of

electrolysis on the electrodes. Reversed electromotive force of polarisation. Deterioration of cells through polarisation.

(97) Secondary Batteries, Storage Batteries, or Accumulators. The gas cell. Storage cells with lead poles.

CHAPTER XVI.

THERMO-ELECTRICITY.

§ 98. **Thermo-electric Currents.***—When two or more conductors of different material are joined together to form a circuit, and one of the junctions is heated, a small current is produced. A moderately delicate galvanometer is necessary for showing the thermo-electric current due to a single pair of junctions.

With such a galvanometer in circuit, join the copper wires coming from the terminals each to one end of a German-silver wire. Let A, B (fig. 163) be the two junctions. First let the junction A be heated in a flame. A current will pass through the galvanometer G, causing a deflection, say, to the right.

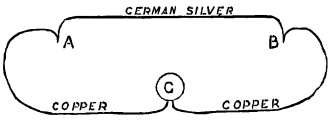


Fig. 163.

Secondly, let A be allowed to cool to its original temperature, and let the junction B be heated. The current will now cause the deflection to go to the left. By inserting a voltaic cell temporarily into the circuit, we find that a deflection to the right means that the current is flowing from B to A along the German-silver. We conclude, therefore, that in a thermo-electric circuit formed of copper and German-silver the current always passes from German-silver to copper through the heated junction, and from copper to German-silver through the cooler junction. It is also at once observed that, in this particular case, the more the one junction is heated, the greater is the current produced.

* First observed by Seebeck in 1821.

Immerse both junctions in pure water, and gradually heat the one to the boiling-point. When this stage is reached, begin to heat the other junction. It will be found that the current will begin to diminish, and will continue diminishing steadily to zero as the temperature of the second junction rises to the boiling-point. When both junctions are at the same temperature, no current will pass. Thus the thermo-electric current depends on the *difference* of temperature of the junctions, vanishing when that difference vanishes.

To get an idea of the magnitude of the current, suppose the junctions to be at 10° C. and 100° C. Then the electromotive force in the copper German-silver circuit will be about ·0015 volt. It would require 1000 pairs of such junctions to give an electromotive force comparable to that of an ordinary voltaic cell. Iron and German-silver similarly arranged will give an electromotive force of ·0027 volt; iron and nickel a little greater; but most other pairs of metals distinctly smaller values. There is one pair of metals, however, which gives an exceptionally large electromotive force—namely, antimony and bismuth under certain conditions. With this pair it is possible to obtain an electromotive force of ·0012 volt for a difference of temperature of only 10° C. It is this couple which is used in the construction of the Thermopile.

§ 99. **The Thermopile.**—This is an instrument suitable, when taken in combination with a delicate galvanometer, for measuring very minute changes of temperature. A rough model, useful for illustration, is easily constructed with strips of iron and German-silver, carefully insulated except at the ends, where they are soldered each to each. The manner in which the successive strips are joined, alternating, so as to form a continuous zigzag conductor, is indicated in fig. 164. As we pass along the conductor in any one direction, we must pass across a junction either from iron to German-silver, or from German-silver to iron; and the whole is so arranged that the junctions of one kind all face in one direction, and those of the other in the opposite direction.

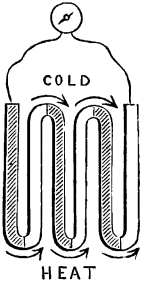


Fig. 164.

If we place the palm of the hand on the one set of junctions, these will all be heated, and a current will be produced. Each pair of junctions will give rise to the same electromotive force; and if there are fifty pairs of junctions, the electromotive force will be increased fifty times.

In the thermopile as usually constructed, small strips of bismuth and antimony are arranged as described; and the general appearance of the essential part of such an instrument is shown in fig. 165. The terminals, by which the pile is introduced into any circuit, are in connection with the binding screws *n*, *p*. The thermopile is particularly useful in experiments on radiation.

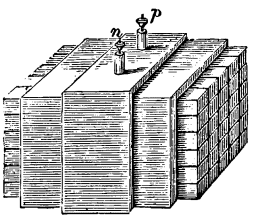


Fig. 165.

§ 100. **Neutral Points and Thermo-electric Inversion.**—With the copper German-silver pair originally described, the electromotive force increases continuously as the temperature of either of the junctions is increased, even up to a bright red heat. But it is not so in all cases. Consider, for example, a circuit of copper and iron, and let the one junction be gradually heated in a flame. As the temperature begins to rise, the current flows from copper to iron through the heated junction. As the temperature continues to rise, the current increases, but not indefinitely. It soon reaches a maximum, and then as the heating continues, begins to diminish, and goes on diminishing, until at length the galvanometer deflection passes through zero and changes sign. This shows that the current now passes from iron to copper through the heated junction; and it passes in the same direction at all higher temperatures. Careful experiment shows that the temperature at which the maximum current occurs is about 240° C. The change in direction of the current is found to occur when the temperature of the heated junction is about as much higher than 240° C. as the temperature of the cold junction is lower.

It matters not what the temperature of the one junction may be, the current always passes through a maximum value as the temperature of the other junction passes through 240° C. This critical temperature is known as the Neutral Point.

The existence of the neutral point shows that whereas at ordinary temperatures the current will flow from copper to iron through the warmer junction, it will flow from iron to copper through the warmer junction if the temperatures of both junctions are higher than the neutral point, or if the temperature of the warmer junction is high enough. This change in the direction of the current is called *thermo-electric inversion.* An iron cadmium couple shows the phenomenon very well, since the neutral point for this pair is about 165° C. If we take the circuit composed of any two metals at the ordinary temperature of the air, say 10° C., and then slightly heat one junction, the current will pass in a definite direction through the warmer junction. The metal from which the current passes is said to be thermo-electrically positive to the other, into which the current passes. We may arrange the metals in order, so that a more positive one always precedes a less positive one. The order at 10° C. for ten of the metals is nickel, palladium, aluminium, lead, copper, silver, zinc, gold, cadmium, iron. At 200° C. the order becomes nickel, palladium, aluminium, lead, copper, gold, silver, zinc, iron, cadmium. At 300° C. the order is palladium, nickel, lead, aluminium, iron, copper, gold, silver, zinc, cadmium. Between 10° and 200°, gold has neutral points with silver and zinc, and iron has a neutral point with cadmium; and between 200° and 300°, iron has neutral points with copper, gold, silver, and zinc.

§ 101. **The Peltier Effect.**—The Peltier Effect is closely connected with the theory of thermo-electricity. It is a thermal effect produced at the junctions of a heterogeneous circuit, round which a current is flowing. The phenomenon may be very well shown by means of the model thermopile of stout iron and German-silver rods described above (p. 215).

To facilitate the making and breaking of contacts, arrange a commutator of six mercury pools (fig. 166). To the centre pair lead the terminals of the model pile W. To the pair on one side connect the terminals of a delicate galvanometer, G; and to the other pair the poles of the battery B. It is clear that we may, by short connecting wires, join up the pile with either the galvanometer or battery. First join up the pile and galvanometer. Then, if there be no difference of temperature between the sets of junctions, there will be no current. Now break connection with the galvanometer, and put the pile in circuit with the battery. After allowing the current to run for a few minutes, disconnect the battery, and again join up the pile with the galvanometer. A current will at once declare itself, and will gradually diminish as time goes on. This current is a thermo-electric current, due to a difference of temperature between the sets of junctions. This difference of temperature has been produced by the battery current flowing across the successive junctions of the pile. In the particular case imagined, there is heating when the current passes from iron to German-silver, and cooling when it passes from German-silver to iron. Doubtless, in such an experiment, there is heating of the conductor as a whole in virtue of Joule's Law; but this may be almost inappreciable, if we use stout rods of low resistance. In any case, however, even though the junctions on one side do not really cool, they will not heat so quickly as the junctions on the other side. Thus if H units of heat are generated in unit time at the junctions because of resistance, and if h is the measure of the Peltier effect; then at the one set of junctions $(H + h)$ units of heat will be generated, and at the other $(H - h)$. By reversing the voltaic current

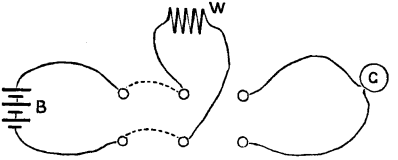


Fig. 166.

through the pile, we reverse the Peltier effect at all the junctions; and the resulting thermo-electric current will pass in the opposite direction through the galvanometer.

SUMMARY.

(98) Thermo-electric Currents. Electromotive force small compared to that of ordinary voltaic cell.
(99) The Thermopile.
(100) Neutral Points and Thermo-electric Inversion.
(101) The Peltier Effect.

CHAPTER XVII.

MAGNETO-ELECTRICITY.

§ 102. **The Production of Induced Currents.***—Lay a coil of wire, A (fig. 167), on a table some little distance from a delicate galvanometer, G. First, let the coil and galvanometer be quite disconnected. Hold a bar magnet, M, over the coil, and then suddenly introduce its north or positive pole into the heart of the coil. If the galvanometer is not too near, no effect will be produced on it.

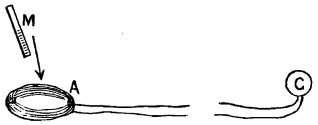


Fig. 167.

Remove the magnet; still no effect. Now join up the coil and galvanometer in circuit, and go through exactly the same operations with the magnet as in the first instance. It will then be found that each time the magnet is introduced or withdrawn, a sudden deflection will be produced on the galvanometer. This is due to a current, which passes round the circuit at the moment the magnet is moved. It is a transient current; for the galvanometer needle is not permanently deflected, but returns to zero when the magnet is brought to rest. This transient current is called the *induced*

* The fundamental laws of magneto-electricity were discovered by Faraday in 1831.

current. If on introducing the north pole we get a deflection to the right, withdrawal of the same pole will produce a deflection to the left. On the other hand, when the south pole is introduced, the deflection is to the left; and to the right when the same pole is withdrawn. In other words, an approaching north pole produces an induced current in the same direction as a receding south pole; and *vice versâ.*

Suppose now that we rest the bar magnet horizontally on a block, and then move the coil A to and fro opposite either pole. Currents exactly similar to those just described will be produced in the circuit. They will last only so long as the motion is kept up; and their direction will depend upon the kind of pole that is being approached or receded from. Suppose that the current produced when the coil is moved up to the north pole causes a deflection to the right. Turn the coil round so that it presents the other face to the magnet, and repeat the experiment. The current now produced by motion towards the north pole will cause a deflection to the left.

If we turn the coil so that it is edge on to the magnet, and move it to and from either pole, no current will be observed. If, however, we rotate the coil so as to make it pass from the edge-on position to the end-on position with reference to the magnet, an induced current will again be obtained. This induced current will have the same direction as that produced by approach of the coil in the same end-on aspect.

Instead of using a permanent magnet, we may use an electro-magnet, with or without an iron core. The presence of an iron core greatly intensifies the inductive action.

To show the phenomenon to best advantage, place the two coils co-axially, that is, either as in *a* (fig. 168), where the coils lie end to end, or as in *b*, where the one coil lies wholly inside the other. Introduce along the common axial core a bar of iron or a bundle of iron wires. Join the terminals of one coil to the battery, and the terminals of the other to the

galvanometer. The coil through which the battery current is to be made to flow is called the primary coil, and the other is called the secondary. The two circuits are similarly distinguished as the primary and secondary circuit.

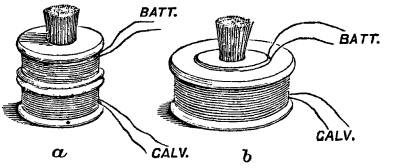


Fig. 168.

Now, every time the battery circuit is completed, an induced or secondary current is produced for the moment in the galvanometer circuit. This is a transient current, which vanishes as soon as the primary current, or current in the primary circuit, has attained its steady value. This transient current we will call the 'induced current at make.' When now the battery circuit is broken, so that the primary current ceases to flow, an induced current again passes round the galvanometer circuit. This is the 'induced current at break.' In all cases, the induced currents at make and break flow round the secondary circuits in opposite directions. Again, if we reverse the direction of the primary current in its circuit, the induced current at make (and therefore at break) is also reversed.

If we keep the primary current steady, and remove the secondary coil from its proximity to the primary coil, induced currents will still be produced.

It is evident, in fact, that we are simply performing with an electro-magnet the experiments first described as performed with a permanent magnet. The establishment of the primary current in a given direction is equivalent to bringing up a particular pole of a permanent magnet.

§ 103. **Relation between Induced Currents and Change in Magnetic Field.**—We have seen that it is possible to place a coil in such a position relatively to a magnet that no induced current is produced when either is moved in a particular way. In the same way, it is possible to arrange the positions of the

primary and secondary coils, so that, on making and breaking the primary, no induced current is obtained. When the two coils are so arranged that no induced current is produced in the secondary when the current is made or broken in the primary, they are said to be *conjugate coils.*

There are an infinite number of positions in which two coils are conjugate to one another; such positions are found very easily by experiment.

Thus, set the two coils in any relative position in which at make of the primary an induced current is produced in the secondary. Then it is possible, after rotating the secondary coil round an appropriate axis, to obtain the induced current in the opposite direction, the primary current being made to flow in exactly the same direction as formerly. Somewhere between these two positions of the secondary coil a position can be found, for which there is no induced current at make of the primary. This conjugate position is readily found by trial.

When an induced current is made to flow in a circuit, it is always because one of two things happens. Either the magnetic field in which the circuit lies changes, or the circuit itself moves so as to change its position relatively to the field. Being produced by a relative change of magnetic field, an induced current is appropriately called a *magneto-electric* current.

A magnetic field is fully described by means of its lines of force, which are closer in the stronger field. The field is indeed measured by the number of lines of force which pass through unit area, set perpendicular to the direction of the lines. Let the area be that enclosed by a single coil of wire, and let this coil be set perpendicular to a given uniform magnetic field, so that it embraces the greatest number of lines of force possible. If this coil be translated without rotation, the number of lines of force embraced by it will not change, and the most delicate galvanometer will indicate no induced current. But if the coil be rotated, the number of lines of force embraced by it will be diminished, and an induced current will set round it.

Or, again, let the coil lie in a field which varies in strength from

point to point, that is, a field whose lines of force are not parallel, but converge towards one aspect and diverge towards the opposite aspect. This is the relation in the second experiment of § 102. Then it is evident that in this case a pure motion of translation will effect a change in the number of lines of force embraced by the circuit. Here, also, an induced current is produced. Instead of translating the circuit, we may move the magnet, which is the nucleus of the field, or intensify the field by passing an appropriate current somewhere in the neighbourhood.

In brief, when by motion of circuits or magnets, or by change of primary currents in the neighbourhood, the number of magnetic lines of force embraced by a closed circuit is changed, an induced current is invariably produced in this circuit. This current may be ascribed to an induced electromotive force, which acts only so long as the change is being effected, and which is measured by the rate at which the number of lines of force embraced by the circuit is changing. Usually the change is made quickly, and we need consider only the total change in the number of lines of force, and the consequent total flux of electricity round the circuit. If we multiply (as in Ohm's Law) the total flux of electricity by the resistance of the circuit, we get a quantity which we may conveniently call the *electromotive impulse* acting round the circuit.

The direction of the induced current is always such as to produce a magnetic effect opposing the change which has caused the induction. This is Lenz's Law, so called after the experimenter who first fully established it. It may be shown to be a necessary consequence of the law of the conservation of energy. As an illustration, take the case of a coil approaching the north pole of a magnet. The induced current is found to be such as to cause repulsion of the north pole, that is, it makes the nearer aspect of the coil a north end. Or, let the two coils, A and B, be set close to each other, and let the primary current be made to flow in A in the direction shown in fig. 169. The induced current at make is found

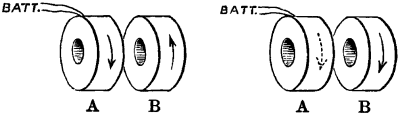


Fig. 169.

to flow round B in the opposite sense, in this way acting against the inducing change.

Again, when the primary in A is broken, the induced current in B flows so as to retard the disappearance of the field, due to the primary current. In other words, the secondary current at break has the same general direction as the primary had just before it was broken.

§ 104. **Mutual and Self Induction.**—For simplicity, consider the case of two coils which have no iron cores. Experiment shows that for any given configuration of the coils, the induced current produced in one by make and break of a current in the other is proportional to the strength of the primary current. This is so because the magnetic field at any point due to a coreless electro-magnet is proportional to the current that constitutes the electro-magnet. Suppose that, when unit current flows in A, the number of lines of force brought into existence in the region enclosed by B is measured by M. In taking the number of lines of force embraced by a coil, we must add together the numbers embraced by the turns taken severally. Thus a coil of 100 turns, and average area 50, will embrace 5000H lines of force of a uniform field, H, perpendicular to the coil. If, then, the coil B embraces M lines of force when unit current flows in A, it is found that the coil A will embrace M lines of force when unit current flows in B. The quantity M is called the coefficient of mutual induction of A and B, or simply the mutual inductance.

If the current I is made to flow in A, then the quantity MI will be the number of new lines of force embraced by B, and this will measure the total electromotive impulse acting round B. If S is the resistance of B, the total flux of electricity will be MI/S. Similarly, if R is the resistance of A, and if the current I is started in B, the total flux of electricity induced in A will be MI/R.

Or suppose that the primary current, whether it be made to flow in A or in B, is derived from a battery of constant electromotive force, E, and negligible resistance; then in either case the total flux of electricity induced in the secondary circuit will be ME/RS.

The mutual inductance of two coils is found to depend solely

upon their form and position relatively to each other. It will be the greatest possible when the two coils are set co-axially, so as to be as near as possible to each other. If the coils are joined together so as to make a continuous coil, there is still mutual inductance between the two parts. Indeed, between every pair of turns of any coil there is a mutual inductance. But instead of calling this inductance *mutual*, as between different portions of one coil, we regard it as part of a quantity called the *Self-inductance* of the coil. This self-inductance is measured by the number of lines of force embraced by the coil when unit current flows in it. There will be corresponding currents of self-induction in the coil when a current is started in it or made to cease. But the experimental detection of these currents is not so easy as in the case of mutual induction, inasmuch as there is the primary current itself in the same coil.

There is, however, a very familiar phenomenon directly due to the self-induced current at break. When any circuit in which a strong current is flowing is broken, a spark may be noticed to pass between the ends as they are being drawn asunder. If an electro-magnet with iron core forms part of the circuit, this spark is unmistakable; and if the bared ends are held in the fingers, a pretty sharp shock may be felt. This spark and shock are due to the self-induced current at break, or to the *extra current*, as it is frequently called. When the main current is broken, the associated magnetic field disappears, and a certain number of lines of force embraced by the circuit pass out of existence. An induced electromotive force acts round the circuit in the direction of the vanishing current; and this is sufficiently intense to cause the extra current to pass as a spark through the air.

The self-induced electromotive force at make acts against the main current, and its effect is to retard the growth of this current from zero to its final steady value. If a powerful electro-magnet be included in the circuit, the time

the current takes to attain its steady value is much greater than if no such electro-magnet were included.

§ 105. **The Induction Coil.**—This is the name given to a special form of apparatus useful for illustrating many of the properties of induced currents. It consists of two coils, the Primary and Secondary. The former is constructed of a comparatively few turns of stout wire wound close round a bobbin. The secondary is wound over it, and consists of a great many turns of very fine wire, each turn of which must be carefully insulated from its neighbours. To intensify the action, a core of iron wires is inserted in the heart of the coils. When a current is passed through the primary, the core becomes magnetised. In virtue of this sudden change in the number of lines of magnetic force embraced by each turn of the secondary coil, an induced electromotive impulse acts round it. Since there are usually several thousand turns, it follows that the total electromotive impulse will be several thousand times greater than the average electromotive force acting round each turn. At break of the primary a similar effect is produced. After a succession of makes and breaks, the number of lines of force called into existence at make is exactly equal to the number of those that vanish at break. Hence the electromotive impulses at make and break are equal, but in opposite directions; and if the secondary circuit is completed through good conductors, the total fluxes of induced electricity will also be equal and in opposite directions. In the familiar form of galvanic battery for giving shocks, the secondary circuit is completed through the human body, and at every make and break of the primary current a shock is felt.

That an induced current may pass, it is not necessary for the secondary circuit to be closed or completed by conductors. The ends of the secondary coil may be separated by an air-space, or by a space in any other dielectric medium; but between them an electric discharge may take place identical in all its properties with the electric discharge familiar in electrostatic experiments. The length of spark obtained in

this way depends on the dielectric strength of the medium, the mutual inductance of the coils, and the strength of the primary current, at whose sudden cessation the spark passes.

The induction coil is particularly useful in studying the electric discharge through rarefied gases. For this and other purposes it is convenient to have an automatic 'make and break' in the primary circuit, similar in principle to the adjustment already described in the electric bell (§ 83).

The terminals of the secondary coil are then joined to the platinum wire electrodes of a vacuum tube. The passage

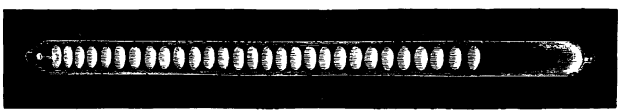


Fig. 170.

of the induced current through the rarefied gas is accompanied by beautiful and remarkable optical effects. In fig. 170 is shown a simple case, in which the luminous striæ so characteristic of this kind of discharge are very distinctly seen. When the gas is made rarer, the striæ become fewer, and at length disappear when the vacuum is 'high.' In vacuum tube discharges, the negative electrode can always be distinguished from the positive electrode by the dark space which surrounds it, and is separated from it by a patch of light of a deep blue colour. This dark space increases as the vacuum is made higher.

§ 106. **Current Eddies.**—When a conducting sheet is moved about in a magnetic field so that the lines of force passing through it are altered in number, or when the same alteration is effected by variation of the field itself, currents are produced in the sheet. These currents circulate somewhat after the fashion of eddies in water, and are consequently known as current eddies. Their presence is easily recognised by their reactionary effects.

For example, suspend a magnetic needle by a fine untwisted thread immediately over a copper plate, and set it oscillating. The oscillations will gradually decrease in amplitude, and soon cease altogether. Now remove the plate and set the needle oscillating to the same original extent as before. The oscillations will again gradually decrease in amplitude, but much less rapidly, so that, long after the time during which the motion died away in the first case, the needle will still be oscillating. In fact, when the copper plate is near, there exists a retarding influence. The energy of motion of the needle is comparatively rapidly abstracted from it. The reason simply is that as the needle swings to and fro, the magnetic field traversing the copper plate is changing correspondingly, currents are induced in the plate, and the electrical energy so developed is transformed into heat in accordance with Joule's Law. According to Lenz's Law, the magnetic reaction of the induced currents is such as to resist the change that induced them, that is, to retard the motion of the needle. Any conductor placed near a vibrating needle has the same *damping* effect, as it is called. Such a conductor placed close to a suspended needle is called a damper. It is an important part of many kinds of galvanometers, serving to retard the oscillations of the needle, and so to facilitate the operation of reading the deflection.

If the conducting plate be rotated below a magnet freely suspended above it, but not in contact with it, it will be drawn round after it. This experiment is known as Arago's Rotating Disc Experiment. The phenomenon was observed in 1824, but was not explained till Faraday discovered the induction of electric currents.

The general explanation of this effect may be given as follows, bearing in mind the fundamental law that the magnetic effect of the induced current is opposed to the magnetic change that produces it. In fig. 171 let SN be the magnet and ABCD the disc rotating in the direction shown. We may divide the disc into four quadrants symmetrically arranged with regard to the magnet and its magnetic field. In quadrants I. and III. each

portion of the disc is moving *from stronger to weaker* fields; hence the reactionary magnetic effect of the induced currents will be on the whole to *increase* the field in these quadrants. On the other hand, in quadrants II. and IV. each portion of the disc is moving *from weaker to stronger* fields, so that the reactionary magnetic effect will be to diminish the field. These effects are indicated in the figure by the algebraic signs *plus* and *minus*. Now, if we consider the general configuration of the lines of force as they pass from N to S, we shall see that the asymmetry which the reactionary magnetic effects have imposed on the magnetic field is similar to the effect of superposing a field in direction DB. Into this direction accordingly the magnet will tend to turn.

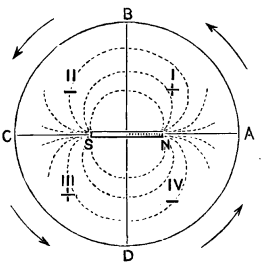


Fig. 171.

When a metal disc is rotated between the poles of a strong magnet, the existence of these current eddies is indicated in two ways. In the first place, there is experienced a very evident resistance to the motion of the disc, which may be so great as to have all the semblance of a powerful friction brake. Then there is a marked heating of the disc. The first effect is an illustration of Lenz's Law, and the second an illustration of Joule's Law.

SUMMARY.

(102) Production of Induced Currents. Magnet moving near a closed coil. Coil moving near a magnet. Same effects with electro-magnets. Primary and secondary circuits.

(103) Relation between Induced Currents and Change of Field. Conjugate coils. Appropriateness of name Magneto-electric. Induced electromotive force measured by the rate of change of field. The direction of the induced current. Lenz's Law.

(104) Mutual and Self Induction. Inductance, mutual and self. The extra current. Slow growth of primary current in circuit of high self-inductance.

(105) The Induction Coil. Discharge through vacuum tubes.

(106) Current Eddies. Damping of a vibrating magnetic needle. Arago's rotating disc experiment.

CHAPTER XVIII.

IMPORTANT APPLICATIONS.

§ 107. **The Electric Telegraph.**—The operation which makes an electric bell ring (§ 83) may obviously be accomplished far from where the bell itself is. All that is needed is a sufficiently long circuit, with a strong enough battery included in it. When this circuit is broken at a point far away from the bell, the current ceases to flow altogether, and the bell ceases ringing. When the circuit is completed again, the bell begins to ring, and we may keep the bell ringing as long a time, or as short a time as we please, by keeping the circuit closed correspondingly. This is essentially the principle of the electric telegraph. Instead of the electric bell, there is some other electro-magnetic arrangement for showing when the current passes.

In the earliest form of telegraph, that of Wheatstone and Cook, this electro-magnetic arrangement was a galvanoscope, the needle of which moved to one side or the other, according to the direction of the current that was sent from the distant station. By combinations of these two signals, all the letters of the alphabet could be symbolised, and any word spelled out.

In the Morse system of telegraph the two signals are got by making the current pass for a short and a long time. At the receiving station the current traverses the coils of an electro-magnet, which draws towards it an armature. This armature is attached to the end of a lever, which tilts slightly towards one side when the current is flowing, and springs back again when the current ceases to flow. The other end of the lever bears a steel point or style. When the current is flowing, the style presses against a ribbon of paper, which is drawn past it by clockwork. When the current is allowed to flow for a short time, the mark imprinted on the paper is a dot; when the current flows for a longer time, the mark is a dash. By combinations of the dot and dash, all the letters of the alphabet are symbolised. The Morse alphabet is as follows:

A · —	J · — — —	S · · ·
B — · · ·	K — · —	T —
C — · — ·	L · — · ·	U · · —
D — · ·	M — —	V · · · —
E ·	N — ·	W · — —
F · · — ·	O — — —	X — · · —
G — — ·	P · — — ·	Y — · — —
H · · · ·	Q — — · —	Z — — · ·
I · ·	R · — ·	

The following diagram (fig. 172) shows how two stations are 'joined up' on the Morse system. B and B_1 are the batteries at the stations S and S_1; k, k' are the transmitting keys, whose action will be described immediately; n, n' are the electro-magnets which receive and record the signals; g, g' are the galvanometers

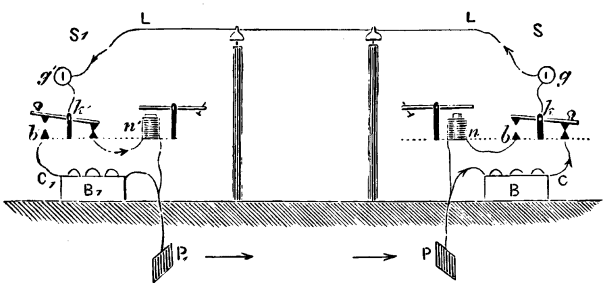


Fig. 172.

which show to the sender that his signals are being transmitted; LL, the line-wire insulated on posts; P, P_1, the earth plates. Instead of the complete wire circuit referred to above, there is only one wire, whose ends are brought into good connection with the earth. The earth, in fact, completes the circuit. The Morse transmitting key is simply a lever which completes the required circuit when it is pushed down. Let the key, k, at the station S be depressed. This completes the circuit, so that the current passes from the copper pole, C, of the battery, B, through the key k, and the galvanometer g, to the line LL, thence through the galvanometer g', the key k', the coils of the electro-magnetic recorder n', and so to earth. If a message is to be sent from S_1 to S, the key k must be left to itself, when it will spring back upon the point b. Then on k' being depressed, the current will flow from B_1

through LL, to the recorder n of station S, and so to the earth. When both keys are left to themselves, the line, galvanometers, and recorders are in circuit with the earth. In this position the galvanometers will show the presence of earth currents.

On long circuits the direct current from one station may not be strong enough to work the recorder at another station. In this case a local battery or *Relay* is introduced. The line current passes through an electro-magnet, which moves a light lever-armature in the familiar way. This completes the circuit containing the relay and the recorder; and thus the recorder is worked by the relay, and not by the line current directly.

For an account of the numerous modifications and improvements which the electric telegraph has undergone since it was first invented, recourse must be had to the technical books on the subject.

§ 108. **The Telephone.**—In its modern form, the telephone is an instrument for transmitting articulate speech to a distance. It consists of two parts, the transmitter and receiver. The original Bell Telephone can be used in both capacities; but, practically, it is used only as a receiver. In fig. 173 a sectional sketch is shown of two Bell telephones arranged in circuit so as to transmit articulate speech. Each telephone consists of a cylindrical bar magnet,



Fig. 173.

a; a small bobbin, b, filled with a good many turns of fine insulated copper wire; a thin circular disc of iron, c, close to, but not touching, the one pole of the magnet, and a mouthpiece (or earpiece), d. The small coils on the two telephones are joined up into one circuit.

Suppose a person to speak into A, which in this case is the transmitter. The sound-waves set the disc c into vibration. These vibrations produce corresponding variations in the magnetic field surrounding the bobbin b; and these again give rise to induced currents in the circuit of the two coils. In the far telephone B, the induced currents produce corresponding variations of field in the neighbourhood of the disc c'. This disc is accordingly set into vibrations, and gives off sonorous waves which will be heard by the

ear placed at the opening, d', of the receiver. Thus the vibrations originally communicated to d are in great measure given off again at d'.

Instead of the Bell Telephone, it is usual to use as transmitter a different instrument altogether, invented by Hughes, and known as the Microphone. The microphone consists of sticks of carbon in loose contact. In a complete circuit connecting two stations, there are two microphone transmitters and (at least) two Bell telephone receivers, one of each at each station. These are included all in the same circuit with a battery of cells. The current flows across the loose contacts of the microphone; and any sound uttered near the microphone produces, in some way not clearly understood, variations in the resistance at the contacts of such a character that the corresponding variations in the current set the iron disc of the telephone receiver into vibrations, which give off to the air very faithfully the original articulate sounds spoken in front of the transmitter.

§ 109. **Dynamo-electric or Magneto-electric Machines.**—When a closed coil is rotated continuously in a magnetic field about an axis not parallel to the lines of force, induced currents are produced, first in one direction, then in the other, as the coil changes its aspect relatively to the field. By attaching the terminals of the coil, insulated carefully one from the other, to the axle about which it is rotated, we may by means of pressure contacts connect these terminals to fixed wires, and so complete the circuit altogether outside the rotating part of it. If the pressure contacts or *brushes*, as they are termed, remain each always in contact with the same terminal of the rotating coil, then an alternating current will be produced in the rest of the circuit as well as in the coil. By a very simple device, known as the *commutator*, the brushes may be made to interchange their connections at the very instant that the electromotive force is changing its direction in the rotating coil; and in this way a current is produced always in the same direction through the rest of the circuit. One form of commutator is shown in fig. 174, which represents a section of the axle of the rotating coil at the place where the brushes press upon it. The dark parts are conducting. H, K are the brushes pressing upon the insulated metal parts which form the terminals of the rotating coil. It is clear that, as the axle rotates, H and K interchange their connections every half turn.



Fig. 174.

The description just given contains the complete statement of the

principle on which all dynamo-electric machines act. The essential parts of such machines are

(1) The Field Magnets, between the poles of which is produced the powerful field necessary for the induction of currents in the moving coils or armature;

(2) The Armature, which consists of coils and iron cores, and rotates in the field. Originally the field-magnets were powerful permanent magnets; but since the discovery of the principle of

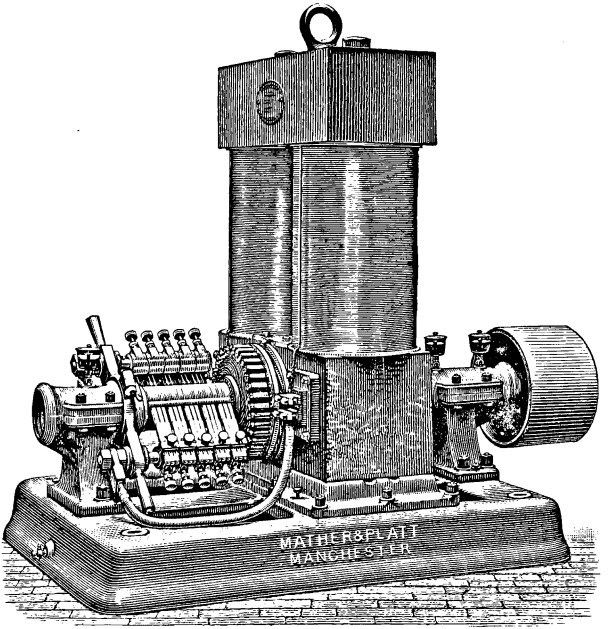


Fig. 175.

self-excitation, the field-magnets have been electro-magnets with cores of soft iron. The coils that embrace the field-magnet cores are included in the circuit of the armature. There is always sufficient residual magnetism in the cores to initiate a feeble current in the wires of the rotating armature. This feeble current develops more magnetism in the cores, which in its turn develops more current in the circuit; and so the process goes on until the field-magnets attain a high magnetisation. As this stage is being

reached, the armature becomes more and more difficult to drive. The electric and magnetic energy is thus derived directly from the mechanical or dynamical work spent in driving the machine—whence the name *Dynamo* familiarly applied to this kind of current generator. A dynamo may be either a continuous current machine, or an alternating current machine, the former being the more usual type. The latter is useful in certain kinds of electric lighting.

Fig. 175 illustrates the Edison-Hopkinson dynamo, which may be cited as an excellent instance of modern construction. Here a drum armature is used, not a ring; and in this instance the armature coils, instead of being of wire as they are in smaller machines, are formed of copper bars insulated with mica, each pair of opposite bars being joined to form a loop, the ends of which are connected to opposite segments of the commutator, as well as to the loops which come next in order. The field-magnets are shunt-wound, and are set vertically with the pole-pieces at the bottom. Machines of this class are made of sufficient size to give a current of 660 ampères, with a potential of 105 volts; the output of electrical energy is therefore at the rate of 69,300 watts or ampère-volts, or over 92 horse-power. There are five brushes on either side of the commutator, giving a large area of contact, and these are separately removable to allow of their being trimmed or cleaned while the machine is running.

§ 110. **Electromotors, or Electro-magnetic Machines.**—The function of these machines is to convert the energy of the electric current into mechanical work. They may be regarded as the converse of the dynamo-electric machines. In the dynamo the current is produced by the rotatory motion of a part of a circuit in a magnetic field. If we pass a current derived from some external source into the dynamo, it will cause that same part of the circuit to rotate.

In electromotors, as in dynamos, the magnetic field may be produced either by permanent magnets, or by electro-magnets. A very simple form of the former type is shown in fig. 176. NS is a fixed permanent magnet; *ns* is an electro-magnet, with iron core capable of rotating about the vertical axis *e*. The commutator *c* is of the kind shown in fig. 174; *a*, *b* are springs pressing on the commutator, and connected to the terminals of the battery. In the position represented in fig. 176, the current magnetises the electro-magnet, so that the nearer pole is a south pole, and is accordingly attracted by the north pole of the permanent magnet; while the south pole of the permanent magnet acts similarly upon the north pole of the electro-magnet. When the electro-magnet is

brought into a line with NS, the springs interchange their connections, and the current in the electro-magnet is reversed. What was at first a north pole becomes a south pole, and *vice versâ*. Hence the end of the electro-magnet, which was at first attracted by the north pole of the permanent magnet, is now repelled. Similarly, the other end, at first attracted by the south pole of the permanent magnet, is now repelled. But in virtue of its acquired momentum the electro-magnet retains its rotation, and each end moves past the pole which is just ceasing to attract and beginning to repel it. The repulsion that then comes into play acts so as to keep up rotation in the same direction. In this way, by reversal of the polarity of the electro-magnet every half revolution, a continuous rotation is kept up. We may suppose this rotating armature to be used for doing useful work, for example, working fans, or sewing-machines, or even much heavier machinery.

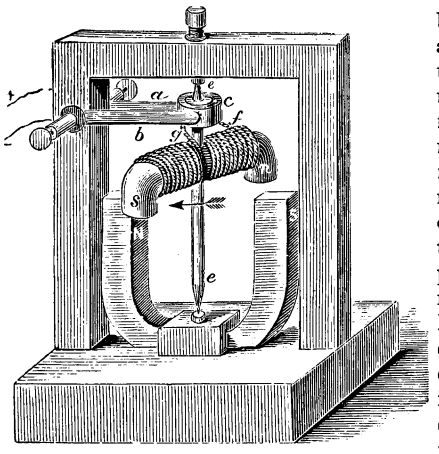


Fig. 176.

Instead of using a permanent magnet, we may arrange the apparatus so that the current supplied magnetises the field-magnets as well as the armature. Thus the current derived from one self-exciting dynamo may be used to drive another dynamo used as a motor. The electromotor finds one of its most important applications in the Electric Railway and Electric Road Cars.

SUMMARY.

(107) The Electric Telegraph.
(108) The Telephone.
(109) Dynamo-Electric or Magneto-Electric Machines.
(110) Electromotors, or Electro-Magnetic Machines.

RADIANT ENERGY.

CHAPTER XIX.

LIGHT.

§ 111. **Familiar Facts concerning Light.**—Light, in its primary significance, is that which affects our organs of vision. In the absence of light there is darkness, and we can see nothing. When we look at an object we turn our eyes so that light coming from the object may enter them and produce the necessary sensation. All we really know is the direction of the light as it enters our eye; but experience teaches us that in this direction lies also the object whose presence is made evident by the light coming from it. By use of a mirror we are able to see the *image* of an object in a direction quite different from that in which we know the object to lie. In this case the light coming from the object is received on a polished surface (glass, mercury, water, metal, mahogany, &c.), and turned aside or *reflected* in a direction depending upon the aspect of the surface. It is easy to convince ourselves, by interposing opaque screens, that the light coming from the object travels in straight lines through the air, in one straight line from the object to the mirror, in another straight line from the mirror to the eye. By experiments of this nature we prove that light travels in straight lines through any homogeneous medium.

The distinction between *opaque*, *translucent*, and *trans-*

parent bodies is of great importance. Strictly speaking, the difference is one of degree, depending upon the manner and extent of the absorptive action of the body for light. In the case of opaque bodies, the light that is not reflected is wholly absorbed, and none passes through. With translucent bodies a certain amount of luminosity gets through, but it has lost by absorption so much of the character and definiteness with which the light originally fell upon the surface that no distinct image is visible. With transparent bodies there is still a loss of light due to absorption—a loss which often means a great change in colour—but what is transmitted brings to the eye a distinct image of the luminous or illuminated object. The same substance may be opaque, translucent, or transparent according to its thickness. Pasteboard is opaque, ordinary paper is translucent. It is possible to get gold-leaf so thin that it becomes not merely translucent but transparent. Clearly, then, the transmissiblity of light through any portion of matter depends upon the condition in which the matter is.

Every substance interposed in the path of a ray of light casts a *shadow* of some kind, from the slight diminution of brightness produced by a transparent body to the completely dark shadow that demonstrates opacity. When the light radiates from a luminous source of small dimensions (like the electric arc) the shadows cast by opaque bodies are crisp in outline, and reproduce the contour of the body as it would appear to an eye placed where the source of light is. In this we have another proof of the rectilinear propagation of light.

When the light comes from an extended surface like the sun, the shadow is not crisp in outline, but is bounded by a hazy margin called the *penumbra*. Each point in the penumbra is, in fact, illuminated by rays which come from some part of the luminous body. A particularly interesting case of penumbra and true shadow is given by an eclipse of the moon. This is due to the moon passing into the

shadow cast by the earth in the presence of the sun. In fig. 177, S may be taken as representing the sun, and T as representing the earth. The dark cone OBB′ is the true shadow — the sun is completely hidden from all points within it. On the other hand, from every point in the shaded part of the cone CO′C′ and outside the dark cone BOB′ a portion of the sun will be visible, and a portion only. Within the region referred to the illumination will be partial, diminishing from the extreme boundaries towards the cone

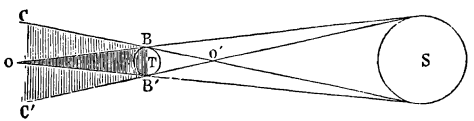


Fig. 177.

BOB′. During an eclipse the moon first enters the *penumbra*, losing so much brightness, so that when it enters the real shadow, the contrast is not between one part in shade and one part in full sunshine, but between one part in shade and another part in partial shade. Similar combinations of shadow and penumbra are obtained with an ordinary gas-jet.

All light comes originally from a luminous body, such as the sun, a star, an electric light, a gas-jet, a candle or other flame of combustion, a phosphorescent substance, &c. But the light by which the eye recognises the form and colour of an illuminated body is not inherent in that body, but has been borrowed from some self-luminous source. Thus the moon and planets shine by reflected light, and almost everything we see by day is in like manner illuminated. Except in the case of phosphorescence, a self-luminous body is invariably found to be at a high temperature. In other words, a body which is giving out light is also giving out heat. It is only after a body (a poker, for example) has been heated for some time that it begins to glow or radiate visible rays. As the metal is allowed to cool from a bright-red condition, the glow gets duller and duller and at length

disappears. But the metal is still a radiating body, producing on our nerves the sensation of heat; and its condition as a cooling but dark body differs only in degree from its previous condition as a cooling and bright body. This suggests that what is popularly called light may only be a part of the actual radiation of a luminous body; that part, namely, which affects our organs of vision. We have seen that waves of compression and dilatation in air form *audible* waves only when their frequency lies between definite limits. Dynamically considered, these waves are all alike; their audibility depends upon their wave-length. As we shall find, a similar identity and difference exist between non-luminous and luminous radiations.

§ 112. **The Speed of Light.**—The speed with which light travels through air is very great, so great as to be, for ordinary terrestrial distances, practically instantaneous. That light travels much faster than sound is proved by many familiar experiences. The flash and report of a gun at even a few hundred feet off are separated by an appreciable interval of time, and at the distance of a mile by fully 5 seconds. To the spectator a batsman at cricket is seen to hit the ball a little sooner than the instant at which the sound of the impact reaches his ear. The interval of time that elapses between the lightning-flash and the beginning of the accompanying peal of thunder is a sure indication of the distance the flash is from us. For we may assume the light to travel instantaneously, whereas the sound originated simultaneously takes a second to travel every 1100 feet.

The earliest measurements of the speed of light through space were obtained from astronomical phenomena. The eclipses of Jupiter's satellites were observed to occur sometimes sooner, sometimes later, than the calculated times. This discrepancy was found by Römer (1676) to be closely related to the distance Jupiter happened to be from the earth, and was fully explained on the hypothesis that light took 16 minutes 36 seconds to pass across the mean diameter of the

earth's orbit. This gives a speed of about 300,000 kilometres, or 186,000 miles, per second. Fifty years later Bradley deduced the same number from the phenomenon of the aberration of light of the fixed stars. When the velocity of the earth in its orbit has a component perpendicular to the ray of light from a star, the star is seen as if displaced in the direction of the earth's motion, the angular displacement being equal to the ratio of the magnitude of this perpendicular component to the speed of light. An analogous phenomenon is the apparent obliquity of a vertical shower of rain to one advancing through it. The greatest displacement of a star due to aberration of light is very small, only $20\frac{1}{4}$ seconds of angle, the arcual measure of which is about $\frac{1}{10000}$. Hence the speed of light is, roughly, 10,000 times the speed of the earth in its orbit.

The speed of light through the air was first experimentally measured by Fizeau (1849), who employed for this purpose a rotating toothed wheel and a mirror at a distance of about 5 miles. Matters were so arranged that a ray of light, after passing through the gap between two teeth and traversing the 5 miles, was received perpendicularly on the surface of the mirror and reflected straight back again. The returning ray was observed by the eye placed behind the toothed wheel. With the wheel at rest, the returning ray will pass through the same gap through which it was originally projected. But when the wheel is set in rotation, and the rate of rotation steadily increased, a time will come when the returning ray will be stopped by the tooth occupying the position which the gap occupied when the ray was projected through it. Hence the observer will observe nothing; for when the gap comes in front of his eye, there is no returning ray to pass through it, and when there is a returning ray, the tooth stops it. Let the rate of rotation be now doubled, then the returning ray will be visible to the observer; for when it reaches the wheel it will pass through the gap next again to the gap through which it was originally projected. In other words, the time during which the wheel turns through the angular distance of two contiguous gaps is equal to the time taken by the light to travel to the mirror and back again. Knowing the rate of rotation of the wheel and the distance of the mirror, we can easily calculate the speed of light.

Analogous in principle is Foucault's method, in which a rotating

mirror takes the place of the rotating wheel. A ray of light is thrown upon the surface of the rotating mirror. Once every revolution, when this mirror is in a certain position, the reflected ray falls perpendicularly upon the fixed mirror, is reflected back to the rotating mirror, and is then thrown off in a direction which makes with the original direction of the incident ray a small but measurable angle. This angle is, in accordance with the law of reflection, twice the angle through which the revolving mirror has turned, as the ray has travelled from it to the fixed mirror and back again. Hence, knowing the rate of rotation of the one mirror and the distance between the mirrors, we can calculate the speed of light.

The most recent values for the velocity of light in vacuo give a mean of 29,990 kilometres, or 186,400 miles, per second. Thus light takes $1\frac{1}{4}$ seconds to come from the moon to us, and $8\frac{1}{4}$ minutes to come from the sun. From the nearest fixed star it takes $3\frac{3}{4}$ years, while from the great majority of stars its time of flight is to be reckoned in hundreds of years. The light-year, or distance travelled in one year by light, is a convenient length in which to express the enormous distances which separate star and star in the universe.

§ 113. **The Nature of Light.**—It will greatly facilitate further discussion to consider at this stage the true nature of light. It is obviously an emanation of some kind from the luminous source. It may be something material shot out from the source, or it may be a disturbance originating at the source and passing out through a medium filling all known space. Most of the ordinary facts can be equally well explained on either hypothesis. The two rival theories of light just stated are generally known as the *Corpuscular* or *Emission Theory* and the *Undulatory* or *Wave Theory* of light. The latter was discarded by Newton because it did not appear fitted to explain the existence of sharp shadows; the waves would bend round an obstacle. With the advance of scientific knowledge this difficulty has been removed; and the Wave Theory is now universally accepted.

A medium, called the Ether, is assumed to fill all space not occupied by the ultimate particles of matter. When these

particles are heated they are thrown into rapid vibrations, which are then transmitted to the ether as waves, very much as a vibrating bell transmits sound-waves to the air. This implies that the ether in the immediate neighbourhood of the particle is connected in some way to it. Thus the presence of matter is to load the ether in which it rests; and all the phenomena of transparency, translucency, opacity, and colour depend upon the manner of this loading.

On this theory, again, the great speed of light admits of a very simple explanation. In the case of an elastic solid, the speed of an elastic wave is measured by the square-root of the ratio of two quantities, the coefficient of elasticity and the density. By diminishing the density we at once increase the speed. Now the ether, if it be in any way comparable to an elastic solid, must have a very small density, or we should expect to find it offer an appreciable resistance to the motion of planets and satellites through it. It should be remembered, however, that the waves of light may not really be elastic vibrations. A very remarkable theory, broached and elaborated by Maxwell, regards the vibrations as oscillatory periodic changes in electric or magnetic condition. We shall find it convenient, however, to form in our mind the rather crude conception of the ether as being of the nature of an elastic incompressible solid of very small density. The waves are therefore purely distortional waves, and the motions of the elements of the ether take place at right angles to the direction of the ray. The vibrations take place in the wave-front.

Reflection and refraction of light follow the same laws as reflection and refraction of any wave-motion. The bending of the ray towards the normal as it enters the denser medium is at once explained if we assume that the speed of light is less in the denser medium. In Newton's form of the corpuscular theory, it was necessary to assume that the light corpuscles travelled faster in the denser medium. Here, then, was a crucial experiment to decide between the claims of the

two rival theories: measure the velocities of light in air and in water. This was effected by Foucault and Fizeau, who proved by independent experiments that light travelled slower in water than in air. The corpuscular theory was thus emphatically disproved. The physical explanation of the fact that light travels slower in the denser medium is to be found in the effect of loading the ether. The light in passing through a transparent substance really passes through the ether permeating this substance. The molecules may be regarded as increasing the effective density of the ether, and so diminishing the speed of light by an amount depending very directly upon the density of the material.

If light is propagated as wave-motion, then it must possess qualities corresponding to the two fundamental properties of waves—the *amplitude* and the *wave-frequency*. Now, just as the ear distinguishes sounds by their intensity and pitch, so the eye distinguishes in light great varieties of intensity and colour. Intensity obviously has to do with the energy of vibration, which is proportional to the square of the amplitude. And that colour depends on wave-length, or (since the speed in ether is the same for all wave-lengths) on frequency, will be demonstrated again and again as the subject is developed.

§ 114. **Photometry** means the numerical comparison of the intensities of different kinds of light. The intensity of light due to a given source, say a candle, depends upon the distance from the source of the point where the intensity is being considered. From a point-source light will radiate out in straight lines and the wave-front will be a spherical surface, carrying energy with it in all directions through the surrounding ether. The same amount of energy will pass in unit time across any imaginary fixed spherical surface with the point-source as centre, and the area of this surface is directly as the square of the radius. Hence the amount that passes across unit area on any such surface will be inversely as the square of the distance from the source.

Let us then take two sources of light, say a candle and a gas-jet, and arrange them in such a way that each will cast a shadow of some opaque object upon a screen close to it. By adjusting the relative distances of the two sources we get the two shadows to be apparently of the same intensity. The brighter source of light must be placed at the greater distance; and, in accordance with the principle just stated, the intensities of the sources of light will be directly as the squares of the distances from the screen. This shows the principle on which all so-called *photometers* act. The sources of light are so arranged with reference to the photometer that they seem to give equal illuminations. This mode of comparison is sufficiently satisfactory when the lights do not differ much in colour; but when there is a marked difference in colour all attempts to compare intensities are rough approximations.

In Bunsen's photometer a sheet of white paper with a grease-spot on it is placed between the two sources of light and moved about until the grease-spot is no longer distinguishable. The paper is then equally illuminated on both sides, so that the greater transparency of the greased part of the paper to transmitted light is compensated for by the greater reflecting power of the rest of the paper. The amounts of light from the two sources are, as before, directly as the squares of their distances from the sheet of paper. For ordinary purposes this simple form of photometer is very serviceable; but for more accurate work experimenters have devised other forms of photometers of more or less ingenious complication.

A very important consideration in photometry is the standard light with which all lights are compared. In this country the standard light is the sperm-candle burning at the rate of 120 grains per hour. A flame is then spoken of as being so many candle-power.

The International Conference of Electricians adopted (in 1884) as standard the light from 1 square centimetre of melted platinum at the melting-point.

§ 115. **Refraction and Dispersion.**—We have already seen that refraction is a necessary property of light, if light is a form of wave-motion propagated with different velocities in different media. But when a ray of sunlight is made to fall obliquely on the surface of some transparent medium, there is found accompanying the refraction another phenomenon known as **Dispersion.** The originally white beam of light becomes, after refraction, coloured at two of its sides, the one side being bluish, the other reddish. The experiment is best performed by allowing the ray of light to pass through a prism of glass or other transparent material. The arrangement adopted by Newton is shown diagrammatically in fig. 178, in which, to bring out the essential character of the phenomenon, the dispersion has been greatly exaggerated.

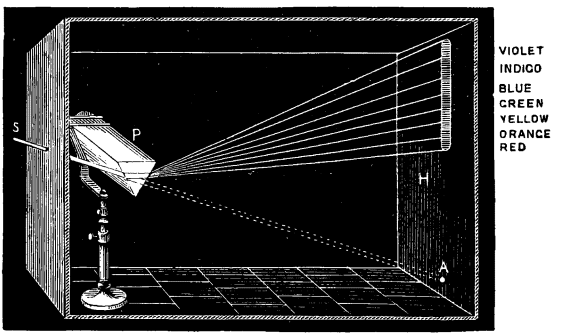


Fig. 178.

A beam of circular section, SA, is introduced into a dark chamber, and in its path is interposed the glass prism P. The rays are bent in towards the normal to the first face as they enter the prism, and are bent away from the normal to the second face as they leave it. Hence the ray as a whole is bent upwards. But, in addition to this refraction, the circular beam is drawn out in a vertical direction, so that when it is received on a screen it is a long narrow ribbon of varied colour. The colours succeed one another from red, through

yellow and green, to blue and violet—the red ray being the least refracted, the violet ray the most. This drawn-out band of colour is called the *Spectrum* of sunlight; and any light similarly treated gives its appropriate spectrum. The experiment proves the composite character of white light, and shows, moreover, that the differently coloured rays of light are differently refracted. This separation of the different coloured components of any ray of light is called *dispersion*. According to the wave theory, the different amounts of refraction—in other words, the different *refrangibilities* of the differently coloured rays—mean that these travel with different speeds in any material medium. In free space—that is, in the ether—all rays of light travel with the same speed; for if it were not so, a sudden outburst in a distant star (e.g. the *Nova Aurigæ* of 1893) would appear first of one homogeneous tint, say red, which would then gradually brighten to a composite white light as the rays of smaller velocity came to us after their many years' flight. Of this there is not the slightest evidence. Nor has there been any direct evidence certainly obtained that different coloured rays travel with different speeds through air, although there is a slight but measurable dispersion accompanying the refraction of a ray passing from the ether into a gas. With solid and liquid substances there is a very pronounced dispersion, and the different speeds of the various coloured lights are regarded as due to the different effects of the grained structure of matter upon the wave-lengths, which are associated with the different colours. See Table B at the end of the book.

§ 116. **Colour.**—Colour, strictly speaking, is a sensation, being the resultant effect upon our organs of vision of the rays coming from the object seen. Each part of the spectrum of white light produces in us the appropriate colour-sensation, red, orange, yellow, green, blue, violet, with all the intermediate gradations of tint. For convenience we speak of a particular ray as being a red or a green ray, although the redness or greenness does not belong to the ray but to the

subjective impression produced by the ray on us. The colour of a ray no more belongs to the ray than the pain produced by the impact of a stone upon our body belongs to the stone. That colour is really a sensation is proved by the existence of colour-blindness. Certain persons, with otherwise perfect vision, are unable to distinguish, say, between what to normal eyes appear as red and green. Others again cannot distinguish between yellow and blue. To such the spectrum appears to be of only two colours, and certain marked contrasts of colour are indistinguishable, or, if they appear to be of different shade, are still of the same tint.

The colour any object appears to have depends on the rays of light it emits or allows to pass through it. Red glass is red because it absorbs most of the green and blue light, so that *red largely predominates*. A black object sends back to our eye practically no light—it absorbs all kinds completely. A white object sends back the different components of light in much the same relative proportions, so that the resultant sensation on the eye is practically unchanged. A white object will appear red if illuminated by red light, blue if illuminated by blue light, and so on. A red object will appear black, or nearly so, in blue light, and probably orange in yellow light. In the consideration of such cases we must bear in mind that the colour of an object is determined by the predominating factor; the other colours may be present, although in much feebler intensity. For example, how do we explain the production of green by a mixture of yellow and blue pigments? The answer is, that green is common to the mixtures of rays given by both the blue and the yellow pigment. A ray of white light, after being reflected from a blue particle, has been robbed of all but green, blue, and violet components. It then falls on a yellow particle, which robs it of the blue and violet. Or, the ray falling on the yellow particle is robbed of all but its orange, yellow, and green. It then falls on a blue particle, which robs it of its yellow and orange. In either case green

is left. Were the original pigments a pure blue and a pure yellow without any admixture of green, it would be impossible to obtain green by mixing them together.

If yellow and blue be made to act simultaneously on the eye, so that neither can be distinguished from the other, the resultant colour-sensation is a more or less dusky pink. The combination is most simply effected by the rapid rotation of a disc which is coloured partly yellow and partly blue. Other colours may be combined in the same way. It is then found that to every colour there corresponds another such that if the two are combined the resulting sensation is a neutral tint. These colours are said to be complementary. Together they impress the eye as white or colourless light. Particular tints of red and green, of orange and blue, of yellow and violet, form in this way pairs of complementary colours.

If we look fixedly for some time at a red-covered book, with a strip of white paper stretched over it, and then turn our eyes upon a sheet of white paper, we see, instead of the white strip with a red background, a red strip with a greenish-blue background. The explanation is that, in looking at the red surface, the eye, as regards its sensitiveness to red light, becomes fatigued. It is no longer equally sensitive to all components of white light, but is more sensitive to those components which produce the colour complementary to the red. Hence the white paper appears bluish, all except the part corresponding to the position occupied by the white strip across the red cover. This by the effect of contrast appears complementary to the rest of the sheet. The reason for this effect of contrast is probably physiological, depending upon the mutual action of neighbouring parts of the eye. A closely related phenomenon is the bluish shadow cast by an object on a white screen illuminated by a red light. A beautiful modification of this experiment is to project two shadows on the screen by means of a white light and a coloured light. The shadow cast by the white light is illuminated by the coloured light, and is of that

colour; the other shadow appears of the complementary colour, although it is really illuminated by the white light.

§ 117. **The Primary Colour-Sensations.** — The ordinary names given to the successive portions of a spectrum of white light do not nearly exhaust the different distinguishable tints. By comparing contiguous strips of colour, Kœnig has shown that the normal eye can distinguish about 300 different tints between the red and violet ends of the spectrum. Eyes differ in their range of sensitiveness to light, just as ears differ in their range of sensitiveness to sound. It is highly probable, indeed, that different animals may have different ranges of sensitiveness, so that rays which are visible to one species are invisible to another. But whatever be the range of colour to which the normal human eye is sensitive, it is possible to express every colour-sensation in terms of three, which are known as the Primary Colour-Sensations. This theory, originated by Young, was developed in a modified form by Von Helmholtz. The physiological assumption lying at the base of the theory is, that the sensitive part of the human eye responds to three colour-sensations, and that any particular colour-sensation is the resultant of these three combined in definite and experimentally ascertainable proportions. Whatever may be the physiology of vision, it is certain that every colour can be matched by a combination of a particular red, green, and blue, none of which can be matched by a synthesis of any other colours. These accordingly are known as the primary colour-sensations. Colour-blindness, in this theory, is explained by the lack of colour-sensitiveness to one (or, possibly in rare cases, to two) of the primary sensations, or to the blending of two of them into one. The facts of colour-blindness are indeed a powerful argument in favour of a theory of this kind; and the occurrence of cases in which only one eye is colour-blind shows that colour-sensation is a function of the eye.

A rival theory has been elaborated by Hering, who reduces all visual perceptions to six fundamental sensations grouped

in pairs—white-black, red-green, yellow-blue. An eye blind to red is also blind to green, and the so-called red sensation is really yellow; and similarly with the other pairs. Mathematically, Hering's theory is an algebraic transformation of the Young-Helmholtz theory. Neither physically nor physiologically has the more recent theory a broader basis. Invented to meet some of the difficulties of the older theory, it has its own peculiar difficulties, which necessitate modification in the theory as originally laid down. The Young-Helmholtz theory is the simpler of the two, and in its latest form is equal to all demands.

SUMMARY.

§ 111. Light. Travels in straight lines. Opaque, translucent, transparent. Shadows. Lunar eclipse. Luminous and illuminated bodies.

§ 112. The Speed of Light. Römer, Bradley, Fizeau, Foucault.

§ 113. The Nature of Light. Corpuscular and Undulatory Theories. Ether compared to an elastic incompressible solid of small density. Speeds in different media.

§ 114. Photometry. Intensity inversely as square of distance. Candle-power.

§ 115. Refraction and Dispersion. Newton's experiment with prism. Solar spectrum.

§ 116. Colour. A subjective impression. Colour-blindness. Colour of mixed pigments. Complementary colours.

§ 117. Primary Colour-Sensations. Young-Helmholtz Theory. Hering's Theory.

CHAPTER XX.

FORMATION OF IMAGES.

§ 118. It has long been customary to discuss the properties of light under two headings, *Physical* and *Geometrical Optics*. In the latter the experimental laws of reflection and refraction are postulated, and from them are deduced by pure geometrical reasoning the properties of mirrors, prisms, lenses, caustics, microscopes, and telescopes. No account is taken of the absorption of light, or of its transformation into other forms of energy. The purely mathematical discussion of Fresnel's wave-surface might also be regarded as a higher branch of Geometrical Optics; but everything else comes under the heading of Physical Optics. The division is not a logical one, although there is a certain convenience in having a definite name for those parts of the subject which have to do most directly with the construction of optical instruments.

The subject-matter of geometrical optics is the formation of images by means of reflection or refraction of pencils or rays of light. A pencil is a bundle of rays which are nearly or wholly parallel. The simple experimental laws of reflection and refraction are postulated; and, to begin with, the pencil is supposed to be one of homogeneous light, so that every ray is equally refracted as it passes from one medium to another. The angles of incidence, reflection, and refraction are the angles which the incident ray, the reflected ray, and the refracted ray make with the normal to the surface at the point of incidence. The plane containing the normal (NN′) and the incident ray (AP) contains also the other rays. The angle of reflection NPB is equal to the angle of incidence APN; both will be represented by the letter i. The angle

of refraction N'PC (or r) is such that the ratio of the trigonometrical sines of the angles of incidence and refraction is constant for the same two media. Thus, if PA and PC (fig. 179) be made of equal length, the ratio of Aa and Cc, the perpendiculars from A and C upon the normal, is constant. If the first medium is the ether, then this ratio is the refractive index of the second medium. For two media whose refractive indices are known, the ratio of the sines of i and r is equal to the ratio of the refractive indices. The refractive index of air differs very slightly from unity, so that practically the refractive index of a substance like water is what is measured when the ray passes from air to water.

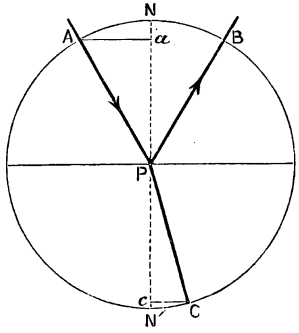


Fig. 179.

§ 119. **The Plane Mirror.**—A mirror is a surface which reflects light. A highly polished silver surface is the best example of a true mirror; but for purposes of illustration

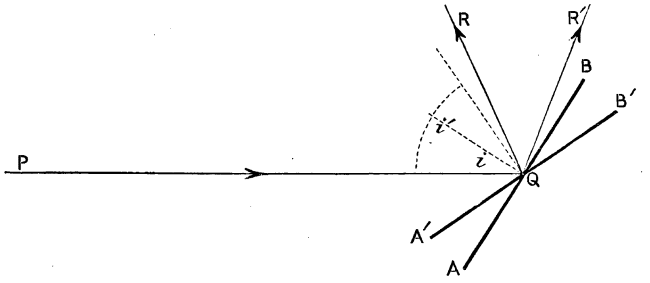


Fig. 180.

glass mirrors silvered behind are the most convenient. We must remember, however, that the principal reflection takes place off the second surface within the glass.

Let PQ (fig. 180) be a ray falling on the plane mirror AB.

It is reflected along the line QR, which makes the angle $2i$ with the original direction PQ. Now let AB be rotated about Q through the angle α into the position A′B′. The normal will move through the same angle, so that the angle of incidence will become $i+\alpha$. Hence the reflected ray QR′ will make angle $2(i+\alpha)$ with the original direction PQ, and therefore angle 2α with the direction QR. Thus, the reflected ray rotates through double the angle of rotation of the plane mirror.

Let Q be a source of light from which rays proceed and fall upon the plane mirror CB (fig. 181). Let AR be one of the reflected rays, so that QAR gives the real course of the ray. To an eye placed at R, the light will seem to come from the direction of A. The exact point from which the reflected rays seem to come is where two contiguous rays meet if produced backwards. But that is easily seen to be the point q, which lies *behind* the mirror at distances from the points of reflection equal to the distances of Q from the same points. Joining Qq, we have AQ equal to Aq, AN common, and by the law of reflection the contained angles equal. Hence the triangles ANQ, ANq are equal in every respect. That is to say, the image of Q is at q, which lies behind the plane the same distance that Q lies in front and on the line drawn through Q perpendicular to the plane. Thus the position of q can be at once found without any attention being paid to the paths of particular rays. All rays from Q after reflection seem to come from q. Since they do not really pass through q, the image is called a *virtual* image.

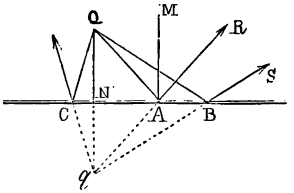


Fig. 181.

To find the path of the ray which after reflection meets the eye at S, join Sq by the straight line cutting the mirror in B. Join QB. Then QBS is the real path of the ray.

The simple construction which gives the image of a

point in a plane mirror gives the image of an extended object. Thus *am* (fig. 182) is the image of AM, every point in the image being at the same distance from the mirror as the corresponding point in the object. The pencil of diverging rays by which the eye sees the point N is shown in the figure. Notice that the eye may see the image of the object in a mirror which does not reach as far as the object. Thus, to see a full-length image of one's self in a mirror it is not necessary to use a full-length mirror.

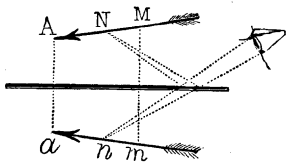


Fig. 182.

An object placed between two parallel mirrors has theoretically an infinite number of images, for every image in the one mirror has itself an image in the other, and so on indefinitely. Practically, the loss of light which accompanies every reflection causes a steady diminution of intensity in each successive image, so that only a finite number is appreciable to the eye. This multiplication of images due to the effect of parallel reflecting surfaces is easily observed in an ordinary glass mirror with silvered back. The explanation is as follows: Let AB (fig. 183) be the reflecting back of the mirror, CD the front of the glass plate. A ray from P is partly reflected from CD and partly refracted. The refracted part is wholly reflected at the point (1), and after emerging from the glass plate is the principal reflected ray by which the image of P is formed. But a part of this ray, when it meets again the surface CD, is reflected back towards AB, and a portion of it finally emerges as ray (2). This goes to form a second image. In exactly the same way a third image is formed, and a fourth, and so on until the diminishing intensity makes them too feeble to affect the eye. The best object to choose for seeing the succession of images is a gas-jet, which will appear to be bordered by several 'ghosts' of itself.

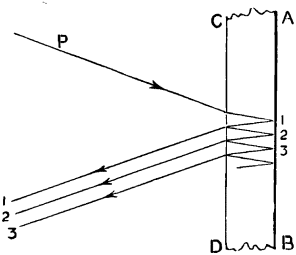


Fig. 183.

The *Kaleidoscope* is a scientific toy which illustrates the law of successive reflections from two mirrors, inclined at an angle to each other. The angle is usually 60°, one-sixth of a complete revolution; but other angles may be used. Let OA, OB (fig. 184) be two plane mirrors containing an angle of 60°, and let P be any point of an object placed between them. Each and every image of P in the two mirrors will obviously be at the same distance OP from O. Draw then a circle with radius OP. In this circle all images of P will lie. P_1, Q_1 will be the first images of P in the mirrors A and B respectively. But P_1 will have itself an image, Q_2, in the B mirror; and Q_1 will have an image, P_2, in the A mirror. Similarly, Q_2 will have an image, P_3, in A; and P_2 an image, Q_3, in B. If, as we have supposed, the angle between the mirrors is 60°, P_3 and Q_3 will coincide, and succeeding images will be repetitions of previous ones. Thus there will be one object and five images arranged in a threefold radial symmetry. If the angle of inclination of the mirror is 45°, there will be seven images of the one object; and, in general, if the angle is $1/n$th of 360°, there will be $n-1$ images of an object placed between them. If the angle cannot be expressed as a proper fraction of four right angles, the successive images go on indefinitely and no two ever come into coincidence.

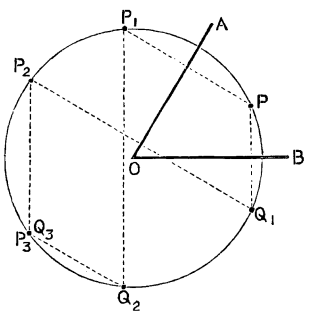


Fig. 184.

§ 120. **Reflection at Curved Surfaces.**—From a curved surface reflection takes place exactly as from a plane surface —the angle between the incident and reflected rays being bisected by the normal at the point of reflection.

Let PAC (fig. 185) represent a spherical concave surface, whose centre is O and axis OA; and let a pencil of rays from a point, U, fall upon the surface in the neighbourhood of the line drawn through U and O. Consider the ray falling on the point P the normal at which is the radius OP. The reflected ray PR will make with OP an angle OPR equal to the angle

OPU. This ray will cut the axis in a point V; and since OP bisects the vertical angle of the triangle UPV, we have

$$\frac{VO}{VP} = \frac{UO}{UP}.$$

For the most important practical applications, attention is limited to pencils of rays which nowhere deviate far from the

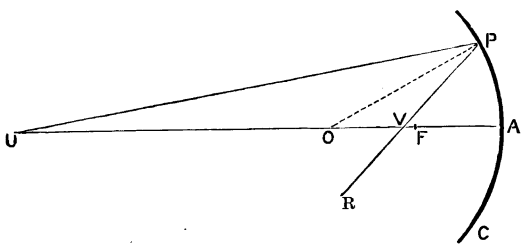


Fig. 185.

axis, so that UP is to a first approximation equal to UA and VP to VA. In other words, the angle POA is always small; and the relation just given becomes approximately

$$\frac{AO - AV}{AV} = \frac{AU - AO}{AU}$$

or, if u, v are put instead of AU and AV, and if $AO = r$, we find, after a simple transformation,

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{r}.$$

Now every ray from U will pass approximately through the point V; hence V is the point from which the rays seem to come after reflection. V therefore is the *image* of U, and it is a real image, because the rays really pass through it. Evidently, if we place the object at V, its image will be at U. That is to say, image and object are interchangeable.

If U is removed to infinity, so that the rays falling on the mirror are parallel to one another, then $u = \infty$ and $v = \frac{1}{2}r$. This point, half-way between the centre O and the pole A, is called the *Principal Focus* of the concave mirror. Any pair

of points whose distances from A satisfy the equation given above are called *Conjugate Foci.* As U is brought up from infinity towards O, V moves out from the principal focus F to O, at which point object and image coincide. If U is brought still nearer, V moves out beyond O, until when U is brought to F, V has passed off to infinity. If we take the source U between F and A, the reflected rays do not come to a focus at all, but remain divergent, and *seem to come from a point behind the mirror.* The equation, in this case, gives

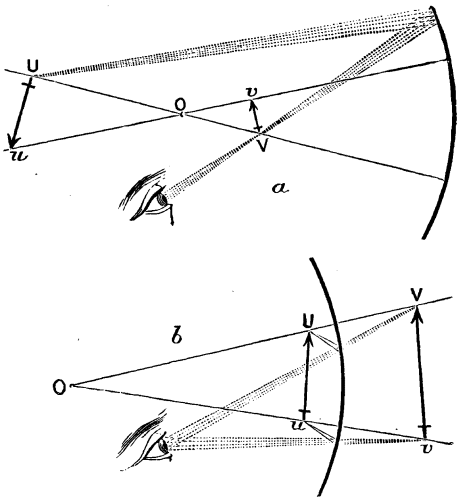


Fig. 186.

negative values to v, indicating that the point V is to be taken on the other side of the surface. Two cases then present themselves, according as the distance of the object from the mirror is greater or less than the distance of the principal focus. In the former case the image is real; in the latter case, virtual. To find the position of the image of Uu, draw through every point of the object a line through the centre, and in this line take the image of that point. The position of the image-point in this line through the centre may be calculated, or it

may be found graphically by a simple construction similar to that for lenses described below in § 126. For, since every ray parallel to the axis of the mirror is reflected so as to pass through the principal focus, let us draw such a ray from the object-point to the mirror and back through the principal focus. This reflected ray, produced if necessary, meets the line drawn through the object-point and centre in the image-point. The various image-points obtained in this way will build up the image of the object. In the one case, when Uu lies farther from the sphere than half its radius, its image Vv will lie on the same side, will be real and inverted, and will be magnified if the distance of the object is less than the radius, but diminished if the object lies at a greater distance (fig. 186 *a*). In the other case, when Uu lies nearer to the spherical surface than half its radius, the image Vv will be behind the mirror, and will be virtual, direct, and always magnified (fig. 186 *b*).

The convex mirror may be discussed in exactly the same way. If divergent rays from any point fall upon it, they will after reflection appear to diverge from a point behind the mirror. The images are always virtual. The relation between the distances of object and image is

$$\frac{1}{v}+\frac{1}{u}=-\frac{2}{r}.$$

The formation of an image is indicated in fig. 186 *b*, if we consider Vv to be the object, Uu the image, and the right-hand side of the arc the reflecting surface. The image is always virtual, erect, and diminished.

A very important use of a concave mirror is to bring parallel or divergent rays to a focus so as to illuminate an object placed there. In the reflecting telescope a concave mirror of large radius is used to form near at hand images of distant objects.

Obviously, cylindrical mirrors have properties very similar to those of spherical mirrors. They are not of much practical use.

§ 121. **Refraction at a Plane Surface.**—There are two cases to be considered according as the ray of light passes from the rare to the dense, or from the dense to the rare medium. A convenient way of illustrating the characteristic

phenomena is indicated diagrammatically in fig. 187. The essentials are a fairly long trough of water with, preferably, glass sides, two plane mirrors, and a pencil of light either from the sun or from the electric arc. One mirror C is placed at the bottom of the trough with its one end a little higher than the other. A second mirror A is so manipu-

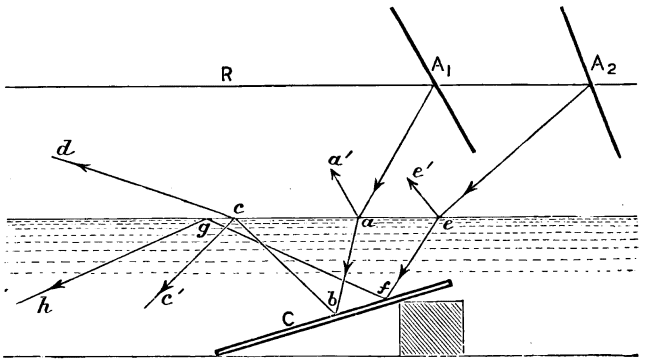


Fig. 187.

lated that the ray R, after reflection, falls on the water surface, and the refracted part is received in the face of the mirror C, and reflected back again through the water. By shifting the movable mirror from one position to another, say from A_1 to A_2, and adjusting it so that the ray always falls on some part of the fixed mirror in the trough, we can vary through a large range the angles of incidence and refraction of the ray as it crosses the surface separating the air and the water. Thus, in position A_1 the ray after one reflection falls on *a* at a comparatively small incidence. Part is reflected along *aa′*, and part refracted towards *b*. It is then reflected to *c*, and because of the inclination of the mirror C, the ray *bc* falls upon the water surface more obliquely than the ray *ab* left it. Here it is reflected along *cc′*, and, if the obliquity is not too great, is partly refracted along *cd*. Now move the mirror to A_2, so that the first angle of incidence is increased,

Then the ray RA_2efg experiences the same changes of direction as the ray RA_1abc; but because of the great obliquity of the portion fg there is *total reflection* at g, and no part escapes into the air. (See Part I., § 146.)

The refractive index of water relatively to air is almost exactly $\frac{4}{3}$ for a mean ray of white light. If i is the angle of incidence *in water* and r the angle of refraction into air, we have

$$\frac{\sin. i}{\sin. r} = \frac{3}{4} \text{ or } \sin. r = \frac{4}{3} \sin. i.$$

Now i may have any value from 0 to 90°—that is, from perpendicularity to grazing incidence. But the *sine* of an angle can never be greater than unity. Hence when i is such that its *sine* is greater than $\frac{3}{4}$, there is no angle r satisfying the equation. In short, there is no refracted ray. The angle whose *sine* is $\frac{3}{4}$ is 48° 35′. This accordingly is the so-called *critical angle* for the passage of light from water to air. When the angle of incidence in water is greater than 48° 35′ there is total reflection within the water, and no part passes into the air.

In general, if μ, μ', be the refractive indices of two contiguous media, μ greater than μ', the critical angle for a ray incident in the denser medium is the angle whose *sine* is μ'/μ. The experimental determination of the critical angle in any case gives at once a datum from which the ratio of the refractive indices may be calculated.

Of the rays which come from any point under water, only those emerge into air which lie originally within a cone whose vertex is at the point, whose axis is vertical, and whose semi-vertical angle is the critical angle.

If we place a thick glass plate over the page of a book the printing seems to be brought a little nearer. Similarly, as we fill water into a basin or tank, the bottom seems to rise, and the water appears shallower than it is. To explain this, consider the pencil of rays with vertical axis which comes from the point P (fig. 188) at the lower surface. They will appear to come from a point P′, which is

the meeting-point of the extreme refracted rays produced till they meet. For small angles of incidence the *sines* differ inappreciably from the *tangents*, and hence it follows at once that the ratio of AP to AP′ is the refractive index of the substance. In water, then, an object looked at vertically appears to be at a depth only $\frac{3}{4}$ of its real depth. If we look at the object more obliquely, its apparent depth is made still smaller. Thus, an oar dipping half in water appears to the eye to be bent; for, instead of seeing BC, we see its image BC′ (fig. 189), every point of the immersed part being at an apparent depth not greater than $\frac{3}{4}$ of its real depth.

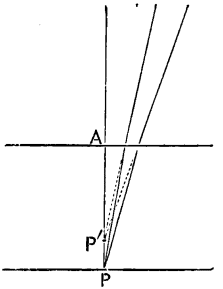


Fig. 188.

If the distance of an object is l, and we interpose between it and our eye a plate of refractive index μ and

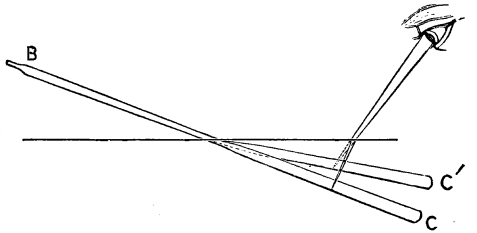


Fig. 189.

thickness t, the farther side of the glass plate is apparently brought nearer by a distance equal to

$$t - \frac{1}{\mu}t,$$

and consequently the apparent distance of the object is

$$l - t + \frac{1}{\mu}t = l - \frac{\mu - 1}{\mu}t.$$

When a ray passes through a medium bounded by parallel sides, the emerging ray is parallel to the entering ray, but is in general displaced by an amount depending upon the angle of incidence, the thickness of the medium, and the refractive index. So far as the bending or refraction of the ray is concerned, the second surface just undoes the effect of the

first surface. The whole course of the ray through such a medium—e.g. a plate of glass—is shown in fig. 190. It is only the normally incident ray Aaa'A' that remains a con-

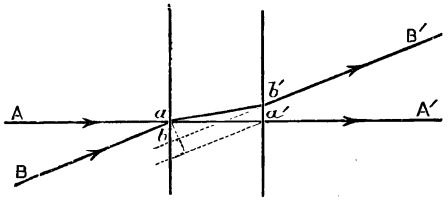


Fig. 190.

tinuous straight line. For a ray of any other incidence, say Bbb'B', b'B' is parallel to Bb, but is displaced laterally through the distance

$$t \sin.(i - r)/\cos. r,$$

where i is the angle of incidence at the first surface, r the angle of refraction, and t the thickness of the plate.

§ 122. **Refraction through a Prism.**—Geometrically a prism is any portion of matter bounded by plane sides intersecting in parallel lines. In this sense the plate is a prism; but in optics the prism is generally understood to be one whose opposite sides are not parallel. Such a case is shown

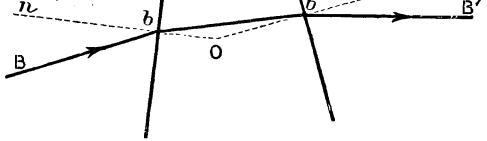


Fig. 191.

in fig. 191, which differs from the last figure in the non-parallelism of the plane faces bounding the medium. The emerging ray b'B' cannot now be parallel to the entering ray

Bb. Draw the normal bn at the point of incidence, then the angle of refraction (r) is known in terms of the angle of incidence. This once-refracted ray proceeds along bb', and falls upon the second face at an angle of incidence equal to $(\alpha - r)$, where α is the inclination of the two faces of the prism. From this second angle of incidence the final angle of emergence i' can be calculated in accordance with the Refraction Law. In short, we have

$$\text{sin. } i = \mu \text{sin. } r \text{ and sin. } i' = \mu \text{sin.}(\alpha - r).$$

The angle of deviation of the ray—that is, the angle between b'B$'$ and bB—is easily seen to be

$$\delta = i + i' - \alpha,$$

and therefore can be expressed in terms of i and α. All the optical properties of the prism can be deduced mathematically from these equations.

It is more instructive, however, to study these properties experimentally. For this end, set the prism, usually of triangular section, in the path of a beam of light, so that the beam falls in the first instance perpendicularly upon one of the faces. The emerging ray we wish to study will appear coloured, so that its position will be easily recognised. As we rotate the prism about an axis parallel to the edges, the emerging ray will alter its position, thus showing that the deviation depends upon the angle of incidence. But it will be found that there is a certain position of the prism, relatively to the incident ray, for which the angle of deviation is a minimum. Rotate the prism in either direction from this position, and the deviation of the emerging ray will increase. When this position of *minimum deviation* is obtained for any ray, the prism is found to be symmetrically situated

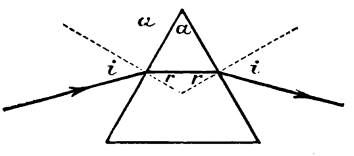


Fig. 192.

with respect to the entering and emerging rays, as indicated in fig. 192. In symbols,

$$i = i',$$
$$r = a - r, \text{ or } 2r = a,$$
$$\delta = 2i - a, \text{ or } 2i = \delta + a,$$

and hence,
$$= \frac{\sin. i}{\sin. r} = \frac{\sin. \frac{\delta + a}{2}}{\sin. \frac{a}{2}}.$$

Now for any given prism a is known, having been accurately measured once for all. The deviation δ can be measured with corresponding accuracy. And in terms of these we can determine the refractive index, μ, of the substance for the particular ray to which the minimum deviation has been adjusted. Because of dispersion, the position of minimum deviation differs for each ray of different colour; but whatever be the position of the prism, there is an equation connecting the refractive index for any ray with the measurable angles of incidence and emergence. Of this equation the one just given is the simplest form.

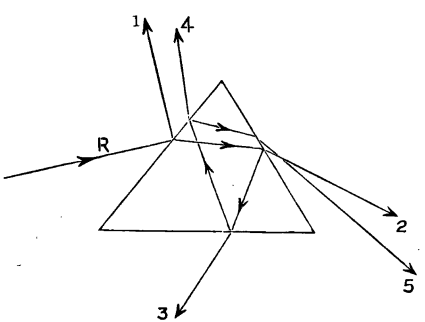


Fig. 193.

The information regarding the refractive properties of any substance is complete only when we know its refractive index for rays of all wave-lengths. The experiment just described gives no clue as to that. The determination of the wave-length corresponding to each definite colour is discussed in the next chapter.

So far, attention has been confined to the ray which traverses the prism and emerges after two refractions. But a portion of a

ray which is suffering refraction is always reflected. In fig. 193 we have an indication of the different rays into which a single ray falling upon a prism may be divided. Ray (1) has suffered one reflection; ray (2) two refractions; ray (3) two refractions and one reflection; ray (4) two refractions and two reflections; and so on. If the angle of incidence at a point within the prism is greater than the critical angle there will be total reflection and no emergent ray at that point.

§ 123. **The Rainbow** is produced by combined refraction and reflection of the sun's rays in raindrops. These are spherical; and when a ray of light falls upon any point of the surface, part is refracted towards the normal, which, for a sphere, is the radius from the centre to the point. A portion of this refracted ray is reflected within the drop, and, again in part, emerges after a second refraction. The course of a ray, which is effective in producing the phenomenon of the rainbow, is shown in fig. 194. The ray SP, after refraction at P, reflection at R, and refraction at Q, finally emerges as the ray QE; and the deviation or angle between SP and QE is expressible in terms of the angle of incidence and the refractive index of water for the particular ray. The dotted ray above SP will, in general, suffer a different deviation, because the angle of incidence is different. Now it can be shown that there is a particular point P such that, if we consider the surface in its neighbourhood, all rays of a particular colour which fall on the surface, either above or below P, emerge above Q, and make smaller acute angles with SP than does the ray QE, which corresponds to the ray falling exactly on the point P. The course of two such rays is shown by the dotted lines in fig. 194. QE is thus a limiting direction for emerging rays of that particular colour, and near it a very large number of these rays pass out, forming to an observer at E a bright-coloured image of the sun in the direction EQ. Every drop, whose angular position with reference to the sun and the observer is the same, will produce a similar

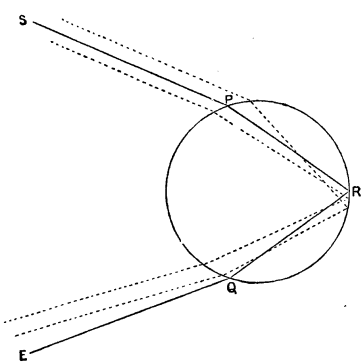


Fig. 194.

image; and all the images of this one particular colour will form a part of an arc of a circle whose centre is diametrically opposite the sun in the celestial sphere. Because of the different refrangibilities of the different coloured rays of white light, the angle between SP and EQ will be smaller for the more highly refrangible rays. In other words, the violet-coloured images will be produced in drops lying at a lower altitude than the drops which are effective in producing the red-coloured rays. To every different colour and to every point on the sun's surface will correspond a different arc of images, which will combine to produce the phenomenon known as the Primary Rainbow, the colours being arranged in the order, red, orange, yellow, green, blue, violet, passing from outside to inside.

The Secondary Rainbow is produced in a similar manner, by rays which suffer two reflections before they emerge. The course of a ray of limiting deviation is shown in fig. 195. This

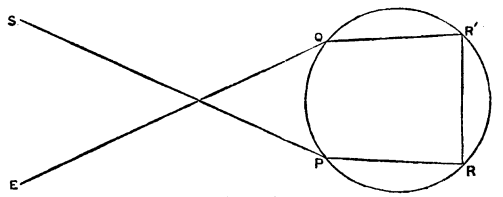


Fig. 195.

bow so formed is of a larger radius than the primary bow, and is much fainter; and the colours are arranged in the opposite order, the violet being outside.

Rainbows produced by rays which have suffered three or more reflections are theoretically possible, and perhaps exist; but they have never been observed. The supernumerary bows which are frequently seen fringing the primary bow are due to Diffraction, the simpler effects of which are discussed in next chapter. When bright and numerous, these supernumerary bows indicate that the raindrops which are effective in producing them are all of nearly the same size.

§ 124. **Thin Lenses.**—A lens is a transparent body (usually glass) bounded by surfaces, one of which must be, and both generally are, curved. The curved surface is almost always spherical, occasionally cylindrical. It may be convex or concave outwards, and the various forms of lenses in practical

use are catalogued under the names double convex, plano-convex, concavo-convex, plano-concave, double concave. The concavo-convex is also called a meniscus lens. Optically considered, lenses may be grouped in two great classes. The double convex lens is the type of the one; the double concave the type of the other. All lenses belonging to the first class can be used as magnifying-glasses—the others cannot so be used. Another important distinction is, that lenses belonging to the first class can alone form inverted real images. We can thus tell at a glance to what type a particular lens belongs. Look through it so as to obtain a clear image of a near object. If the image is erect and magnified, the lens is convex; but if the image is erect and diminished, the lens is concave. If the image is inverted, the lens is convex. Every convex lens produces an inverted image, if the object and observer be far enough away from the lens. Every convex lens produces a magnified erect image if the object be near enough.

The use of the convex lens as a 'burning-glass' is familiar to all. The sun's rays are allowed to pass through the lens, which is adjusted so that there falls upon the object to be burned the smallest possible disc. The sun's rays are in this way concentrated to a point known as the *principal focus* (Latin, fireplace or hearth); and if this focus is held steadily on the same point of a piece of wood or paper the substance will soon catch fire.

This very simple experiment shows clearly the broad characteristic of a convex lens—it makes parallel rays converge. More generally, it makes convergent rays more convergent, divergent rays less divergent, it may be parallel or convergent. The concave lens, again, has exactly the opposite effects—causing parallel rays to diverge, divergent rays to become more divergent, and convergent rays to become less convergent, it may be parallel or divergent.

By applying the law of refraction to the case of a pencil of rays traversing a lens of either type, we can by purely

geometric reasoning establish the properties just stated. That increased convergence or decreased divergence is the characteristic of the convex lens is seen at once from fig. 196*a*. To each of the parallel rays falling non-centrally the lens acts exactly like a prism whose faces are the tangent planes at the points of entrance and of emergence of the ray. In

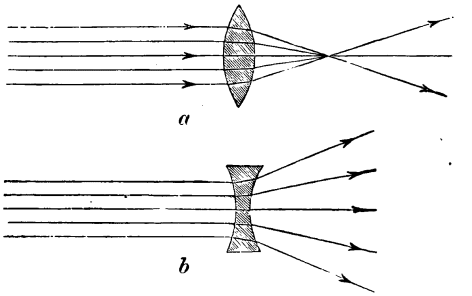


Fig. 196.

other words, the lens in its action upon a pencil of rays parallel to the axis, bends every ray towards the thick part of the lens. The action of the concave lens (fig. 196 *b*) may be described in exactly the same words; but in this case parallel rays, being bent towards the thick parts of the lens, are evidently rendered more divergent.

An ideal lens is one having no thickness, and refracting all parallel rays to a definite focus. No material lens can do this; but lenses, whose greatest thickness is small compared to their diameters and to the other lengths involved, do so approximately for rays nearly parallel to the axes. In what follows, the discussion is almost entirely limited to this class of lenses, generally known as **Thin Lenses.**

§ 125. **Conjugate Foci.**—Let an ordinary hand magnifying-glass be placed a yard or two distant from a bright object, say a gas-jet, so that a pencil of rays may pass through it. If then a sheet of paper (approximately white) be properly adjusted on the other side of the lens, a distinct inverted

image of the gas-jet will be projected on the sheet. A slight displacement of the sheet farther from or nearer to the lens produces a deterioration in the distinctness of the image, and there is no difficulty in making the adjustment which gives the most distinct image. In this condition of things, every point in the image is the focus of rays originally diverging from the corresponding point in the object.

Let the distances of the object and image from the centre of the lens be carefully measured—suppose, for example, that the lens is 3 feet from the jet, and the distinct image 2 feet from the lens. Then, if we place the lens 2 feet from the jet, the image will be found to be 3 feet from the lens. Thus (see fig. 197), if rays diverging from a point U are, after passing through the lens, brought to a focus V, then rays from the point V will be brought to the focus U. U and V are called *Conjugate Foci*. The one is the *real image* of the other, for the rays pass through it.

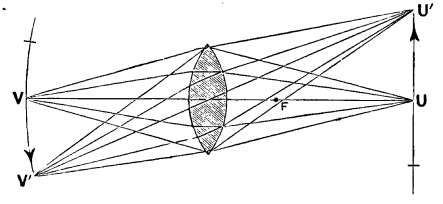


Fig. 197.

If the lens be moved up towards the jet, the sheet of paper must be moved to a farther distance on the other side of the lens, in order that a distinct image may be obtained. But, at length, as the lens is gradually brought nearer to the object, it will not be possible to obtain a distinct image at all. The distance of the image will increase with great rapidity for a very small diminution of the distance of the object. Ultimately, when the object occupies the *principal focus* of the lens, the diverging rays from every point of it will proceed after reflection as parallel rays. The image will be formed at an infinite distance.

If u be the distance of the object, and v the distance of the image from the lens, it can be proved geometrically, and

may be readily established by experiment, that the sum of the reciprocals of these distances is constant, or,

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

where f, the principal focal length of the lens that is being used, is what u (or v) becomes when v (or u) is infinite. It is interesting to note the resemblance this equation has in form to the equation for the concave mirror. The only difference is that in the case of the mirror the principal focal distance is simply half the radius.

In the instance given above, where $u=3$, and $v=2$, we find for f, the focal length of the lens, the value 1·2. This method of finding the position of the principal focus is more accurate than the method which takes advantage of parallel rays from the sun, and has the further merit of being always practicable. Any one who has the use of an ordinary photographic camera with ground-glass back for receiving the image of an object can test the law just given with great ease. It is necessary, of course, to use a singlet lens; with a doublet the formula requires modification.

We must now consider what happens when the distance of the object from the lens is shorter than the focal length—that is, when u is less than f. In this case $1/f$ is less than $1/u$. If, then, we carry $1/u$ to the right-hand side of the equation, we get $1/v$ as a *negative* quantity, always less numerically than $1/u$. Hence v is numerically greater than u, and is measured backwards to the same side as U. The divergent rays from U are rendered less divergent (fig. 198), and appear

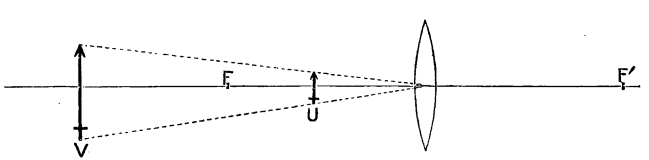


Fig. 198.

to come from V, which is situated behind U. In this case V is a virtual image, for the rays do not really pass through

it. Any object placed at U has an enlarged erect image at V. The lens, in short, is acting as a magnifying-glass.

The formula for the concave lens is got by simply changing the sign of f; for if we regard a convex lens as being positive, then the concave lens will be negative. Thus,

$$\frac{1}{v} = -\frac{1}{f} - \frac{1}{u},$$

a negative quantity, if u be positive, as it always is for a *real* object point U. The image V is then measured backwards towards U, and v must be less than u. The divergent rays from U are made more divergent, and seem to come from V. The image is virtual, erect, and diminished (fig. 199).

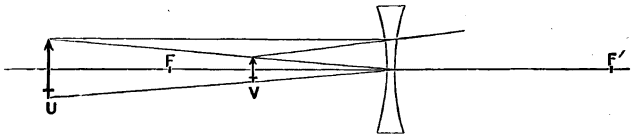


Fig. 199.

The impossibility of locating the exact position of a virtual image (for such an image cannot be projected on a screen) makes the direct determination of the focal length of a concave lens a matter of comparative difficulty. But it may be determined indirectly by using in combination with the concave lens a convex lens whose focal length is known. The focal length of this convex lens must be shorter than that of the concave lens; for the convex lens has to overcome the increased divergence due to the concave lens so as still to produce a real image. The arrangement is shown

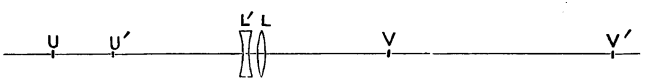


Fig. 200.

diagrammatically in fig. 200. Let U be the object. The convex lens L at distance u will, if the concave lens be removed,

form a real image V at distance v. With the concave lens L′ inserted, the rays will fall upon the convex lens as if radiating from the point U′, distant u' from L. Hence the final image will be V′ at distance v'. If a be the measured distance between L and L′, f the focal length of L, and f' of L′, we have the equations:

$$(1) \qquad \frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

$$(2) \qquad \frac{1}{u'}+\frac{1}{v'}=\frac{1}{f'}$$

$$(3) \qquad -\frac{1}{u'-a}+\frac{1}{u-a}=-\frac{1}{f'}$$

—the negative sign being given to the first term in (3) because u' is measured backwards.

Equation (1) determines f, if it has not been already determined; equation (2) determines u' in terms of f and the measured distance v'; and substituting in (3) the measured distances v' and a, and the calculated distance u', we get the value of f'. If the lenses are thin and placed as close together as possible (as in figure 207), the quantity a may be neglected.

For example, let the object be at distance 3 feet from the lens already referred to, for which, when used alone, the distance of the real image is 2 feet. A concave lens is placed 6 inches distant, as in the figure, and the image is thrown off to a distance of 10 feet. Find the focal length of the concave lens. The focal length of the convex lens is 1·2. Hence equation (2) gives:

$$\frac{1}{u'}+\frac{1}{10}=\frac{1}{1\cdot2} \text{ or } u'=\frac{12}{8\cdot8}=\frac{15}{11}=1\cdot3636,$$

and then from equation (3) we find:

$$\frac{1}{f'}=\frac{1}{1\cdot364-\cdot5}-\frac{1}{2-\cdot5}=\frac{28}{57} \text{ or } f'=2\tfrac{1}{28} \text{ ft.}$$

The measurement of focal lengths of lenses by projection of images on a screen gives roughly approximate results only; but the more refined methods of the Optical Bench in which the image is viewed through a telescope are the same in *principle* as those here described.

§ 126. **Graphical Construction of Images for Thin Lenses.**—In thin lenses there are three important points, which, when given, enable us with great ease to construct the image of any object. These are—(1) The two Principal Foci, one on each side; (2) The optical centre of the lens.

The property of the optical centre of any lens, thin or thick, is, that any ray passing through it suffers no deviation. Draw any two parallel radii from the centres, O, O′, of the spherical surfaces (fig. 201). Join the points (*ab*) where these meet their

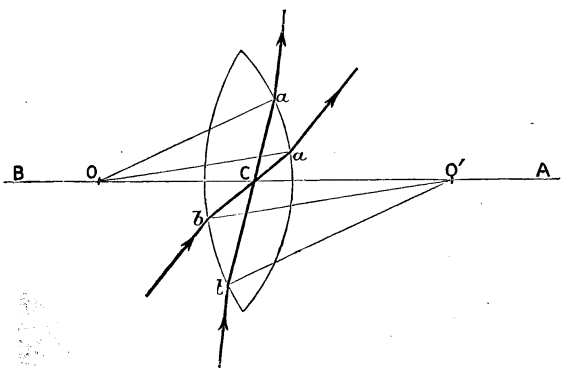


Fig. 201.

respective spheres. Then, because of the equality of the angles which *ab* makes with the normals to the surfaces, it is evident that the ray which, entering at *b*, passes along *ba* will emerge at *a* parallel to its original direction. The point C where *ba* intersects the axis BA is the optic centre; and through this point all rays pass which suffer no ultimate deviation. There is, of course, displacement out of the original line, the entering rays, which after refraction pass through C, seeming to converge to a point on the one side of C, while their parallel continuations seem to diverge from a point on the other side of C. For thin lenses and for rays which make small angles with the axis, these two points and C are

practically coincident. It is easy to show, by means of the similar triangles OaC, O′bC, that

$$OC = OO' \times \frac{r}{r+s}$$

where r is the radius Oa, and s the radius Ob. Similar formulæ hold for other forms of lenses. For double-concave and double-convex lenses the optic centre lies within the lens. If one surface is plane, the optic centre is at the intersection of the axis and the curved surface. For meniscus-shaped lenses, the optic axis lies outside the lens, on the side of the greater curvature. These statements are easily verified by means of the graphical construction.

Let, then, F, F′, C (fig. 202) be the positions of the principal foci

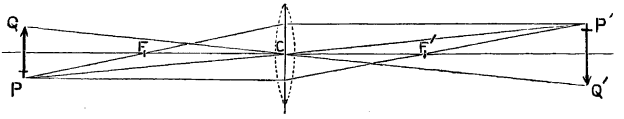


Fig. 202.

and the optic centre, say, of a convex lens; and let the rays pass from left to right. Then F′ is the focus for entering parallel rays, F the focus for emerging parallel rays. It is required to find the image of PQ. Draw a line through C perpendicular to the axis. The ray from P which passes through F becomes, after refraction, parallel to the axis. The ray from P through C suffers no deviation. The ray from P parallel to the axis is refracted so as to pass through F′. These three rays after refraction will meet in the point P′, where P′ is the image of P.

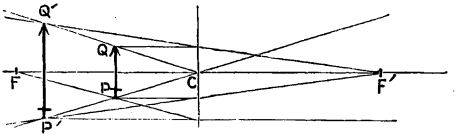


Fig. 203.

The construction for the image when PQ lies within the focus is shown in fig. 203.

From these figures the formulæ already given for the distances of conjugate foci can be deduced by simple geometrical reasoning.

For the triangle PCF is similar to the triangle whose apex is P and whose base is the line through P′ parallel to FF′. Hence, remembering that the pencil of rays is slightly inclined to the axis, we get,

$$\frac{u}{f}=\frac{PC}{FC}=\frac{PP'}{v}=\frac{u+v}{v},$$

where u, v, and f have their former significance, and hence, dividing by u,

$$\frac{1}{f}=\frac{1}{v}+\frac{1}{u}.$$

In the case of the concave lens, the focus for entering parallel rays is a virtual focus, from which the rays after refraction seem to diverge. F′ will then lie to the left, if we suppose the rays to be coming from that side. The construction for the image of any object is shown in figure 204.

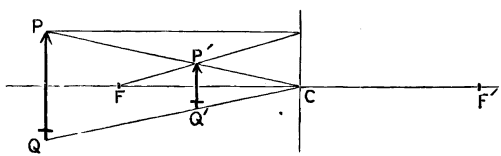


Fig. 204.

When the lens is thick, we must use, instead of C, the two points already mentioned, the so-called *nodal points*, which have the property that any entering ray passing or seeming to pass through the one continues after refraction parallel to its original direction, but so as to pass, or seem to pass, through the other. But this statement is also true for any system of lenses, such, for example, as a microscope or photographic doublet. In all such cases there are two *principal foci*, F, F′, and two *nodal points*, H, H′. The planes drawn through these points perpendicular to the axis or line joining them are called respectively the principal focal planes and the nodal planes. Once the positions of these planes are determined for any system of lenses, the optical constants of that system are fully determined. The distances HF and H′F′ are equal to one another, and measure the principal focal length of the system.

§ 127. The Human Eye.—Optically considered, the eye is a combination of lenses by which an image of an external object is thrown upon a screen called the *retina*. The

manner in which this image is produced is shown in fig. 205. The rays from the object ABC fall on the outer surface of the *cornea*, across which they pass with diminished divergence, and then enter the *crystalline lens* through the pupil. The crystalline lens is a double-convex lens of com-

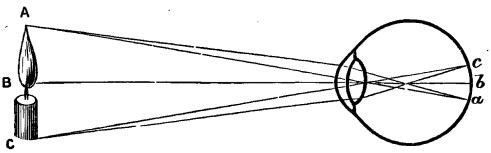


Fig. 205.

paratively short focus, and is, optically, the most important member of the system. After passing through it the rays traverse the *aqueous humour* of the eye, and are finally brought to a focus on the retina. The inverted real image, formed on the retina in exactly the same way as a landscape is projected upon the sensitive plate in a photographic camera, affects the terminals of the optic nerve and produces the corresponding sensation in the observer. All parts of the retina are not equally sensitive. The most sensitive part is the so-called *yellow spot;* and to see any part of an object clearly the observer must so direct his vision that the image of this part falls on the yellow spot.

That the eye may be serviceable in forming clear images of objects at all distances, it must be able to alter its optical constants; for, if distant objects are clearly focussed on the retina, it is impossible for near objects to be so focussed with the eye in the same condition. This adjustment or *accommodation* for distinct vision at different distances is effected muscularly by a change in the shape of the crystalline lens. Young children are able to accommodate their eye for distances less than 3 inches. As age increases, the power of accommodation diminishes. A normal eye in middle life cannot see clearly at a distance of less than about 10 inches;

while in old age the power of accommodation is so limited that reading a book at arm's-length becomes a severe strain. Under these conditions, artificial correction is needed if near objects are to be viewed with comfort. This defect of *long-sightedness*, which is usually experienced by normal eyes with advancing years, springs simply from the diminished power of accommodation for near objects.

A contrary defect is that of *short-sightedness*, which means a lack of accommodation for distant objects. A short-sighted eye can see very near objects with great distinctness; but its range of accommodation is very limited, and objects at a distance greater than a few inches begin to appear hazy in outline, while the details of distant objects are quite indefinite. The crystalline lens is too highly refractive or its distance from the retina is too great, so that parallel rays are brought to a focus in front of the retina.

To correct for each of these defects, a lens must be used which will form the image of the object within the range for which accommodation is possible. The long-sighted person must use a convex lens, so as to get a virtual image at a sufficiently great distance. The short-sighted person must use a concave lens, so as to get images of all distant objects near enough.

In the various formulæ for lenses, it will be noticed that distances and focal lengths enter as reciprocals. To facilitate the calculation of lenses to be used for different kinds of sight, the following convenient nomenclature has been adopted. A lens of 1 diopter is defined to be a positive or convex lens, whose principal focal length is 1 metre. A lens of 2 diopters has a focal length of $\frac{1}{2}$ metre; of 3 diopters, a focal length of $\frac{1}{3}$ metre; and generally a lens of n diopters has a focal length of $\frac{1}{n}$th metre. A lens of $-n$ diopters is a negative or concave lens, whose principal focal length is $\frac{1}{n}$th metre.

If two thin lenses, of m and n diopters respectively, are set close together, the combination is equivalent to a lens of $m+n$ diopters. For example, when a lens of $+4$ diopters is combined with a lens of -2 diopters, the result is an equivalent lens of $+2$ diopters.

A long-sighted person can see clearly at a distance of 1 metre. What lens must he use to read comfortably a book at a distance of 25 centimetres (nearly 10 inches)? We assume the lens to be close to the eye, so that 1 and 0·25 correspond to the distances v and u in the formula:

$$\frac{1}{f}=\frac{1}{u}-\frac{1}{v}=\frac{1}{\cdot 25}-1=4-1=3.$$

Hence the lens required is a lens of 3 diopters—that is, a lens of $33\frac{1}{3}$ centimetres (fully 13 inches) focal length.

A short-sighted person can see clearly at a maximum distance of 15 centimetres (6 inches). What lens must he use so as to see clearly all distant objects? Evidently a concave lens of 15 centimetres focal length—that is, a lens of nearly 7 diopters ($100 \div 15$).

§ 128. **Achromatic Lenses.**—As already pointed out (§ 115), refraction at the boundary of two different media is invariably accompanied by dispersion. Hence, if a single lens be used for forming an image, the margins of the image will appear coloured. The different refrangibilities of the constituents of white light necessitate a different principal focus for light of each colour. The result is a lack of definiteness in the outline of the image. To obtain a clear-cut image, this **chromatic aberration**, as it is called, must in some way be corrected, and an achromatic system of lenses formed.

Now, different kinds of glass differ appreciably in both their refractive and their dispersive powers, and according to different laws. The more highly refractive is not necessarily the more highly dispersive. Hence it is possible, by suitable combinations of prisms or lenses, to get rid of the deviation due to refraction and yet retain dispersion, or to get rid of dispersion and still retain refraction. The former is ex-

emplified in the direct-vision spectroscope (see below, § 147), the latter in all forms of achromatic lens.

The two kinds of glass known as flint and crown glass differ so greatly in their dispersive powers that a flint-glass prism, whose angle is about half that of a crown-glass prism, produces as long a spectrum. (See Table B at end.) Hence, by combining these, as in fig. 206, we can obtain a refracted

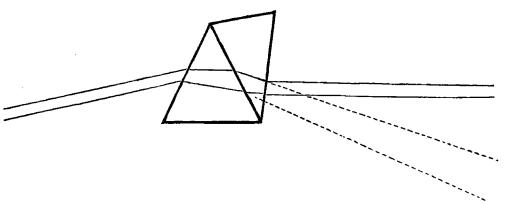


Fig. 206.

beam of white light almost colourless. Flint and crown glass prisms are manufactured so as to give in combination an almost accurate achromatism.

Lenses are combined for achromatic purposes somewhat similarly, the chromatic aberration of, say, a convex crown-glass lens being corrected by means of a concave flint-glass lens of less focal length. In fig. 207 the points r, v are the principal

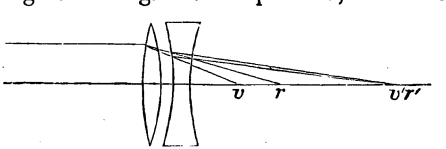


Fig. 207.

foci for parallel rays of red and violet light passing through the convex lens only. The effect of the concave lens is to diminish the convergence of these rays in such a way that the second pair of foci, r' v', are made to coincide. Hence the two lenses in combination bring the red and violet rays to the same focus—there is, in short, no relative dispersion of these rays. But though the chromatic aberration is destroyed, so far as red and violet are concerned, it might not be so for

some intermediate colour, such as yellow or green. For not only do substances differ as regards the total range of dispersion throughout the visible spectrum, but there is no necessary proportionality between the lengths of the differently coloured parts of the spectrum. With some substances, for example, the yellow and green will be, in proportion to the whole spectrum, longer than with other substances. Occasionally, indeed, as in instances of so-called *anomalous dispersion*, the colours do not follow the usual succession, but the violet may appear between the red and the green.

Practically, it is found that if chromatic aberration is corrected for the three colours, red, green, and violet, the achromatism is perfect. This in general will require three lenses, two convex and one concave, or two concave and one convex. All optical instruments of any value, such as telescopes, microscopes, photographic lenses, &c., are fitted with achromatic combinations. The shorter the focus used, the greater the necessity for correcting the chromatic aberration.

§ 129. **Caustics: Spherical Aberration.**—In all that precedes, it has been assumed that a pencil of rays, after reflection or refraction, passes or appears to pass through a single point called a *focus*. But this is only a first approximation to the truth—an approximation, however, which is accurate enough for nearly axial pencils falling centrally on thin lenses or mirrors of small curvature.

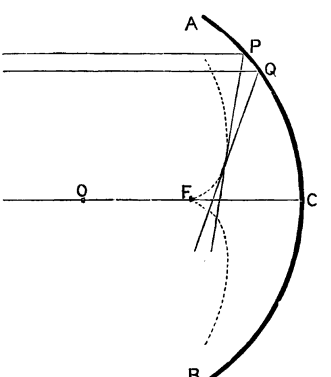


Fig. 208.

Consider, for example, a concave spherical mirror, on which parallel rays are falling; and suppose, for simplicity, that these rays are parallel to the axis. Let ACB (fig. 208) be any central section. As already shown, rays in the neighbourhood of C are reflected so as to pass through F. But it is evident from the construction that

rays incident on points somewhat removed from C do not after reflection pass through F. Any contiguous pair, such as those falling on P and Q, intersect at a point which is not even near F. But it is found that the points of intersection of successive contiguous pairs of rays lie on a curve, to which every reflected ray is a tangent. This curve is called a **Caustic.** It has a cusp at the principal focus F, through and near which many more reflected rays pass than through or near any other part of the caustic. The caustic curves in the various central plane sections of the spherical mirror combine to form a caustic surface shaped somewhat like the surface of an apple near the point of attachment of the stalk. The caustic, being the curve along which successive rays intersect one another, is the conspicuously bright edge of the region illuminated by the reflected rays. The brightest part of the caustic is where the reflected rays crowd together most—that is, in the neighbourhood of the cusp or focus. Caustics may frequently be observed formed on the surface of tea or coffee in a cup. They are produced by the reflection, from the inner surface of the cup, of rays from the windows or the gas-jets.

Caustics are formed by refracted rays in precisely the same way. With lenses of comparatively short focus, rays passing near the margins of the lens are, because of the spherical form of the surfaces of the lens, refracted so as to pass to one or other side of the focus. This is known as **Spherical Aberration.** Such rays are optically lost so far as the production of a clear image is concerned. They may be stopped by using appropriate diaphragms, and the definition improved at the expense of brightness. But there is a further effect due to the spherical form of the surfaces of the lens—namely, the distortion of the image. Straight lines in the object become curved lines in the image. This defect is easily corrected by using a combination of lenses, so that the spherical distortion of the one lens is balanced by that of another. For example, two exactly equal plano-convex lenses placed at a distance from one another less than their focal length, and with their curved surfaces facing opposite ways, will produce rectilinear images of rectilinear objects. This is the principle on which the photographic doublets known as *Rectilinears* are constructed.

§ 130. **Change of Form of Wave-front by Refraction: Determination of Refractive Index.**—When a plane wave-front (Part I., § 146) falls on the bounding surface of two different media it continues in the second medium as a plane wave-front, moving in a different direction. In other words, parallel rays remain parallel. But when, by passage across a curved surface, parallel rays

become convergent or divergent, it is clear that the wave-front has changed form. For example, after passing through a thin convex lens, the wave-front, originally plane, becomes a part of a sphere whose centre is the principal focus. Or, to take the more general case of conjugate foci, the spherical waves radiating from the object-point become, after refraction, spherical waves converging upon the image-point. Every part of each wave-front takes exactly the same time to pass from the one point to the other point. Those parts which enter the lens first travel the greatest distance with diminished speed in the denser medium. Now suppose we have two lenses, A, B, of the same geometrical form, but made of substances of different densities, that of A, say, being the greater. Then in A the wave-front will travel more slowly than in B, and will emerge a spherical wave of greater curvature. In other words, A will have the shorter principal focal length and will be the stronger lens. The ray which passes axially through a lens from one of a pair of conjugate foci to the other takes the same time as the ray which passes in any other of the many possible paths. Let the lens be a plano-convex lens of thickness h and diameter $2a$, and let r be the radius of the spherical surface. Let u, v be the distances of the conjugate foci from the middle of the plane face. Then, since the refractive index μ is the reciprocal of the speed in the lens, that in air being taken as unity, we find the time taken by the ray to pass along the axial line from U to V to be

$$u - h + v + \mu h ;$$

similarly, the time taken to pass from U to V by the ray which just passes through the rim of the lens is

$$\sqrt{u^2+a^2}+\sqrt{v^2+a^2}=u+v+\frac{a^2}{2}\left(\frac{1}{u}+\frac{1}{v}\right)$$

if a be small compared to u and v.

Equating those two expressions which represent equal times, we get

$$\frac{a^2}{2}\left(\frac{1}{u}+\frac{1}{v}\right)=(\mu-1)\,h=(\mu-1)\,\frac{a^2}{2r}$$

since h is the height of a small segment of diameter, $2a$, of the sphere whose radius is r. Finally,

$$\frac{1}{u}+\frac{1}{v}=(\mu-1)\,\frac{1}{r}.$$

If the lens is double convex, the radius of the other surface being s, then it is easy to see that the expression becomes

$$\frac{1}{u}+\frac{1}{v}=(\mu-1)\left(\frac{1}{r}+\frac{1}{s}\right)$$

a formula which enables us to calculate the refractive index of the

substance forming a lens when the focal length and radii of curvature of the lens have been determined. The formulæ for other forms of lens are obtained from this by appropriate change of sign of either or both of the radii of curvature. The refractive indices of various substances are given in Table B at the end of the book.

§ 131. **The Microscope and Telescope.** — The most important optical instruments are the microscope and telescope; and it is evident that a thoroughly efficient instrument of either class must produce clear achromatic images as free as possible from all distortion. The construction of suitable lenses is an art of the highest technical character. Here we must be content with a description of the broad principles of construction.

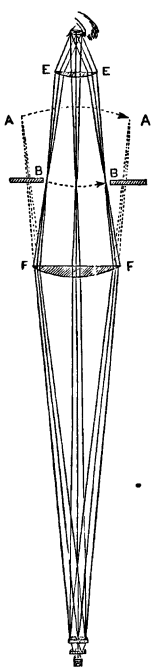


Fig. 209.

The Simple Microscope may be a single convex lens used as a magnifying-glass; or it may be a combination of lenses equivalent in their action to a single lens. The object must lie within the principal focus, and the eye must be adjusted so as to see the image at the apparent minimum distance of distinct vision. The shorter the focus of the lens, the nearer can the object be placed to the eye, the farther away, *relatively*, will the image be thrown, and the greater will be the magnification produced.

It is not practically possible to get more than moderate magnification with a simple microscope. A much more powerful and efficient instrument is the *Compound Microscope*, in which an image of the object, and not the object itself, is viewed through a magnifying lens or system of lenses. One form of compound microscope is shown in fig. 209. It consists essentially of two parts, an object-glass and an eye-piece. The object-glass is a lens, or system of lenses, of

very short focus. The object is placed just beyond the focus, and an inverted and magnified real image of it is formed on the other side of the object-glass. This image is then viewed by the magnifying eye-piece, which, in the form figured, consists of the field-glass FF and the eye-glass EE. The function of the field-glass is to catch the rays before they have converged to form the image AA and make them converge more rapidly to form the somewhat smaller image BB. This real image is then viewed through the lens EE.

The *Astronomical Telescope* is constructed on the same principle as the compound microscope. The main difference is that, whereas the object-glass of the microscope is of short focus and forms a magnified inverted real image of the object placed near it, the object-glass of the telescope is of long focus and forms a diminished inverted real image of an object at a considerable distance. Astronomical objects are so distant that the rays of light from them form parallel pencils. Hence the image is formed at the principal focus of the object-glass. This real image is then viewed through an eye-piece which is virtually a simple microscope.

For astronomical purposes the inversion of the image observed with this form of telescope is of no consequence; but it is otherwise with a telescope intended for sighting terrestrial objects. In this case the eye-piece must be constructed so as to invert a second time. In short, the eye-piece must be of the character of a compound microscope rather than of a simple microscope. Usually it is composed of four distinct achromatic lenses.

The *Galilean Telescope* is the combination familiar in binocular field or opera glasses. It differs from the astronomical telescope in having a concave instead of a convex lens for the eye-piece. The concave lens is placed a certain distance within the focus of the object-glass. Thus there is no real image of the object formed. The concave eye-piece receives the converging rays before they have met and gives them the divergence necessary for distinct vision.

In the early days of telescope construction, before the method of making achromatic lenses was discovered, the reflecting telescope was an unsatisfactory instrument. Astronomers accordingly turned their attention to the construction of *reflecting* telescopes, in which a concave mirror takes the place of the convex object-glass. Newton's form of reflecting telescope, shown in fig. 210, may be taken as a simple type. Here the parallel rays from a star or planet falling on a slightly concave mirror are reflected towards a focus. But before they have quite converged to this point, they are received on a small plane mirror and deflected to the side of the tube, where they form an image to be viewed through the eye-piece.

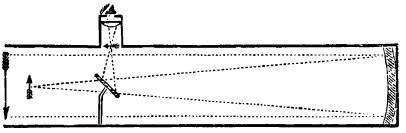


Fig. 210.

In microscopes and telescopes the question of **magnification** is of great importance. A glance at the various cases of optical images pictured in §§ 125–6 shows that with thin lenses the ratio of the linear sizes of object and image is simply the ratio of their distances from the optical centre of the lens. In using a single lens as a magnifying-glass (fig. 203), the operator by practice adjusts the lens until the image is at the least distance of distinct vision. If we suppose the eye placed at the optical centre (a *possible* position with a highly curved meniscus lens), the object and image will subtend the same angle, and therefore be of the same apparent size. But at this distance the object cannot be seen clearly, whereas the image has been projected to the distance of most distinct vision. To estimate the magnifying power we must compare the apparent size of the image with the apparent size which the object would have *if placed at the least distance of distinct vision.* In other words, the magnifying power is the ratio of the angle subtended by the image to the angle subtended by the object removed to the same

distance and viewed with the naked eye. To a first approximation this is the ratio of the distances of image and object. If D be the least distance of distinct vision, and f the focal length of the lens, the distance x at which the object must be placed is found from the equation:

$$\frac{1}{x}-\frac{1}{D}=\frac{1}{f} \text{ or } \frac{1}{x}=\frac{D+f}{Df}.$$

Whence the magnifying power is

$$\frac{D}{x}=\frac{D+f}{f}=\frac{D}{f} \text{ approximately,}$$

since in any strongly magnifying lens the focal length f is small compared to D, the distance of distinct vision. The expression shows that a given lens is more efficient as a magnifier to a long-sighted than to a short-sighted eye.

In the case of the astronomical telescope, the real image of the distant object formed by the object-glass in the principal focus subtends at the object-glass the same angle which the object itself does (see fig. 202), and this is practically the angle subtended at the eye. This real image is then viewed through a magnifying eye-piece; and if the eye be at the optical centre the magnification will be the ratio of the distances of the real image from the object-glass and eye-piece. The eye-piece must be adjusted to distance x from the real image so as to project the final virtual image to the least distance (D) of distinct vision. Hence the magnifying power of an astronomical telescope is

$$\frac{F}{x}=\frac{F}{f}\,\frac{D+f}{D}=\frac{F}{f}\left(1+\frac{f}{D}\right)$$

where F, f are the focal lengths of the object-glass and eye-piece respectively. For a long-sighted eye the fraction f/D is small, and the magnification is practically the ratio of the focal lengths of the two lenses. For a short-sighted person the magnification is a little greater. Thus the longer the focal length of the object-glass, and the shorter the focal length of the eye-piece, the greater the magnification. The

same principle is applicable to other forms of telescope and to the compound microscope; but in these, because of the use of *thick* lenses or of systems of lenses instead of single lenses, the very simple theory sketched here is insufficient.

§ 132. **Atmospheric Refraction: Mirage.**—The tremulous appearance which objects often have on a warm day, especially if viewed over a highly heated surface, is due to the heterogeneous character of the air. Currents of hot rarefied air are streaming up through cooler, denser portions, and the refractive power of any particular region is undergoing fluctuating changes, which, though slight, are sufficient to produce appreciable distortions in the images of objects viewed through it. The scintillation of the distant stars is due to the same time-fluctuation in the optical quality of the air through which the ray is coming, taken in conjunction with the fact that the eye is dealing with a very slender pencil of rays. The image of a star obtained with a large telescope does not twinkle, since the telescope collects a very large pencil of rays, whose average effect is practically unchanged by the varying atmospheric conditions.

The most important effect of atmospheric reflection is the increased apparent altitude of all heavenly bodies. As a ray of light enters the upper regions of the atmosphere it begins to suffer refraction, and as it penetrates to deeper and denser regions becomes slightly curved. The direction the object appears to be in is tangential to the ray as it enters the eye. A little consideration will show that the lower a star is in the heavens, the greater will be the effect of atmospheric refraction. On the other hand, a star in the zenith will not have its position altered, because the rays strike the successive layers of air normally. A star when it appears to be setting is already below the horizon 34 minutes of arc, or fully half a degree. At any point where there is a clear horizon, the whole disc of the sun or moon will be visible, although its true direction is really below the horizon. All heavenly bodies will appear to rise some minutes of time earlier and

set some minutes later than they would appear to do if the earth had no atmosphere. Thus, one effect of atmospheric refraction is to lengthen the day and shorten the night.

The refractive index of air depends upon its density, which, by Boyle's and Charles' Laws, is determined by its pressure and temperature. Both these quantities change as we ascend in air. Hence, in general, those rays from an object at the earth's surface which start with an upward inclination will become curved. It can be shown by a simple enough calculation that the refractive index will not change with height if the temperature diminishes by about 1° C. for every hundred feet of ascent. For this particular rate the rays are straight lines. For any higher rate of fall off, we should have an unstable condition—a denser layer above a lighter layer. For any lower rate of diminution of temperature, or for any possible rate of increase of temperature with height, the rays will be curved with concavity downwards. And it is possible, under exceptional conditions of the atmosphere, for certain rays to be deflected back to the earth's surface. One case, greatly exaggerated, is shown in fig. 211. Here



Fig. 211.

certain rays from a distant object AB are, so to speak, reflected back towards the eye at E, and the image is seen as at A′B′. Because of the curvature of the earth or the intervention of a hill, AB might not be visible at all in its true direction. Another case is shown in fig. 212, in which the peculiar optical condition does not begin till a certain height is reached, indicated by the dotted line. The rays from B are sent back along a lower line than those from A. The image A′B′ is accordingly inverted.

These phenomena of *Mirage* have been observed by Scoresby and others in high latitudes. Some examples given by Scoresby are reproduced in fig. 213. The different cases figured did not occur at the same time; they are different mirage appearances of the same ship on different occasions.

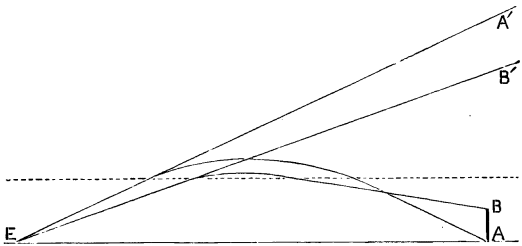


Fig. 212.

These can all be fully explained by one or more intermediate strata between the cold air over a frozen sea and a warm stratum 50 or 100 feet above. The phenomenon known as *looming*, in which an object appears drawn out to a height

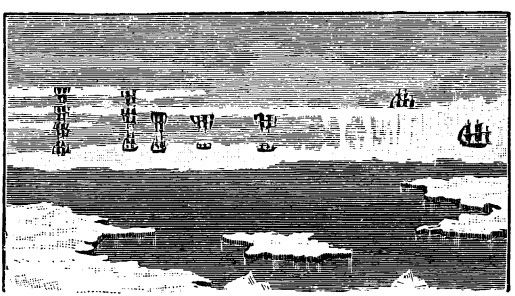


Fig. 213.

exceeding its natural height, is a case of mirage also, produced under less extreme conditions.

The ordinary mirage of the desert is due to the highly heated lower layers of air, so that rays which start with a downward inclination pursue a curved path concave upwards. The result is an apparent *reflected* image below the real object,

which is at the same time seen directly. The traveller gets the impression of the existence of a sheet of water between him and the object.

The effects of mirage may be reproduced by gently introducing a layer of pure water over a strong solution of salt contained in a glass vessel. The boundary between the liquids of different density will soon assume a condition capable of showing all the ordinary phenomena of looming and mirage when an object is viewed horizontally through it. Wollaston studied the phenomena by looking through the stratum of air close to a very hot bar of iron.

EXERCISES.

1. Show that, when the law of reflection is satisfied, the course pursued by the ray (e.g. QAR in fig. 181) is the path of shortest time from Q to R.

2. A person is standing at any point in a square room with plane mirrors on the four walls. Consider the positions of the images of himself which he may see.

3. A traveller seated in a railway carriage at night observes the reflection of the carriage interior in the window, and notices that the brightest part of the image of the partition does not occupy the same relative position in it as the brightest part of the partition itself, both brightest parts being due to the reflection of the lamp-light in the polished wood. Explain this.

4. An object and its *real* image in a concave mirror are distant 6 feet and 2 feet respectively from the mirror. Find the radius of curvature, and calculate the position of the image when the object is placed at 6 inches distance from the mirror.

5. A convex lens has a focal length of 2 feet; at what distance must it be placed from an object so as to produce a distinct image on a screen 10 feet from the object?

6. By use of the same lens, an image of an object is formed at a distance of 3 feet. A second thin lens is then placed in close contact with the first, and the image is formed at 2 feet distance. Calculate the focal length of the second lens.

7. What would happen if a concave lens of the same focal length were substituted for the second convex lens of the last question?

8. A long-sighted person has to use a lens of 4 diopters to see clearly at a distance of 20 centimetres. What is the range of clear vision of his unaided eye?

9. Construct graphically, in the scale of an inch to the foot, the image of an object (1) 3 feet, (2) 8 inches, distant from a concave mirror of 2 feet radius. Substitute a convex mirror of 1 foot radius for the concave mirror, and construct the images of the same object at the same distances.

10. A convex lens of 3 feet focal length is used to produce an image of an object at 6 feet distance. Give the graphical construction for the position of the image, drawn to any convenient scale.

11. Compare the magnifications obtained with a lens of half-an-inch focal length, when used by two persons whose least distances of distinct vision are 12 and 5 inches.

12. The object-glass of a telescope has a focal length of 10 feet, and the eye-piece a focal length of half-an-inch. Compare the magnifications obtained by the two persons of last question.

SUMMARY.

§ 118. Physical and Geometrical Optics. Reflection and Refraction of Pencils of Light.

§ 119. The Plane Mirror. Virtual Image. Parallel Mirrors. Kaleidoscope.

§ 120. Reflection at Curved Surfaces. Principal Focus. Conjugate Foci. Real and Virtual Images.

§ 121. Refraction at a Plane Surface.

§ 122. Refraction through a Prism. Minimum Deviation.

§ 123. The Rainbow.

§ 124. Thin Lenses. Principal Focus.

§ 125. Conjugate Foci. Real and Virtual Images. Combinations of Lenses.

§ 126. Graphical Construction of the Images for the Lenses. Optic Centre and Focal Points. Focal and Nodal Planes.

§ 127. The Human Eye. Correction for Long-sightedness and Short-sightedness.

§ 128. Achromatic Lenses. Chromatic Aberration.

§ 129. Caustics: Spherical Aberration. Caustics by Reflection and by Refraction. Distortion of Image. Rectilinear Lenses.

§ 130. Change of Form of Wave-front by Refraction. Retardation in Denser Material. Focal Length of Lens in terms of Refractive Index and Curvatures.

§ 131. The Microscope and Telescope. Simple and Compound Microscope. Astronomical Telescope. Galilean Telescope. Newton's Reflecting Telescope. Magnification.

§ 132. Atmospheric Refraction. Mirage. Looming.

CHAPTER XXI.

ETHER WAVES.

§ 133. We now pass to the consideration of those phenomena, many of them of great beauty, which are intimately involved in the wave theory of light, and which, indeed, form a cumulative argument in its favour positively irresistible. It should be clearly borne in mind from the outset that the acceptance of this wave theory in no way commits us to any crude conception of the mechanism of the wave-motion. A certain group of properties of light, included under the name *Polarisation*, requires the vibrations or displacements which constitute the wave-motion to be executed in planes perpendicular to the direction of the ray; and many of these properties can be explained in terms of the simplest kind of wave-motion having this characteristic. But of the essential character of an ether wave nothing definite is known, except its wave-length.

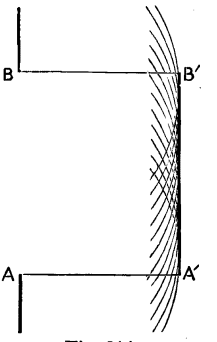


Fig. 214.

The mode of propagation of a wave-front of light (Part I., § 146) was first clearly stated by Huygens. Each disturbed portion of the medium is regarded as a centre of disturbance, from which a new set of spherical waves is started. Consider a plane wave-front a portion of which is allowed to pass through the opening AB (fig. 214). From each point of the wave-front AB, spherical waves will be propagated; and at a definite instant later, these spherical waves will have their ultimate intersections along A′B′, the new position of

the wave-front. No doubt certain waves spread out into the region outside the part BB'A'A, but these waves do not conspire to produce a resultant wave. It can be shown, indeed, that the resultant of the detached wavelets which diverge to either side is too feeble to produce light. The intensity is great along the common tangent to the simultaneous equal-phased spherical wavelets. So long as the aperture through which the light is coming is great compared to the wave-length, the description just given is sufficiently accurate. But when the aperture is made of the same order of smallness as the wave-length, a new phenomenon comes into evidence, namely, *diffraction*. There is no longer a distinct shadow edge, but colour-fringes are observed where we should expect the shadow to begin. Grimaldi, Newton, and others made careful measurements of these fringes, which suggested to them the bending in of the different coloured rays as they passed the edge of an obstacle—whence the name *inflexion*, also applied to this phenomenon. Young was the first to show that diffraction is a necessary result of the interference of ether waves; but the complete explanation was given a little later by Fresnel, to whom, more than to any other single man, the development of the wave theory is due. Anything like a sound discussion of the effects of interference in light requires the aid of the higher mathematics. Here we must be content with an indication of how the method is applied in the explanation of such familiar phenomena as the colours of soap-films, wings of insects, mother-of-pearl, the coronæ seen round the moon shining through a fleecy cloud, and so on.

§ 134. **Interference of Light.**—Two tuning-forks tuned to nearly the same pitch produce, when sounded together, the interference phenomenon known as *beats* (Part I., § 154); but it is not possible to get interference of light with two rays from different sources. The interfering rays must be parts of the same original ray. The following experiment, a simple modification of one due to Young, is easily made. Rule

with a sharp penknife two fine parallel lines, say $\frac{1}{32}$ of an inch apart or less, upon the silvered back of a piece of ordinary looking-glass, or (what is even better) the opaque film of a spoilt photographic plate. This will give two fine parallel slits through which light will pass. On another plate rule similarly a single fine line. If a gas-jet be viewed through the single slit held close to the eye, there will be seen a succession of coloured fringes spreading out on each side of the jet. These are the so-called diffraction fringes. But if the gas-jet be viewed through the double slit, there will be seen, in addition to the diffraction fringes, a series of alternately bright and dark lines on the image of the jet. The effect will be better observed if the single slit be interposed in front of the gas-jet, some 10 or 12 inches from the eye. These fine bright lines, separated by dark spaces, are due to the interference of the two rays into which the double slit divides the original single ray from the luminous source.

A pretty modification of this experiment is obtained by ruling the two fine slits so slightly oblique that they meet near the one end, while at the other they are, say, $\frac{1}{50}$ of an inch apart. While the gas-jet (or, better, a single vertical illuminated slit) is being looked at, move the double slit vertically in front of the eye. As the slits become closer, the interference lines widen out, finally disappearing at the instant the slits coalesce. Only the diffraction fringes are then to be seen. Moreover, since the single slit, due to the coalescing of the two, begins broader than it ultimately becomes, we see by this simple experiment not only that the interference bands are wider when the two slits are nearer, but also that the diffraction fringes are wider when the single slit is narrower.

In order to get rid of the diffraction fringes and obtain a pure interference effect between two parts of the same original ray, Fresnel devised two ingenious forms of experiment. In the one experiment two plane mirrors AB, BC (fig. 215) are adjusted so as

to be almost, but not quite, in the same plane. Each mirror will reflect rays from the luminous slit represented by the point P as if they were radiating from the image of P in the mirror. There will therefore be two virtual images O, O′, from which similar rays appear to proceed.

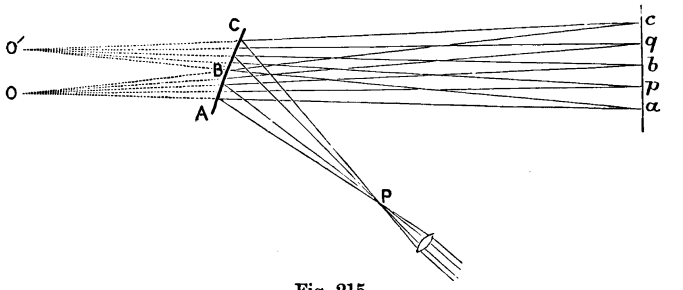


Fig. 215.

If the distance Oq is longer than the distance O′q by half a wave-length, or any odd multiple of half a wave-length, the crests of the waves of one ray will coincide with the troughs of the waves of the other, and darkness will be the result. At b, the central point, crest will fall with crest, and trough with trough. As we pass along the line bc, the ray from O will increase in length, and that from O′ diminish in length. Hence there will be a continuous succession of alternating points of maximum and minimum brightness, the former corresponding to positions for which the rays differ in length by 1, 2, 3, 4..... complete wave-lengths, the latter to positions for which the rays differ in length by 1, 3, 5, 7..... half wave-lengths.

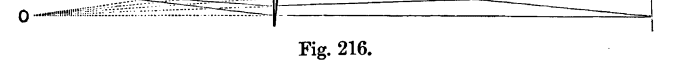


Fig. 216.

In the second form of experiment, an isosceles prism of very large vertical angle is used—an angle differing very slightly from two right angles. The rays from a luminous point P fall normally on the base (fig. 216). Some are refracted so as to appear to come from the virtual image O; and others, as if they proceeded from the image O′. The phenomenon is otherwise identical with the former.

Let $2d$ be the distance between the virtual images O, O′, and x the distance between the central bright band and the next maximum to either side; then, if D is the distance between OO′ and ac, the difference between the lengths of the two rays will be

$$\begin{aligned} & \sqrt{D^2+(d+x)^2}-\sqrt{D^2+(d-x)^2} \\ = \quad & \tfrac{1}{2}\left(\frac{d+x}{D}\right)^2-\tfrac{1}{2}\left(\frac{d-x}{D}\right)^2 \\ = \quad & \frac{2dx}{D^2} \text{ approximately.} \end{aligned}$$

But this difference, expressed in easily measurable quantities, is the wave-length of the most luminous component of white light.

If we experiment with different coloured rays of light, we find that the distance between the successive bands varies with the colour. Blue light gives the bands narrower and closer than yellow light or red light. Hence, when white light is used the bands are blue on the side nearer to the centre, and red on the side farther off. One of the most homogeneous of lights is the yellow light given out by common salt burning in a colourless flame, and generally known as sodium light. If the interference experiment be made with this light, a great number of alternating black and yellow bands will be observed. When white light is used, the overlapping of the bands due to the differently coloured rays soon destroys all appearance of variation of intensity, so that the interference bands are comparatively few in number.

If by any means one of the interfering rays is slightly accelerated or retarded, this will virtually shorten or lengthen its course, and a corresponding change will be observed in the positions of the interference bands. When a transparent plate of thickness t is interposed in the path of one of the rays, the path is virtually lengthened by an amount equal to $t(\mu-1)$, where μ is the ratio of the speeds of light in air and in the substance of the plate. The change produced in the position of the interference bands can be measured; and the direction in which the bands are displaced proves that

light moves more quickly in air than in solid or liquid transparent substances.

The phenomena of interference not only find an extremely simple explanation in terms of the wave theory of light, but they give us an experimental method of measuring the wave-lengths. Thus the wave-length of yellow light is about $\frac{1}{40000}$ of an inch. The number of waves of yellow light which pass per second through any point will be the number of times this length is contained in the distance travelled by light in one second, namely, about 500 million million (5×10^{14}).

§ 135. **Mother-of-pearl: Diffraction Grating.**—The colours of mother-of-pearl are at once explained on the interference principle. That they are due to the configuration of the surface was proved very simply by Brewster, who found the same play of colours on a wax mould of the shell. The surface is very finely furrowed; and when it is viewed in any light the tops of the furrows become sources of disturbance of spherical wavelets.

Let a ray of sunlight fall normally on the finely furrowed surface AB (fig. 217)—(the figure, of course, is greatly magnified). Each ridge will become a source of spherical waves, which all start in the same phase. Let p, q be two contiguous ridges. Then an eye in the direction of E will receive two rays pE and qE of different length. The difference of length is ph, where qh is perpendicular to the rays. If this difference of length is a whole wave-length of a particular kind of light, say of blue light, then that kind of light will be greatly intensified, and will be the prevailing tint of the light seen from the direction pE. Let now the eye be shifted in position so that the rays

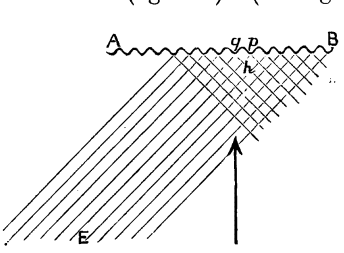


Fig. 217.

from p, q, &c., are increased in obliquity. Then clearly ph becomes longer, and will correspond to the wave-length of, say, yellow light. From this new position yellow will be the prevailing tint. For a still higher obliquity of rays, red will be the prevailing tint. Thus the same part of the surface will appear of a tint which depends on its obliquity to the line of vision.

The principle is usefully applied in the *Diffraction Grating*, which is a surface ruled artificially with a great number of parallel lines at very minute equal intervals apart. If the grating is formed on the surface of a glass plate, the light to be studied is transmitted through it; but if the grating is formed on the surface of a metal plate, then the light is reflected as in the last example. Let AB (fig. 218) be a magnified representation of part of the section of the ruled surface, the dark strokes representing the ruled lines end on to the paper. The intermediate spaces are either the transparent (because unruled) parts of the glass surface or the reflecting ridges of the metal surface. In the former case the incident beam of parallel rays is supposed to come from above; in the latter case, from below. A simple application of the principle of interference shows us that the direction in which a particular ray has its first maximum intensity is a simple function of the wave-length and the distance between any two contiguous lines of the grating. Thus, in fig. 219, if PO is the direction of the normally transmitted or reflected ray, and V the position of the first violet light as projected

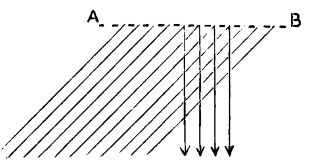


Fig. 218.

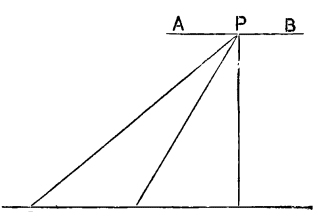


Fig. 219.

on a screen set parallel to AB and a little distance from it, then it is easy to see that

$$\frac{\text{the wave-length}}{\text{distance between the lines}} = \frac{\text{VO}}{\text{VP}} = \text{sin. VPO.}$$

But this angle can be measured with great accuracy, and hence the wave-length is obtained at once in terms of the distance between the lines of the grating. For example, if there are 4000 lines to the inch on the grating, the deviation of the ray whose wave-length is $\frac{1}{40000}$ inch is the angle whose sine is $\frac{1}{10}$—that is, nearly 6°. This colour of ray will be seen a second time in a direction making with the normal an angle whose sine is $\frac{2}{10}$ or $\frac{1}{5}$—about 11½°. It will be seen a third time in a position corresponding to the angle 17½, whose sine is $\frac{3}{10}$; and so on indefinitely. If sodium light is used, there will be an endless succession of yellow lines recurring at definite intervals on both sides of the central line PO. When a narrow beam of white light falls on the grating, the successive spectra begin to overlap after the first two or three, and finally destroy one another. Since, in any experiment as carried out, the distance of all parts of the first spectrum from the grating is practically the same, the colours of the spectrum are arranged so that their wave-lengths are proportional to the distances from O. A spectrum fulfilling this condition is called a *Normal Spectrum*. The spectrum obtained by refraction through a prism is never normal—as a rule the blue part of the dispersion spectrum is more drawn out relatively to the red than the corresponding portion of the diffraction spectrum.

This experiment demonstrates that differently coloured rays of light differ in wave-length from $\frac{1}{34000}$ of an inch for dull red to $\frac{1}{70000}$ of an inch for violet.

It is on the same principle that we explain the coronæ round the moon when it is viewed through a fleecy cloud, the small particles of the cloud taking the place of the lines of the grating. The smaller the particles the larger the rings. Young constructed an instrument called the Eriometer, by means of which the dimensions

of small particles are measured in terms of the size of the diffraction rings they produce round a luminous point.

§ 136. **Colours of Soap-films and Thin Plates: Newton's Rings.**—The principle of interference explains with great simplicity the colours of thin films, such as soap-films, thin layers of oil on water, thin skins of oxide on metal surfaces, fine slices of mica, the wings of the house-fly, and so on. In all these cases, parts of the incident pencil are reflected from the two surfaces bounding the film, pursue paths of slightly different length, and produce interference effects accordingly. Let AB (fig. 220) be the thin film greatly magnified; and let RP represent an incident ray of sodium light, part of which is reflected to the eye in the direction PE. Another part is refracted to Q, reflected to S, and finally sent out along SE parallel to PE. But the time taken by the light to pass along the path RPQSE is longer than the time taken to pass along the path RPE. Hence the eye is acted upon simultaneously by two disturbances which in the original ray RP lay apart in time by an interval equal to the time taken by the ray to travel from P to S within the film. In other words, the wave-fronts of PE and SE which coalesce were originally different wave-fronts in RP. If, then, the original distance between these ultimately coalescing wave-fronts is equal to half a wave-length of the light constituting the ray, the two rays PE and SE will tend to destroy one another, the crests of the one wave-motion filling up more or less the troughs of the other. In other words, if the two interfering rays differ in phase by an amount corresponding to half a wave-length, or any odd multiple of half a wave-length, the resultant intensity of light will be greatly diminished, and in the special case of equality of amplitudes, reduced to zero. On the

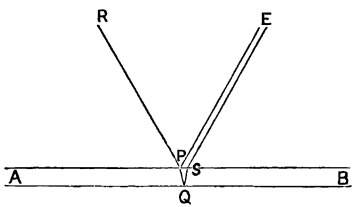


Fig. 220.

other hand, if the thickness of the film and inclination of the rays are such as to make them differ in phase by an amount corresponding to a whole wave-length, or any multiple of a whole wave-length, the resultant intensity of light will be greatly increased, crest falling with crest and trough with trough. For other differences of phase, the resultant intensity will be intermediate to the extremes just stated. But that such variations in the difference of phase will exist in the case of a uniform film illuminated by yellow light is evident when we consider that rays reaching the eye from other parts of the film have, in general, different angles of incidence and reflection, involving a different length of course PQS within the film, and a corresponding different phase-difference. For example, if we suppose the rays PE and SE to have exactly half a wave-length difference of phase, the rays from more distant points will differ by more than a half wave-length; and as the eye glances along the surface the appearance will be a gradually increasing intensity of yellow light, until the maximum intensity is reached, the difference of phase corresponding then to a whole wave-length. Still farther on, when the difference of phase corresponds to a wave-length and a half, a second region of minimum intensity will be reached; still farther on, a second maximum with two wave-lengths' difference of phase; still farther on, a third minimum with $2\frac{1}{2}$ wave-lengths' difference of phase; and so on indefinitely.

These alternate bands of increased and diminished brightness will come more rapidly if the thickness of the film increases gradually in one direction. A soap-bubble generally thins off towards the top, but in an irregular manner, so that the bright and dark bands are of irregular form, and are, besides, constantly shifting. For a careful study of the phenomenon, there is nothing better than Newton's original form of experiment known as *Newton's Rings*. A convex lens of long focus is laid on a plane glass surface, as indicated in fig. 221. The rays reflected internally from the convex surface

at a, and externally from the plane surface at b, will interfere, the resultant intensity depending upon the thickness of air traversed by the ray. For equal angles of incidence, this thickness diminishes to zero at the point of contact. If the arrangement be viewed in yellow light, there will be seen a series of concentric bright and dark rings. The rings of maximum brightness correspond to the positions from which

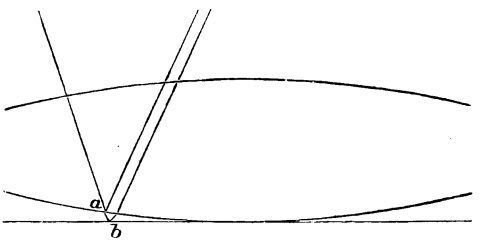


Fig. 221.

come the interfering rays whose phases differ by amounts corresponding to 1, 2, 3, 4.....complete wave-lengths. If, again, the arrangement be viewed in red light, there will be seen a series of concentric red rings, separated by dark bands, but the positions of the rings of maximum red brightness will be quite different from those of maximum yellow brightness. In like manner, green or blue light will produce its own particular set of rings. On the wave theory of light, the explanation of this difference in the positions of the differently coloured rings is simply that the different coloured rays of light have different wave-lengths.

In the usual form of experiment with Newton's rings, the arrangement is viewed in white light. The result is a series of concentric rings of different colours. For every point there will be a particular wave-length, the light corresponding to which will have its maximum intensity. This particular colour will be greatly intensified; while other colours will not be so intensified, and some may, indeed, be practically obliterated.

If the surfaces are pressed as close together as possible, there is complete absence of light at the region of contact. The explanation of this is dynamical, and depends upon the fact that the one ray is being reflected from the rare medium back into the dense, while the other is being reflected from the dense back into the rare. The phases of the vibrations are changed at both reflecting boundaries, but in opposite directions, and in such a way that the *relative* change of phase is half a wave-length; hence, where the thickness of the separating layer of air is negligible compared with a wave-length, all kinds of rays suffer a similar interference—all are practically obliterated. On the margin of the 'dark spot' the first light that appears is blue, for its wave-length is shortest, and the crest-to-crest condition for maximum intensity will be first produced for it.

The colours, as observed by Newton, were as follows, beginning with the dark central rings: 'black, blue, white, yellow, red; violet, blue, green, yellow, red; purple, blue, green, yellow, red; green, red; greenish blue, red; greenish blue, pale red; greenish blue, reddish white.' Newton also measured carefully the positions of the rings of least brightness, beginning with the central dark ring. Knowing the curvature of the convex surface, he was able to calculate the thickness of the layer of air corresponding to each of these dark rings. These thicknesses he found to be, in fractions of an inch,

$$\frac{2}{178000},\ \frac{4}{178000},\ \frac{6}{178000},\ \&c.$$

If λ be the wave-length of the light quenched, we have, approximately (remembering that a half wave-length difference of phase is already accounted for by the different conditions of reflection),

$$\lambda = \frac{4}{178000} = \frac{1}{44500} \text{ inch}$$
$$= 5710 \times 10^{-7} \text{ millimetres.}$$

This is the wave-length of light in the most luminous part of the spectrum—that, namely, between the yellow and the green.

Newton's measurements are, historically, the first of sufficient accuracy to give an approximate estimate of the wave-lengths of light. (See Table A, p. 338.)

§ 137. **Double Refraction.**—When an object is viewed through certain transparent crystals, two images are seen. This phenomenon, known as *Double Refraction*, is specially well marked in Iceland Spar. It was discovered by Bartholinus late in the seventeenth century, was fully described by Huygens in 1690, and by Newton in 1704.

When a ray of light is incident normally upon one of the plane faces of an ordinary rhombohedral crystal of Iceland spar, it emerges from the opposite parallel face as two rays. The one ray is a continuation of the original ray, and is found to obey the ordinary laws of refraction. It is accordingly called the *ordinary ray*. The other, which, though incident normally, is refracted out of the normal as it enters the crystal, is called the *extraordinary ray*. The plane containing these two rays bears a constant relation to the configuration of the spar, so that if the crystal be rotated, the extraordinary ray rotates about the ordinary ray. This is very simply demonstrated by placing the Iceland spar on a sheet of paper with a mark (a cross, say) on it, and noting how the one image rotates about the other as the crystal is rotated about a vertical axis. The extraordinary image is at once recognised.

Huygens explained the propagation of the extraordinary ray by assuming the disturbance to be propagated in the form of a spheroid instead of a sphere. The physical significance of this is that, so far as one part of an ordinary ray of light is concerned, the velocity of propagation is not the same in all directions; the medium is, in fact, æolotropic. The molecules of the substance seem to 'load' the ether, so that a disturbance is propagated more easily in one direction than in another. And yet the existence of the ordinary ray, which is propagated by means of spherical waves, shows that this one-sidedness of the medium does not affect all kinds of light.

Let now two pieces of Iceland spar be placed similarly, with the surfaces of the one all parallel to the corresponding surfaces of the other, and let a ray of light be passed normally through both. Then it is found that the ordinary ray in the first behaves as the ordinary ray in the second, and the extraordinary ray in the first as the extraordinary ray in the second. But if the one crystal be rotated through a right angle about the normal ray as axis, the ordinary ray in the first is refracted as the extraordinary ray in the second, and the extraordinary ray in the first continues as the ordinary ray in the second. Further, should the angle of rotation be less or greater than a right angle, each ray from the first crystal is broken up into two in the second crystal. Consequently with two crystals suitably arranged we can get *four* images of an object viewed through them.

Newton, in his discussion of these curious phenomena, asked the question, 'Have not the Rays of Light several sides, indued with several original Properties?' It was from the undulatory theory, as developed by Fresnel, that a complete affirmative answer was obtained. It is all involved in the conception that the vibrations or displacements constituting light take place at right angles to the direction of propagation of the ray.

§ 138. **The Nicol's Prism.**—It is very important to obtain complete separation of the two rays in a crystal of Iceland spar, so as to be able to study either to advantage. The natural method of doing this is to use sufficiently long crystals; but such specimens are rare and costly. Practically we have recourse to the *Nicol* or *Nicol's Prism*, so called after its inventor (1828). A rhomb of Iceland spar is cut in two, along the longer diagonal plane, and the pieces cemented together again with Canada balsam. Because of the relations between the refractive indices involved, the ordinary ray suffers total reflection from the Canada balsam and is thrown to one side, while the extraordinary ray passes through. A second Nicol placed similarly to the first will allow this extraordinary

ray to pass through it also. The two Nicols are then said to be 'parallel.' But as the second Nicol is made to rotate, less and less of the light from the first Nicol passes through till the rotation amounts to a right angle. In this position the Nicols are said to be 'crossed,' and in combination they are, if well constructed, perfectly opaque to strong light. The reason is, that the extraordinary ray from the first Nicol, passing into the second Nicol as an ordinary ray, is completely stopped, because the second Nicol, like the first, stops all ordinary rays. In other words, the first Nicol imposes upon the ray a particular 'sidedness,' in virtue of which it cannot be transmitted through the second (crossed) Nicol.

§ 139. **Plane Polarisation.**—The ray which has acquired this 'sidedness' by passing through a Nicol prism is said to be a *plane polarised* ray. The general term Polarisation applied to this peculiar condition of light, and to other analogous conditions, is not happily chosen; Newton's original description of the ray having *sides* is infinitely more appropriate, especially from the point of view of the wave theory of light.

A close consideration of the doubly refracting properties of Iceland spar shows that each of the two rays, the ordinary and the extraordinary, is polarised. But this is proved experimentally at once by viewing the two images through a Nicol prism. As the Nicol is rotated, the images vanish alternately. The position for which the ordinary ray is stopped is exactly 90° from that for which the extraordinary ray is stopped; and each ray is stopped twice every complete rotation. In any other than these four critical positions, both images are seen in varying intensities as the position is altered. As the one increases in intensity during the rotation of the Nicol the other diminishes in intensity; and when the one is at its maximum the other is invisible. The ordinary and extraordinary rays produced by double refraction are therefore both plane polarised rays, but *polarised in planes perpendicular to each other.*

The assumption that the vibrations or displacements which constitute light take place at right angles to the direction of

propagation of the ray gives an immediate explanation of these phenomena. We have simply to suppose, in the first place, that in a ray of ordinary light the vibrations occur, on the average, as frequently in one direction as in another; and then, in the second place, that a Nicol's prism is a kind of sieve which allows the transmission of vibrations only in one definite direction—that is, in one definite plane containing the ray. All vibration at right angles to this plane is suppressed. Or, to put it more accurately, every vibration may be regarded as the resultant of two components in lines perpendicularly to one another, and of these only one is allowed to pass through the Nicol. The action of the Iceland spar is, of course, to treat these components differently, passing them on as independent rays whose vibrations take place in planes perpendicular to one another. The action of two *crossed* Nicols is at once evident. If the first Nicol be set so as to transmit, say, vertical vibrations, the second is set so as to transmit only horizontal vibrations. What passes through the one is stopped by the other. For any other position of the second Nicol there will be transmitted a corresponding proportion of the vibrations which have passed through the first Nicol.

The polarisation of light is usually introduced with reference to the property of the doubly refracting crystal known as Tourmaline. In slices of tourmaline cut parallel to the optic axis (the direction in the crystal along which the ordinary and extraordinary rays are not separated) the one ray suffers much greater absorption than the other. The ray which suffers less absorption will be practically all that is left, if a suitable thickness of tourmaline be chosen. Two such slices of tourmaline may be used exactly as two Nicols. Two placed parallel let through nearly as much light as one; but two 'crossed' are practically opaque to transmitted light. The disadvantage of the tourmaline is, that the transmitted light is greatly altered in colour and intensity because of absorption, whereas the Nicol is extremely transparent.

It is usual to call the Nicol or tourmaline which produces the polarised ray the *Polariser*. The second Nicol or tourmaline is called the *Analyser;* by means of it the character of the ray, as regards its polarisation, is demonstrated.

§ 140. **Polarisation by Reflection.**—As discovered by Malus in 1802 light may be polarised by reflection. For example, sky-light reflected from the surface of a glass plate,* when viewed through a Nicol's prism, is found to change greatly in intensity as the Nicol is rotated about the reflected ray as axis. There is, indeed, a certain angle of incidence, known as the polarising angle, for which the reflected ray is almost as completely polarised as if it had been transmitted through a Nicol. The intensity will be reduced to an almost vanishing minimum when the analyser is set in the proper position. For other angles of incidence there is always a considerable amount of light transmitted through the Nicol, in whatever position it may be; but, in all cases, the intensity passes through a minimum twice every rotation of the Nicol. We may suppose this *partially polarised* light to be a mixture of plane polarised light and of ordinary unpolarised light. When the Nicol is placed so as to obliterate the former, part of the latter passes through the Nicol, and emerges from it as a plane polarised ray.

A very remarkable relation exists between the polarising angle and the refractive index of the medium from whose surface the light is reflected. It is found that the reflected ray is almost completely polarised when it makes a right angle with the refracted ray. Let AP (fig. 222) be the incident ray, PC the refracted ray, and PB the reflected ray. Then, if BPC is a right angle, PB is (very approximately) a plane polarised ray of light. The *plane of polarisation* of the ray is defined to be the plane containing these various rays. Let ABC be a circle with centre at P, and let Aa, Bb, Cc be perpendiculars on the surface. Clearly, Aa and Bb are equal, and because BPC is a right angle, Pb is equal to Cc. Hence, if μ is the refractive index, we have

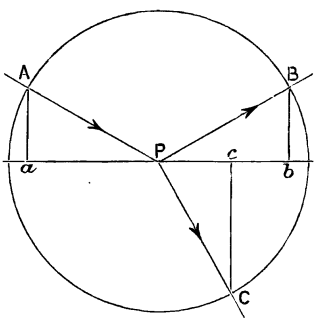


Fig. 222.

$$\mu = \frac{Cc}{Aa} = \frac{Pb}{Bb} = \text{tangent PB}b.$$

That is, the tangent of the polarising angle is equal to the refractive index.

* It is better not to use an ordinary mirror for this experiment, but a glass plate blackened on the one surface is very effective.

Just as a reflecting surface can be used as a polariser, so also it can be used as an analyser. Set a glass plate on a table so as to reflect light from a window to the eye. Take a second plate, and hold it close up to the eye so that the first plate is seen by reflection in it. Then slowly rotate the second plate conically about the eye so that in all positions the first plate may be seen by reflection. As this is being done, the brightness of the light so reflected from the two plates in succession will vary from a maximum when the final reflected ray is in the first plane of incidence to a minimum when the final reflected ray is perpendicular to the first plane of incidence. If the angles of incidence on the two plates are equal to the angles of polarisation for these plates, the ray of minimum brightness will be a ray of nearly complete extinction.

The blue light of a cloudless sky is thrown back by reflection from small particles, and is in consequence partially polarised, as may be easily demonstrated by viewing it through a Nicol, or after reflection from a glass plate. The polarisation is most pronounced in light coming from a part of the sky distant 90° from the sun. This is the explanation of the peculiar black aspect of certain parts of the sky when viewed by reflection on the surface of a lake. The water surface in this case plays the part of an analyser.

Not only is the reflected ray polarised completely or partially, as the case may be, but so also is the refracted ray. Moreover, the refracted ray is polarised in a different plane, its plane of polarisation being perpendicular to the plane of incidence. If we regard the vibrations of light as being displacements analogous to those of an elastic solid, there is evidence that the plane in which the vibrations of a plane polarised ray of light are executed is perpendicular to what has been defined as the plane of polarisation. That is, if a ray of light is reflected from the surface of a transparent medium at the polarising angle, the vibrations in the reflected ray are believed to take place at right angles to the plane of incidence, and those of the refracted ray in the plane of incidence.

Light reflected from the surface of an opaque body, such as polished metal or wood, is also in a sense polarised; but it is not possible to get an approximately plane polarised ray of light. Whatever be the angle of incidence, a good deal of

light will always pass through the Nicol in every position of it. For this reason ordinary mirrors are not convenient for the study of polarisation by reflection.

§ 141. **Elliptic and Circular Polarisation.**—In general, when a plane polarised ray of light is reflected, it becomes what is known as elliptically polarised. Viewed through a rotating Nicol, the ray varies in intensity, but in no position wholly vanishes. Elliptically polarised light may be regarded as composed of two plane polarised rays of light, polarised in perpendicular planes, and differing in phase but not in period. The significance of the name and the essential character of the phenomenon are best realised by means of a simple analogy. The bob of a simple pendulum capable of oscillating in all vertical planes may be compared to a particle of ether, the vertical equilibrium position of the pendulum being the direction in which the ray of light is supposed to pass. The pendulum, when oscillating in a definite plane, illustrates the essential nature of plane polarised light. As it reaches the extremity of a swing let a slight impulse be communicated to it in a direction perpendicular to the plane of motion. The pendulum will then begin to move conically, the bob tracing out a curve which for small deviations from the plumb is an ellipse, or it may be a circle. This elliptic motion is the resultant of two simple harmonic motions in different lines of equal period but different phase.

When the phases differ by a multiple of two right angles the elliptic motion becomes straight line motion, the minor axis vanishing. When the phases differ by a right angle the elliptic path has its least eccentric form, and becomes truly circular when the amplitudes are equal. In short, plane polarised light and circularly polarised light are the limiting cases of elliptically polarised light.

When a plane polarised ray of light is reflected at the polarising angle from the surface of a transparent medium, it remains a plane polarised ray only when the plane of reflection is the plane of polarisation. For any other position of

the reflecting surface, the ray, if it has any intensity at all, is elliptically polarised. For, in general, the incident plane polarised ray may be resolved into two, polarised in the plane of incidence and in the perpendicular plane respectively, and equal in phase. But these two components, when reflected, suffer different alterations of phase, and consequently recombine into a single ray elliptically polarised.

It is evident that elliptically polarised light cannot be cut off by use of a Nicol as analyser. While the Nicol is being rotated the light varies in intensity from a maximum to a minimum, and back again to a maximum, and so on in alternation twice every revolution. In the case of a circularly polarised ray there is no change in intensity during rotation of the Nicol. It is therefore impossible by means of a Nicol prism to distinguish between common unpolarised light and circularly polarised light, or between partially polarised light and elliptically polarised light. Other properties of polarised light, however, enable us to make the distinction with ease. It will suffice, here, briefly to indicate the principle of one method of distinguishing between common light and circularly polarised light. In the latter the rectangular components are equal in amplitude, but differ in phase by 90°. If, then, the circularly polarised ray be subjected to any treatment which would have produced it at first from a plane polarised ray, the difference of phase will be increased by another 90°, and plane polarised light will be obtained. This the Nicol will at once prove. With common light, on the other hand, it will not be possible to produce plane polarised light by this treatment. Fresnel's Rhomb is one form of apparatus for transforming a plane polarised ray into a circularly polarised ray and a circularly polarised ray into a plane polarised ray. It consists of a parallelepiped of glass so cut that a ray passes through it after two internal total reflections at particular angles of incidence.

§ 142. **Rotation of the Plane of Polarisation.**—Let P (fig. 223) be the polariser and A the analyser, so adjusted

that no light passes through—in short, let them be crossed Nicols. If a piece of quartz, Q, be placed between P and A so that the plane polarised ray from P passes along the axis

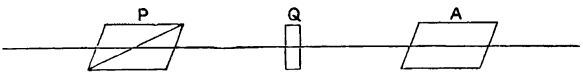


Fig. 223.

of Q, it is found that some light passes through the analyser. With a sufficiently thin piece of the crystal the light that so passes has a distinct colour. If the analyser or polariser be rotated the colour changes; and if the quartz be slightly inclined to the direction of the ray a change of colour is also produced.

A beautiful variation of the experiment is to use a crystal of Iceland spar instead of the Nicol prism P (fig. 223). The beam of light is divided into two, polarised in perpendicular planes. Let the analyser, A, be adjusted so as to cut out one of the rays. Then if a thin piece of quartz be introduced, at once two images are seen coloured with complementary tints—e.g. red and green, or yellow and blue. Their complementary character is beautifully demonstrated if the two images overlap as in fig. 224. The part, *w*, common to the two images is white; the separated parts, *r* and *g*, are coloured. When the analyser is rotated the separate coloured parts change colour, but always so as to remain complementary. The overlapping part remains persistently white.

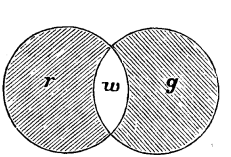


Fig. 224.

The true nature of the effect produced by the interposed quartz is best shown by passing through the polariser a ray of monochromatic light, say a small part of the spectrum of sunlight, or the yellow sodium light. Suppose the latter to be used, and the Nicols to be crossed so that when no quartz is interposed light is completely cut off.

When the quartz is introduced a yellow light is seen, but the rotation of the analyser through a certain angle again produces darkness. Thus the yellow ray after traversing the quartz is still a plane polarised ray, but its *plane of polarisation has been rotated through a certain angle.* The amount of rotation produced by a given specimen of quartz depends upon the colour—that is, upon the wave-length of the ray. Let Aa (fig. 225) represent the direction of the vibrations of the plane polarised ray of white light which has passed through the polariser. The effect of the quartz is to rotate the plane of vibration of each component ray through a certain angle to left or to right, and the amount of rotation on the ray of shorter wave-length is greater than that on the ray of longer wave-length. The red ray vibration may be rotated into the position rR, and the violet ray vibration into position vV, the vibrations belonging to the intermediate wave-lengths being rotated into intermediate positions. Thus the original plane, Aa, of all the vibrations is drawn out fan-like into the sector of rays RV. Any particular ray will be obliterated if the analyser is rotated through the appropriate angle. When this ray is so cut out the light that passes will be of the complementary colour. Rotation of the analyser simply cuts out in order rays of successive tints.

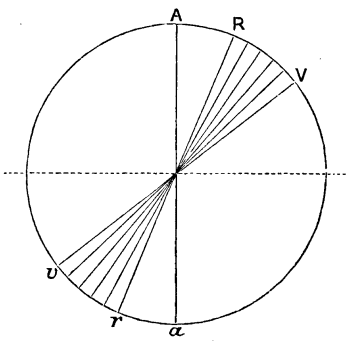


Fig. 225.

Experiment proves that the amount of rotation is proportional to the thickness of the plate, a fact which at once explains the change of colour (referred to above) when the quartz is inclined slightly so that the ray passes through a greater thickness. A quartz plate one millimetre thick

rotates the plane of polarisation of the low red rays through an angle of about 13°, and that of the violet rays through an angle of about 51°. Some specimens of quartz produce a right-handed rotation—that is, the rotation has the same relation to the direction of the ray which the rotation of a right-handed screw has to its forward motion. Other specimens, again, produce a left-handed rotation. These two kinds of quartz are distinguished as right-handed or dextrogyrate and left-handed or levogyrate. The optical difference corresponds to a secondary difference in the crystalline structure.

Similar rotatory effects are produced by certain liquids, such as oil of turpentine, solution of sugar in water, solution of camphor in alcohol. The rotation is in these cases much smaller than that produced by quartz, and to measure it accurately the solution must be enclosed in a long tube placed between the polariser and analyser. A mixture of liquids having rotatory effects has a resultant rotatory effect, which is the algebraic sum of the rotatory effects of the components taken singly. Hence the amount of rotation produced by a given length of solution of sugar is directly as the strength of the solution. This gives a very important method of testing the strengths of sugar solutions, for which a special form of instrument, the Saccharimeter, has been constructed. It consists essentially of a polariser and analyser with an intermediate tube having flat transparent ends, in which the solution is contained. There are several types of saccharimeter, such as Jellett's, Laurent's, Soleil's, descriptions of which will be found in books treating of the subject in detail.

When a plane polarised ray of light traverses certain substances placed in a strong magnetic field, rotational effects are produced. Also, when a ray of plane polarised light is reflected from the polished end of a strongly magnetised piece of iron, a rotation of the plane of polarisation takes place. These, discovered respectively by Faraday and Kerr, are two of the simplest of a remarkable series of magneto-optic facts of a high theoretic interest. In one fundamental respect the rotation of the plane of polarisation in a

magnetic field differs from the rotation produced by transmission through quartz or other substance. If, in the latter case, the ray is reflected back again, the rotation is reversed and the ray emerges as it entered. But, in the magnetic rotation, the ray, after to and fro transmission, emerges with *double* the rotation. The difference points to an essential difference in the mechanism of the two classes of phenomena.

§ 143. **Polarisation of Radiant Heat.**—The dark or invisible rays of so-called Radiant Heat differ from the visible rays of light only in their longer wave-length. It should be possible, therefore, to polarise them. Berard, in 1814, found distinct evidence that the infra-red rays of the spectrum were polarised by reflection. Nothing further was done, however, till Forbes took up the inquiry in 1834. Melloni had in the meantime invented the thermopile (§ 99), and had successfully applied it to investigate the laws of reflection and refraction of radiant heat and the diathermanous properties of matter (§ 148). With this instrument Forbes proved that the polarisation of radiant heat could be effected by exactly the same processes as polarisation of light — namely, by reflection, refraction, and transmission through certain crystals. Double refraction and circular and elliptic polarisation were also established. The most effective demonstration of the plane polarisation of heat is given by Forbes's experiment in which the ray is transmitted through thin slices of mica inclined at the polarising angle. Two such sets of slices placed similarly with respect to a ray of radiant heat allow nearly as much of the energy to pass as passes through one set singly. In this position they behave like parallel Nicols; but when one is rotated through a right angle about the ray as axis almost all the radiant heat is stopped. In this position they act like crossed Nicols or tourmalines.

§ 144. **Interference of Polarised Light.**—It is not possible to do more than indicate very briefly the nature of the exquisitely beautiful phenomena produced by the interference of polarised light. Their importance lies in their applicability to the investigation of the structure of minerals and of certain organic bodies. A complete explanation requires the aid of the higher analysis.

The rotation of the plane of polarisation by quartz is really a case of interference. Quartz is a doubly-refracting crystal, in which the ordinary and extraordinary rays are, in general, elliptically polarised, and are circularly polarised when they are parallel to the axis of the crystal. Consequently, the plane polarised ray, as it passes into the quartz, is decomposed into two circularly polarised rays, the rotations being in opposite senses. But the extraordinary ray travels slower than the ordinary ray. When, therefore, they recombine on emergence, they still form a plane polarised ray, but rotated relatively to the original plane of polarisation by an amount proportional to the lag in phase of the extraordinary ray behind the ordinary ray.

When a ray of plane polarised light passes into a doubly refracting crystal in a direction parallel to its axis, it will in general be decomposed into two, polarised in perpendicular planes, which constitute the ordinary and extraordinary rays. These will travel with different speeds, and on emergence will combine into elliptically polarised light (unless the retardation in phase corresponds to any multiple of half a wave-length). In this condition let the light be viewed through a Nicol or other analyser. This will allow the components of the two rays to pass which correspond to the plane of polarisation of the second Nicol. If the doubly refracting crystal between the polariser and analyser be thin enough to permit the one component to lag a few half wave-lengths behind the other, interference will be produced, and the whole light that passes through polariser, crystal, and analyser will appear coloured. If, with the Nicols crossed, the crystal be rotated about the axis, the *intensity* of the light will change so as to have four maxima and four vanishing points in one complete revolution. If one of the Nicols be rotated, the tint will change so as to show complementary colours at every quadrant distance. The particular pair of complementary colours thus produced alternately as the Nicol is rotated depends upon the difference of phase of the two interfering components, and that depends on the thickness of the plate and the difference of the velocities of the ordinary and extraordinary rays. The thickness of plate traversed by the ray may be slightly increased by inclining the crystal to the direction of the ray. This will produce a change in the colour of the light that is most affected by the interference.

Finally, let the apparatus be so arranged that the polarised light is made convergent or divergent when it is passing through the crystalline plate. As we pass outwards from the axis, the effective thickness of plate traversed will increase, and a different tint will

correspond to each different distance from the axis. The result will be a series of concentric coloured rings at first sight not unlike Newton's rings. In the present case, however, the rings will be crossed by two mutually perpendicular diametrical lines dark or bright according as the polariser and analyser are crossed or are parallel. This dark or bright cross indicates those directions in the crystalline plate in which only one ray is transmitted—those directions, in fact, for which the planes of polarisation of the ordinary and extraordinary rays are parallel or perpendicular to the original plane of polarisation. In other words, along those directions the crystalline plate produces no change on the plane polarised ray passing into it.

We have confined our explanations entirely to what are known as uni-axal crystals, in which there is one optic axis or direction of coincidence of the two rays. But the greater number of crystals—such, for example, as *mica*, *felspar*, *arragonite*—possess two optic axes. These are distinguished as bi-axal crystals. They give rise to similar but more complicated phenomena, an account of which will be found in more advanced treatises on light.

§ 145. **Electro-magnetic Waves.**—A discussion of ether waves would not be complete without a reference to Hertz's experiments on the propagation of electro-magnetic waves through space. There is no difficulty in getting such waves, which we now know to accompany every ordinary discharge of a Leyden Jar or condenser (§ 67); in general, every discharge of the kind known as oscillatory. That a discharge may, under certain conditions, be oscillatory—that is, take place by a more or less rapid alternation of positive and negative discharges of diminishing magnitudes—was first perceived by Henry (1842), who saw in them the explanation of the anomalous and apparently haphazard magnetising effects of discharge currents, studied as early as 1826 by Savart. The complete theory and experimental discussion of the oscillatory discharge we owe respectively to Kelvin and Von Helmholtz. The phenomenon is dynamically analogous to the vibrations of a spring in a viscous medium, or the oscillations of a viscous fluid in a U-tube. The only difference is that the electrical oscillations may take place so rapidly that, however many there may be constituting the one discharge, they exist

for an interval extremely short compared with the period of the quickest vibrations attainable in a mechanical manner. Now, when a spark of this character passes between two conductors of different potential, the system becomes a source of electro-magnetic waves having the same frequency as the electrical oscillations. This frequency depends chiefly upon the capacity of the discharging system, and upon the self-inductance (§ 104) of the channel along which the discharge takes place. By varying either or both of these quantities, we may adjust the frequency through a very wide range of values. The resistance of the channel also has an influence on the frequency; but its effect is mainly appreciable in the rapid decay of the oscillations. To get high frequencies—that is, rapid oscillations—we must make the capacity and self-inductance as small as possible. In some of his experiments Hertz used two brass rods set in line and separated by a small air-space, across which the spark passed. The rods were made the terminals of the secondary coil of an induction machine. This particular 'oscillator' gave, at every spark, electrical oscillations at the estimated rate of nine hundred million per second, the estimate being made from the measured length of the electro-magnetic wave produced, on the assumption that the velocity of light was the velocity of propagation of the electro-magnetic disturbance. In the device for detecting the electrical oscillations lay the novelty of Hertz's method. It was essentially an application of the well-known principle of resonance. Just as a stretched pianoforte wire or a Helmholtz resonator will respond to a note of its own pitch (Part I., § 153), so Hertz's electrical resonators respond to the electrical oscillations produced by the 'oscillators' with which they are, so to speak, in tune. For the oscillator just described the resonator was a piece of copper wire bent into a ring 7·5 centimetres in diameter, with a very minute air-space separating its opposed ends, one of which was pointed, and the other terminated in a small brass ball. This resonator had its capacity, inductance, and

resistance so related that the period of electric oscillation was exactly the same as the period of electric oscillation of the oscillator. In other words, any electromotive force made to act along it for an instant would start in it a current which would decay by a series of diminishing oscillations of frequency nine hundred million per second. If, then, a series of properly-timed electromotive impulses were made to act, the cumulative effect of these would be to work up the difference of potential at the opposing terminals sufficiently to produce a spark across the air-space between. In this way a Hertz resonator suitably placed in the neighbourhood of its appropriate oscillator may be made to spark in response to the sparking of the oscillator.

Following on Hertz's classical experiments, many even simpler arrangements have been devised for demonstrating the existence of electrical resonance. Some of the simplest of these have been described by Lodge. Thus, one Leyden jar may be made to resonate to another which is being charged and discharged at a considerable distance away. Or even simpler still, a spherical conductor, placed between the knobs of two Leyden jars which form the terminals of the secondary coil of an induction coil, will at every spark that passes across the air-spaces give rise to electrical oscillations, which will radiate forth as electric waves. An equal-sized insulated sphere placed at some little distance will be found to give tiny sparks to a sharp point held near it. These sparks are due to the electrical resonance, the impulses being accurately timed to the natural period of the electrical oscillations of the sphere. The effect may be compared to the large oscillations of a liquid surface produced when the containing vessel is kept very gently rocking synchronously with the natural period of oscillation of the liquid.

Sheets of metal or good conductors are found to be opaque to the electro-magnetic waves, partly reflecting them as polished surfaces reflect light. By means of a parabolic reflector, the waves radiating from an oscillator placed at

the focus may be projected in parallel rays and reconcentrated again to the focus of another parabolic reflector. To such rays thick walls are almost perfectly transparent. A prism of pitch may be used to refract the rays; gratings of wire may be used to polarise them. In short, these electro-magnetic rays are found to possess properties identical with the properties of light rays. They may differ greatly in wave-length and frequency; but their velocities of propagation are of the same order of quantity.

In later experiments by various experimenters, many other methods of detecting the presence of electro-magnetic waves have been adopted. Thus, the heating effect of the currents induced by these waves in a very thin wire is sufficient to produce in the wire a change of resistance which can be measured on a delicate galvanometer. A vacuum tube, placed in the path of the electric rays, is lit up. The resistance of a layer or column of iron filings in loose contact is markedly diminished when the electric waves pass through it, the diminution being measured by the increase of current in a circuit containing the filings. By using such forms of detector we are able to study electric rays of much shorter wave-length than was possible with the original form of Hertz resonator. Electrical oscillations in the molecules of a luminous body may well be the source of the electro-magnetic disturbance, which, according to Maxwell, is the real nature of what is commonly called light. That good conductors are opaque alike to light and electric radiations, and that optically transparent bodies are transparent also to these radiations and in many cases good insulators, are broad facts thoroughly in keeping with Maxwell's theory. The optical opacity of many bodies which are transparent to the electric rays means that transparency is a function of wave-length, and is simply another instance of selective absorption (§ 148).

Hertz's experiments were a remarkable corroboration of Maxwell's electro-magnetic theory of light, first published in 1864. According to the Faraday-Maxwell theory of

electric action, electric induction is associated with electricity at rest, and depends upon the dielectric properties of the medium, symbolically upon a quantity K known as the specific inductive capacity or the dielectric constant of the medium (§ 66). Magnetic induction is—if we accept Ampère's theory of molecular currents—associated with electricity in motion, and depends on the magnetic permeability (μ) of the medium. The permeability is defined, in magnetic units, as the ratio of the Induction to the Magnetic Force—that is, the ratio of the number of lines of force passing through the medium to the number of lines of force that would be present if the medium were simply the ether (§§ 16, 17). Excepting in the case of the magnetic metals, the permeability differs slightly from unity, being greater than one for paramagnetic substances and less than one for diamagnetic substances (§ 84). Now Maxwell's theory requires that an electro-magnetic disturbance travelling in a non-conducting medium shall be propagated with a velocity equal to the reciprocal of the square root of the product $K\mu$, namely, $1/\sqrt{K\mu}$. Generally, K is measured according to the electro-static system of units, and μ according to the electro-magnetic system of units; but in the present case both must be measured according to the same system. It matters not which is chosen, the quantity $1/\sqrt{K\mu}$ will come out exactly the same number, which is the number of electro-static units of electricity contained in one electro-magnetic unit. The *quantity* represented by this number is quite independent of the fundamental units of space, time, and mass that are used; and it has a value almost identical with the value of the velocity of light—namely, about 300,000 kilometres per second. According to this view, $\sqrt{K\mu}$ should be for any medium proportional to its refractive index for light, or, more accurately, for ether waves of infinitely long wave-lengths, for 'these are the only waves whose motion can be compared with the slow processes by which we determine the capacity of the dielectric.' Now μ is

very nearly the same for all substances which are sufficiently insulating to have a measurable dielectric constant; for any such substance, therefore, $\sqrt{K}$—K being assumed to be unity for vacuum — should measure its refractive index for infinitely long waves. This conclusion is fairly well supported in the case of gases, good insulating liquids, and certain solid dielectrics of uniform structure. These two striking relations between electrical and optical physical constants and the general considerations referred to in the concluding sentences of last paragraph lent a strong support to Maxwell's view that light is an electro-magnetic disturbance; but not till Hertz (twenty-four years later) showed how to detect the presence of electric waves did this brilliant theory command the assent of Continental physicists. See Introduction to Part II.

SUMMARY.

§ 133. Ether Waves. Huygen's Principle.

§ 134. Interference of Light. Young's Experiment. Fresnel's Mirrors and Bi-prism. Measurement of Wave-length.

§ 135. Mother-of-pearl: Diffraction Grating. Normal Spectrum. Coronæ. Eriometer.

§ 136. Newton's Rings. Colours of Thin Plates.

§ 137. Double Refraction. Iceland Spar. Ordinary and Extraordinary Ray. Huygen's Construction.

§ 138. The Nicol's Prism.

§ 139. Plane Polarisation. Polariser and Analyser.

§ 140. Polarisation by Reflection. Polarising Angle. Sky-light polarised.

§ 141. Elliptic and Circular Polarisation.

§ 142. Rotation of Plane of Polarisation. Quartz, Solutions of Sugar, &c. Rotation produced in Magnetic Field.

§ 143. Polarisation of Radiant Heat. Forbes's Experiment with Mica.

§ 144. Interference of Polarised Light. Phenomena in Divergent or Convergent Light. Uni-axal and Bi-axal Crystals.

§ 145. Electro-magnetic Waves. Hertz's Experiments. Maxwell's Electro-magnetic Theory of Light.

CHAPTER XXII.

RADIATION AND ABSORPTION.

§ 146. Luminous and Non-Luminous Radiations.—The distinction implied in these words is purely accidental, depending on the properties of the normal human eye. The *luminosity* of the radiations emitted by any luminous body may represent a mere fraction of the whole energy that is passing out from the body. In general it is a highly heated body which is luminous, and the higher the temperature the brighter the body. Thus an iron ball heated to a white-heat is emitting, in addition to the radiations of light of all the spectral colours, the so-called dark 'heat-rays,' which are sensible long after the ball has cooled to below dull red. Now, physically, there is no marked distinction between these dark rays and the luminous rays which affect our organs of vision. Both are propagated by wave-motion through the ether with the same velocity; both, as shown in last chapter, can be polarised. As we pass from the lowest appreciable heat-rays to the most refrangible light-rays, the wave-length gradually diminishes, the frequency gradually increases. But the violet rays do not form the limit of possible radiations. With very intense sources of radiant energy, such as the sun, the arc and incandescent electric lights, the magnesium light, &c., there are non-luminous rays of higher frequency than the luminous rays. These it is customary to distinguish as the *Actinic Rays*. It is to these, in combination with the more refrangible luminous rays, that the production of ordinary photographic effects is due.

By the dispersive action of an ordinary prism these various kinds of radiation can be separated out in the form of a

spectrum. The composition of the visible part is obvious to the eye. The dark heat-rays lying beyond the red end of the spectrum may be studied with the aid of a thermopile and galvanometer (§§ 81, 99). The non-luminous actinic rays lying beyond the violet end of the spectrum are readily 'seen' with the aid of what are known as fluorescent substances (see below, § 151).

It is important to bear in mind that all the physical properties of luminous radiation belong also to non-luminous radiation, although the latter are much more difficult to study experimentally. This arises simply from the fact that we have an organ of sense specially well fitted for investigating the properties of light.

§ 147. **The Visible Solar Spectrum.**—The spectrum of sunlight, formed, as Newton first formed it, by the dispersive action of a glass prism, is built up of different coloured images of the hole or slit through which the beam of sunlight has passed. These images necessarily overlap; but the effect of the overlapping is to a large extent minimised by using a very narrow slit parallel to the edge of the prism. A spectrum formed in this way is called a *pure spectrum*, and, when it is closely examined, is found to be not *continuous* in its brightness, but to have gaps or 'dark lines,' indicating the absence or relatively small intensity of certain constituents of white light. These lines were first observed by Wollaston in 1802. In 1817 Fraunhofer, with improved optical apparatus, measured the relative positions of a great number of these dark lines, and named the more conspicuous of them by the early letters of the alphabet. They are shown in the first spectrum in the plate of spectra at the end of the book, and are also indicated in the skeleton spectra of fig. 227. They are the standard lines with which it is usual to compare the characteristics of other spectra.

For the careful observation of these lines the *Spectroscope* or *Spectrometer* has been constructed. It consists essentially of a prism or train of prisms, P (fig. 226); a collimator, C,

at the focus of whose lens, L, is placed a narrow slit, S, parallel to the edge of the prism; and a telescope, T, for producing a magnified image of the spectrum of the illuminated slit. In the form of spectroscope known as the *direct vision* spectroscope, a train of three or more prisms of two different kinds of glass is so arranged that the final deviation of the mean ray is zero. The prisms of the different kinds of glass alternate so as to produce opposite deviations, the deviation due to the 1st, 3d, 5th, &c., being corrected by the opposite deviation due to the 2d, 4th, &c. But because of the different dispersive powers of the two kinds of glass, there is a resultant dispersion, although no resultant average deviation. The combination is the converse of the achromatic combination, in which there is deviation without dispersion (§ 128).

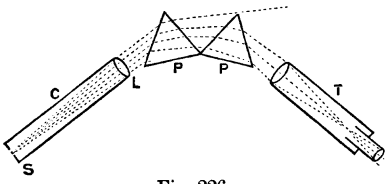


Fig. 226.

Nearly all transparent refracting substances produce similar spectra, the dark lines being arranged in the same order, but usually somewhat differently spaced. That arises from the fact that substances vary in their dispersive as well as in their refractive powers. The optical value of the dark lines is, that they give us the means of accurately determining the refractive indices of different substances for rays of definite wave-length. Now, although refrangibility depends on wave-length, being in general greater for the shorter wave-length,* it does not depend upon it according to any simple law. Hence in prismatic spectra the characteristic lines are not spaced in accordance with any simple relation to the wave-lengths of the corresponding rays. If, however, we substitute for the prismatic part of the

* There are cases of so-called *Anomalous Dispersion* in which this is not true; but for an account of these we must refer to more advanced treatises, such as Tait's *Light*.

spectroscope a diffraction grating (§ 135), we obtain a spectrum in which the rays are spaced according to a law of extreme simplicity, and from which, in fact, we are able readily to calculate the wave-lengths of the rays corresponding to the characteristic lines. Such a spectrum is called a *Normal Spectrum.* In fig. 227, the distribution of the principal

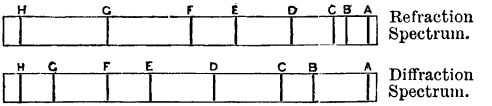


Fig. 227.

Fraunhofer lines in an ordinary refraction spectrum is compared with that in the diffraction spectrum. It will be seen how in the latter the red and yellow part of the spectrum is much more spread out than the blue and violet part.

What evidence have we that, in spite of the different spacing of the dark lines in these two types of spectra, they are identically the same lines in both? In short, what is the true significance of those lines? The following simple experiments give the clue to the correct answer to these questions :—First examine the spectrum of the yellow light given out when a quantity of common salt is burned in a hot flame. The characteristic bright yellow line of sodium vapour will be obtained (see the plate of spectra at the end of the book). Now let a ray of sunlight be passed through the burning sodium vapour into the spectroscope. We shall then see the solar spectrum with the Fraunhofer lines spaced as usual, but the so-called D line in the yellow will be very much darkened. If, instead of sunlight, we use the electric arc-light, we shall find the otherwise continuous spectrum from the incandescent carbons crossed by a dark band in the yellow. Hence the effect of the yellow flame is to *cut out, or absorb, some of the yellow light of the sunbeam passing through it.* And not only so, but the dark absorption band *occupies exactly the same position* as the bright yellow band forming the spectrum of the sodium vapour.

Careful examination in a good spectroscope shows that D is not a single line but a group of several lines, with two conspicuous constituents; and this is so both in the solar spectrum and in the sodium spectrum. Moreover, the bright lines in the sodium spectrum agree exactly with the Fraunhofer D group of dark lines in the solar spectrum. Thus we establish for one case what can be established for many other cases, that the dark lines in the solar spectrum are due to the absorption of these particular rays by cooler substances between us and the source of the solar light. These substances are mainly constituents of the sun's atmosphere, and many of them have been identified, by the simple method of comparison of spectra, with such well-known substances as hydrogen, sodium, potassium, calcium, iron, nickel, cobalt, magnesium—all, of course, in the gaseous condition.

In the spectrum plate the spectra of a number of different substances are shown. If each of these substances, supposed to be in a state of incandescent vapour, were placed in the path of a ray of white light from a brighter source, it would produce in the otherwise continuous spectrum a series of dark absorption lines coinciding exactly in position with the bright lines characteristic of its spectrum.

In this experiment we have the simplest ocular demonstration of the great principle that a substance powerfully absorbs exactly the rays it radiates. This principle was studied, under the name of the **Theory of Exchanges,** by Prevost, and its significance was greatly extended by Balfour Stewart, in the domain of radiant heat. It assumed final complete form in the hands of Kirchhoff and Bunsen, to whom is due the theoretical and experimental foundation of the method of Spectrum Analysis.

§ 148. **Selective Absorption.**—When radiant energy is received by a substance there is always more or less absorption, the energy so absorbed being, in general, transformed into heat. This is true for all grades of conditions from transparent to opaque, and it holds also for molar as dis-

tinguished from molecular vibrations. It is found, moreover, that all kinds of rays are not equally absorbed by any particular substance; in other words, that every absorbing medium exerts a peculiarly powerful absorption upon certain kinds of radiations, and a comparatively feeble absorption upon others. The natural colours of bodies are a case in point. Red glass is red because it absorbs the energy of the blue rays almost completely, but only a comparatively small proportion of the energy of the red rays. This property is called Selective Absorption. The most striking examples are perhaps those dealt with in last paragraph—namely, the production of dark absorption lines in an originally continuous spectrum.

To get a complete view of the whole subject it is necessary to consider the behaviour of matter in relation to the non-luminous radiations. With regard to these, substances will be 'transparent' or 'opaque,' using these terms in a generalised sense. Transparency to a particular radiation means that the substance has for that radiation no selective absorption. Just as red glass is transparent to red light but opaque to blue light, and violet glass opaque to all but the more refrangible luminous rays, so a substance may be, or may be conceived to be, opaque to all luminous rays, but transparent to the infra-red rays of so-called radiant heat. A piece of rock-salt and a piece of colourless glass are about equally transparent to ordinary rays of light. But the glass powerfully absorbs the energy of the dark heat-rays, while the rock-salt allows the greater part to pass through it. Rock-salt is said to be *diathermanous*—that is, transparent to radiant heat.

If there were sentient beings whose organs of vision were a little coarser than ours, so as to be sensitive to the infra-red rays, but incapable of seeing rays of shorter wave-length (as we are incapable of seeing the ultra-violet actinic rays), then to these beings glass would be opaque, and rock-salt would be transparent.

Being a bad absorber of heat-rays, rock-salt is also a bad radiator; a hot piece of rock-salt radiates less than a similar

hot piece of glass. Moreover, to the radiations so emitted by the rock-salt a second piece of rock-salt is much less transparent than it is to the radiations emitted by the hot glass. This is in full accord with the principle already enunciated that the radiative and absorptive powers of a body are equal for every kind of ray.

Another instructive experiment is to heat a piece of coloured glass in a bright-red fire. When the glass is at the same temperature as the coals behind it, it appears colourless, for the rays it absorbs it supplies again by its own radiation. If the glass is cooler than the coals it will appear of its usual colour—the colour of the rays that pass through it; but if the coals get cooler behind it, it will appear of the complementary colour, acting, in short, as a luminous body.

Of great interest, also, is the experiment of heating a strip of tourmaline till it is self-luminous. It is found to give off rays of plane polarised light; but the plane of polarisation is at right angles to that imposed on a ray transmitted through the tourmaline. Thus the crystal radiates exactly what it absorbs.

These are some of the results obtained by Balfour Stewart. The physical basis of the whole theory of exchanges is beautifully illustrated by an analogy from Sound due to Stokes. Imagine a room filled with tuning-forks all tuned to the same note. Let the listener be placed at one end of a room, and a musical instrument be placed at the other. Any note sounded by the instrument will be heard by the listener; but if the note sounded is the note to which the tuning-forks are tuned, the intensity of sound heard by the listener will be much less than for any other note. The reason is, that the tuning-forks vibrate in sympathy with the note to which they are tuned, and consequently absorb a considerable fraction of the original energy of vibration. Notes of a different pitch will not be absorbed in this pronounced way; the forks will simply act as obstacles, and will

not exert any true absorption. In such a case, when the original note ceases to sound there is immediate silence; but when the note that is being sounded is in tune with the tuning-forks, its cessation is not followed by silence, but the same note continues to be emitted by the tuning-forks.

§ 149. **Application of Carnot's Principle.**—In a system of bodies at different temperatures the tendency is towards equalisation of temperature. Once the equalisation of temperature is effected there is no further change of temperature possible until some colder or hotter body is made part of the system. Imagine an enclosure bounded by a perfectly reflecting and therefore impervious wall, and containing a system of bodies in thermal equilibrium. Although, in accordance with Carnot's principle, there is no change of temperature occurring, each body is, according to Prevost's theory of exchanges, radiating and absorbing energy. The fact that thermal equilibrium, once established, continues so to be, obviously requires that each body shall absorb exactly what it radiates.

Suppose, for example, that the enclosure contains two thermometers, one with a dark bulb, the other with a brightly polished silver bulb. It is easy to show, by bringing into the vicinity of these thermometers a source of radiant heat, that the black-bulb thermometer absorbs the heat much more quickly than the bright-bulb thermometer. Also, by heating them to the same temperature and allowing them to cool, we find that the one with the black bulb is the better radiator, and cools more quickly. This shows that the better absorber is the better radiator. The perfect *equality* of the radiating and absorptive powers of each follows at once from the fact that both come to the same temperature when left long enough in one enclosure. For if one radiated more than it absorbed, it would cool to a temperature lower than that of surrounding bodies; and if one radiated less than it absorbed, it would heat indefinitely by drawing heat from colder bodies. Both of these conclusions are inconsistent with Carnot's principle.

The reasoning just given may be extended to prove that there is absolute equality of radiative and absorptive powers for each particular radiation as characterised by wave-length and plane of polarisation.

§ 150. **Radiation as influenced by Motion.**—Waves on water will appear longer or shorter according as the observer is moving in the direction of propagation or in the contrary direction. In the former case fewer waves will pass him in a given time than if he were at rest—that is, the frequency will be less. In the latter case the frequency will be greater.

For a like reason, if the source of sound and the listener are approaching each other, the apparent pitch of the sound will be higher than the real pitch, by an amount depending on the relative velocity of approach. Suppose, for example, that a railway train is approaching at 60 miles per hour, or 88 feet per second, and that the frequency of the engine-whistle is n. In the nth part of a second, the distance between the train and the listener has diminished by $88/n$ feet. Hence, if we take 1100 feet per second to be the speed of sound, we find that two successive condensations of air will reach the listener's ear at an interval less than the normal interval by the amount $\frac{88}{1100}\frac{1}{n}$. In other words, the frequency will be increased from n to $n/\left(1-\frac{88}{1100}\right)$ or $\frac{1100}{1012}n$, corresponding to nearly a full tone in pitch. If, while the whistle is being sustained, the train passes close by the listener, the pitch will appear to fall by about one tone and a half—that is, a minor third. Passengers in the train will hear the whistle at its true pitch.

Precisely similar effects take place in the case of light, if the source and the observer are approaching, or receding from, one another. Because of the very great velocity of light as compared with velocities of bodies, the effect is, however, very slight, and requires for its detection and measurement refined spectroscopic apparatus. We have seen that the

dark lines in the solar spectrum are due to selective absorption by definite gaseous media. They are identified by comparison of the sun's spectrum with the bright-line spectra of these substances. Now, it has been found by careful measurement that groups of lines in the spectra of certain stars, although identical to a first approximation with the lines of some known substance, are in reality slightly displaced towards one or other end of the spectrum. This displacement is due to the relative motion of approach or recession of the star and the observer. If the displacement is towards the violet end, the relative motion is one of approach; if towards the red end, one of recession. This principle, generally known as *Döppler's Principle*, has been applied to measure the velocity of various stars in the line of sight, relatively to the observer.

According to the kinetic theory, the molecules of a gas are darting about in all directions with great velocities. In the case of a gas at rest as a whole relatively to an observer, the great majority of the molecules will be either approaching or receding with various speeds between given limits. The result will be that any radiation of definite wave-length given out by the vibrating molecules of the gas will reach the eye as a mixture of radiations of slightly different wave-lengths, and that the corresponding spectrum line will be slightly broadened.

§ 151. **Fluorescence** is the property possessed by certain substances of absorbing rays of one wave-length, and emitting visible rays of quite a different wave-length. The transformation is from the shorter to the longer wave-length, and corresponds to a degradation of energy. In a certain sense, the phenomenon may be compared with the ordinary transformation into heat of the rays of light absorbed by a body, and the subsequent radiation of part of this energy as radiant heat. But the peculiar feature of fluorescence is the emission of light of a definite refrangibility characteristic of each fluorescing substance when the substance is illuminated by

light of greater refrangibility or is set in the path of invisible ultra-violet rays.

A beautiful example of fluorescence is given by uranium or canary glass, which appears of a shimmering green colour in ordinary white light. When placed in a beam of blue or violet light, the substance emits the same brilliant green glow. The fluorescent shimmer is not seen if the canary glass is held in red or yellow light. It begins to appear when the illuminating light is blue, and becomes more brilliant as the refrangibility of the incident rays increases.

A dilute solution of sulphate of quinine gives in white light a beautiful blue fluorescence. If a solar spectrum be projected on a white screen, and the screen be painted beyond the violet region with a brush dipped in the quinine solution, the spectrum will become greatly extended. The ultra-violet rays are changed by the fluorescing liquid into visible rays of lower refrangibility. By use of such fluorescing substances we are able to study optically the ultra-violet part of the spectrum. Absorption lines and bands are found to exist in it as well as in the ordinary visible part of the spectrum of sunlight. The platino-cyanides of potassium, sodium, barium, and calcium have remarkable fluorescing properties, and have recently come much to the front in connection with the study of Röntgen's Rays (§ 152).

Phosphorescence is a phenomenon closely allied to fluorescence. It is, in fact, fluorescence persisting after the illuminating rays have been withdrawn. 'Luminous Paint' is the most familiar example. Although comparatively few substances are conspicuously phosphorescent, the great majority are so to a slight degree. The luminosity persists for a short time—it may be a small fraction of a second, but measurable nevertheless—after the light has been withdrawn.

The dynamical theory of fluorescence and phosphorescence has been given by Stokes. It depends upon the manner in which a system, having a natural period of vibration, responds to vibratory disturbances falling upon it. The

behaviour is different according as the period of the forced vibration is shorter or longer than the period of the free vibration. In the former case, when the disturbance acts at some point of the system, the disturbance remains as a local agitation; fluorescence is the result. In the latter case, the disturbance passes off through the system; there is no local agitation, no fluorescence.

The phenomena of fluorescence and phosphorescence are not in harmony with the theory of exchanges. For example, chlorophyll is a substance which has a marked absorption band in the red. It also fluoresces with a red glow. But the fluorescent red emitted by radiation does not correspond in its limiting wave-lengths to the absorption band.

§ 152. **Cathode, Lenard, and Röntgen Rays.**—Very beautiful phosphorescence is exhibited, under certain circumstances, by the glass walls of vacuum tubes, through which an electric discharge is passing (§ 105). In a Crookes' Tube, in which the vacuum is so high that the stratification has disappeared, and the dark region round the negative electrode or cathode extends up to, and even beyond, the positive electrode or anode, this phosphorescence is particularly evident. The particles of the highly-rarefied gas seem to be projected with great velocities from the surface of the cathode, forming a molecular stream of charged particles, which, impinging on the walls of the tube, cause them to phosphoresce. The phosphorescence is most intense where the bombardment is greatest. When a screen is interposed between the cathode and the wall towards which the molecular stream is projected, a distinct shadow of the screen is thrown on the wall, the glass round the shadow continuing to phosphoresce brightly. This stream of cathode rays may be turned aside by the action of a magnet, behaving like a flexible conductor carrying an electric current.

Lenard has recently observed that these cathode rays seem to pass through a thin disc of aluminium inserted into the glass walls of the vacuum tube. The presence of radiations

of some kind immediately outside this aluminium 'window' is demonstrated by the fluorescence produced in a fluorescing substance placed in their path. If these Lenard Rays, as they are called, be the continuation outside the tube of the original cathode rays within the tube (and they have many properties in common), then the latter can hardly be streams of gaseous matter, but must be ether waves. It is conceivable, however, that the rapid electrical changes produced in the aluminium window by the bombardment of the charged molecules may originate on the farther side rays having a certain similarity to the true cathode rays. Moreover, there are certain features of the cathode rays which are difficult if not impossible to explain if the hypothesis that they are 'radiant matter' be disallowed.

Differing apparently in all fundamental respects from light rays, cathode rays, and Lenard rays are the rays recently discovered by Röntgen and known by his name. They are produced under conditions somewhat similar to those under which the Lenard rays are produced, but seem to originate at the phosphorescing *glass wall* of the vacuum tube. They can be studied optically by the use of fluorescent screens of the platino-cyanides mentioned above (§151) or photographically by their decomposing action on the sensitive photographic film. They pass with ease through many substances which are optically opaque, but are absorbed by others. For example, the bony structures of the body are much less transparent than the fleshy parts. Hence by placing the hand between the fluorescent screen and the source of the Röntgen rays, we see the shadow of the skeleton of the hand, with a much fainter shadow of the flesh and skin bordering it. These rays have apparently no tendency to be refracted, and they have not as yet been polarised. Possibly they differ from ordinary light and actinic rays in having a much shorter wave-length, in virtue of which they are able to pass through between the molecules of matter with comparative freedom, and are uninfluenced, or very

slightly influenced, by the loading of the ether, in terms of which we explain the ordinary phenomena of refraction and dispersion. They may, however, as suggested by Stokes, be of the nature of irregular impulses, devoid of any true periodicity communicated to the ether and transmitted through it with great velocity.

One of the most curious physical properties of the Röntgen rays is their effect on the insulating power of dielectrics. Air, for example, through which these rays are being transmitted, is for the time being a conductor of electricity, and retains this property for a short time after the rays have ceased to pass through it.

A closer study and fuller knowledge of the properties of these recently discovered forms of radiation can hardly fail to give us a deeper insight into the constitution of matter and ether.

SUMMARY.

§ 146. Luminous and Non-luminous Radiations. Dark Heat Rays. Actinic Rays.

§ 147. The visible Solar Spectrum. Pure Spectrum. Fraunhofer's Lines. Spectroscope. Absorption Spectrums of Sodium. Theory of Exchanges.

§ 148. Selective Absorption. Transparent. Diathermanous. Stokes's Analogy.

§ 149. Carnot's Principle applied.

§ 150. Radiation as influenced by Motion. Motion of Star in Line of Sight. Döppler's Principle.

§ 151. Fluorescence. Phosphorescence.

§ 152. Cathode, Lenard, and Röntgen Rays.

TABLES OF OPTICAL CONSTANTS.

A.—Wave-lengths in Tenth-metres (10^{-10} metre) of Characteristic Bands and Lines of the Solar Spectrum.

[The authorities are Abney for the Infra-Red Rays, Rowland fc the Light Rays (A to K), and Rowland, Kayser, and Runge fc the Ultra-Violet Rays.]

Ray.	Wave-length.	Substance.
ψ_2	27000	
ψ_1		
ϕ_2	12400	
ϕ_1	12000	
Y	8990·4	
	8986·5	
X_4	8806·1	
X_3	8661·4	
X_2	8541·8	
X_1	8497·0	
Z	8226·4	
A	7594·059	Oxygen.
B	6867·461	"
C	6563·054	Hydrogen.
D_1	5896·154	Sodium.
D_2	5890·182	"
E_1	5270·533	Iron.
	5270·448	Calcium.
E_2	5269·722	Iron.
b_1	5183·792	Magnesium.
b_2	5172·871	"
b_3	5169·218	Iron.
	5169·066	"
b_4	5167·686	"
	5167·501	Magnesium.
F	4861·496	Hydrogen.
G	4308·071	Iron.
	4307·904	Calcium.
h	4101·85	Hydrogen.
H	3968·620	Calcium.
K	3933·809	"
L	3820·566	Iron.
M	3727·768	"
	3727·10	"
N	3581·344	"
O	3441·135	"
P	3361·30	"
Q	3286·87	"
R	3181·40	Calcium.
	3179·45	"
r	3144·58	Iron.
S_1	3100·779	"
S_2	3100·415	"
	3100·064	"
s	3047·720	"
T	3021·191	"
	3020·759	"
t	2994·542	"
U	2947·993	"

B.—Table of Refractive Indices for various Substances.

The Refractive Index is given for the ray corresponding to the more refrangible D line of the solar spectrum.

The relative dispersive powers of the various substances are indicated in the columns headed D—A and F—D, which contain the differences of the refractive indices for the rays D, A, and F, D respectively. The last column $\frac{F-A}{D-1} \times 100$, gives a numerical estimate of the relative Dispersions.

Substance.	Density.	Refractive Index for D.	D—A.	F—D.	$\frac{F-A}{D-1} \times 100$.
Hard Crown Glass.....	2·4858	1·51712	·00536	·00603	2·20
Soft Crown Glass.....	2·5504	1·51458	·00562	·00641	2·34
Light Flint Glass.....	2·8664	1·54102	·00696	·00810	2·78
Dense Flint Glass.....	3·8895	1·65037	·01123	·01387	3·87
Diamond.....	....	2·4173	·01489	·01805	2·32
Rock-salt.....	...	1·5440	·00755	·00900	3·04
Sylvin.....	...	1·49031	·00654	·00799	2·97
Alum.....	...	1·45601	·00544	·00539	2·38
Water.....	...	1·3330	·00409	·00415	2·48
Alcohol.....	·800	1·3633	·0037	·0042	2·18
Ether.....	·713	1·3594	·0044	·0047	2·54
Chloroform..	1·501	1·4492	·0052	·0062	2·54
Air.....	·00128	1·0002947	·0000018	·0000021	1·32

C.—Table of Refrangibilities of Infra-Red Rays of given Wave-length.

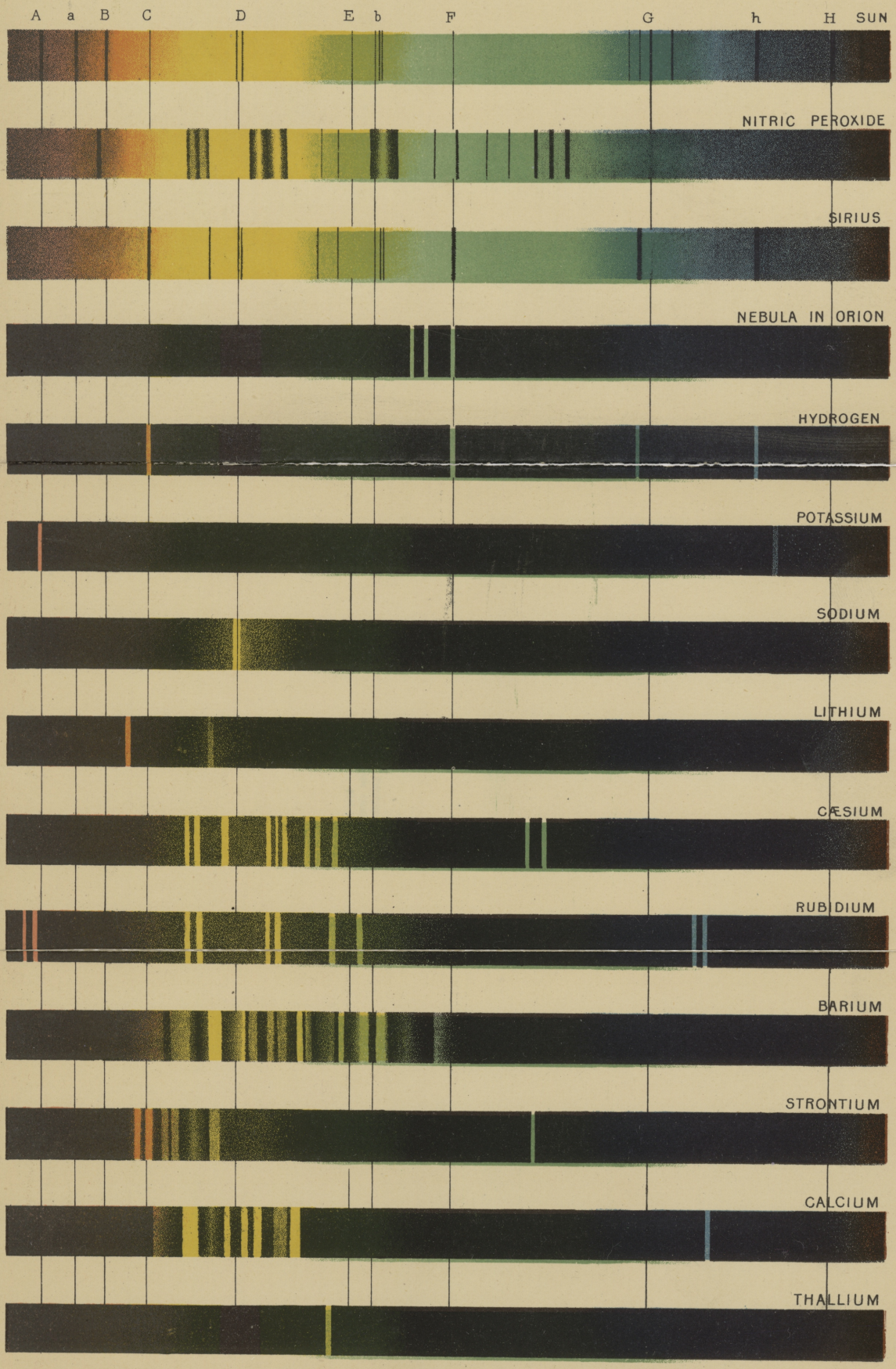
[The wave-length (λ) is expressed in seventh-metres (10^{-7} metre); the first two (shortest) wave-lengths belong to the visible rays F and D emitted by hydrogen and sodium respectively. The second, third, fourth, and fifth columns contain the refractive indices of water, quartz, rock-salt, and fluor-spar for the rays whose wave-lengths are entered in the first column. They are deduced by interpolation from Rubens' results.]

Wave-length.	Water.	Quartz.	Rock-salt.	Fluor-spar.
4·86	1·3373	1·5499	1·5531	1·4372
5·89	1·3330	1·5442	1·5441	1·4340
8	1·3281	1·5385	1·5355	1·4307
10	1·3249	1·5350	1·5317	1·4290
15	...	1·5283	1·5279	1·4265
20	...	1·5210	1·5269	1·4240
25	...	1·5109	1·5255	1·4215
30	...	1·4994	1·5240	1·4187
40	...	1·4678	1·5210	1.4120
60	...	...	1·5154	1·3990
80	...	...	1·5072	1·3535

The longest wave-length measured in the case of water was $12{\cdot}56 \times 10^{-4}$ mm.; with various kinds of glass the measurements could be carried to wave-length 25×10^{-4} mm.; for the measurement of refrangibilities for longer wave-lengths highly diathermanous substances must be used, such as Quartz, Rock-salt, Fluor-spar, and Sylvin.

TABLE OF SPECTRA

Showing the Character of the light obtained from various sources



W. & R. CHAMBERS, LIMITED, LONDON & EDINBURGH.

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*** *Proper names are in heavy type.*

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